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Trajectories of a dust particle in a "sheared slab" unperturbed magnetic configuration

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Abstract

The dust particles diffusion induced by a magnetic field with linear x-dependence of the shear and without turbulence is studied. The solutions of the Newton -Lorentz equation of dust particles are obtained for physically relevant parameter values; we obtained also the trajectories for different values of the shear parameter and the Larmor frequency.

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1 Introduction

We have analyzed in our previous paper [1], the dust particles trajectories induced by a magnetic field with average component and without turbulence. We have solved the Newton-Lorentz equation of dust particles for physically relevant parameter values, namely the shear parameter and the Larmor frequency. Because the impact of dust particles on tokamak walls can create a deterioration of the latter it is very important to study the transport of dust grains at the edge of the plasma. We extended the previous mentioned paper and in the current paper we use a linear x-dependence (i.e. on the radial coordinate x) of the shear specific to tokamak plasma and astrophysics systems.

The paper is organized as follows. The equations of motion for the dust particle in a sheared slab magnetic field are established in section 2. In section 3, the velocities and the trajectories for the dust particles were calculated and represented. The conclusions are summarized in section 4.

2 Equations of motion for the dust particle

The electric field is considered to be irrelevant in our analysis and only the unperturbed magnetic field with shear is considered. The Newton-Lorentz force is:

$$m\frac{d\mathbf{V}}{dt} = q\left(\mathbf{V}\times\mathbf{B}\right) \tag{1}$$

where the unperturbed sheared magnetic field is given by the expression:

$$\mathbf{B}\left(\mathbf{X}\right) = B_0\left[\mathbf{e}_z + XL_s^{-1}\mathbf{e}_y\right] \tag{2}$$

where $\mathbf{X} \equiv (X,\ Y,\ Z)$ and the shear parameter has the order of magnitude of $L_s^{-1} \in [10^{-1}, 10^{-2}]\ m^{-1}$ [4]. The scalar equations corresponding to eqs. (1) combined with the eq.(2) are:

$$m\frac{dV_x}{dt} = qB_0\left(V_y - V_zXL_s^{-1}\right) \tag{3}$$

$$m\frac{dV_y}{dt} = -qB_0V_x \tag{4}$$

$$m\frac{dV_z}{dt} = qB_0V_xXL_s^{-1} \tag{5}$$

We use the following dimensionless quantities:

$$\frac{\mathbf{V}}{v_0} = \mathbf{v} \quad , \quad \frac{t}{t_0} = \tau, \quad \frac{\mathbf{X}}{L} = \mathbf{x} \tag{6}$$

where $v_0 \equiv v_{th} \simeq 10^3 ms^{-1}$, $t_0 \equiv t_s \simeq 10^4$ sec and L is of order of meter. The dimensionless equations that we obtain is:

$$\begin{aligned}
\frac{dv_x}{d\tau} &= \frac{qB_0t_0}{m}\left(v_y - v_zxLL_s^{-1}\right) \equiv \Omega\left(v_y - xK_{sh}v_z\right)\\
\frac{dv_y}{d\tau} &= -\frac{qB_0t_0}{m}v_x \equiv -\Omega v_x\\
\frac{dv_z}{d\tau} &= \frac{qB_0t_0}{m}xLL_s^{-1}v_x \equiv \Omega xK_{sh}v_x
\end{aligned} \tag{7}$$

where $\Omega \equiv \frac{qB_0t_0}{m}$ and $K_{sh} = LL_s^{-1}$. We will consider that the masses of thr dust particles are in the range from $[10^{-11},\ 10^{-10}]\ kg$ and the electric charges are in the range from $[10^{-14},\ \ 10^{-13}]\ C$ [2]-[5]. The order of magnitude of the magnetic field is considered to be of order $10\ T$. The thermal velocity v_{th} is of order $10^3\ m/s$ and the stopping time $t_0 = t_s$ is of order $10^4 s$ if the dimension of the dust grain is $10^{-2}m$. Ω is considered to be of order $[10, 100]$.

3 The trajectories for the dust particle

In this section we represented the solutions and the trajectories for the dust particle for different values of the Lorentz frequency Ω and the shear parameter K_{sh} [6]. In figure (1) were represented the solutions of the system (7) for $\Omega = 10$ and $\Omega = 25$ and a fixed shear parameter $K_{sh} = 5.5$. We also represented the trajectories for the dimensionless Larmor frequency $\Omega = 10$ and $\Omega = 25$ and a fixed shear parameter $K_{sh} = 5.5$ in figure (2). It is obviously that for the same time-interval $\tau \in [0,\ 0.1]$ an increase of the Larmor frequency from $\Omega = 10$ to $\Omega = 25$ gives an increased number of oscillations as we can observe from the figure (2).

In figure (3) were represented the solutions of the system (7) for $\Omega = 30$ and $\Omega = 60$ and a fixed shear parameter $K_{sh} = 10$. We also represented the corresponding trajectories in figure (4). It is obviously that for the same time-interval $\tau \in [0,\ 0.1]$ an increase of the Larmor frequency from $\Omega = 30$ to $\Omega = 60$ gives finally helical trajectories as we can observe from the figure (4). Correspondingly the number of oscillations of the solutions increase if the Larmor frequency increase as we can observe from figure (3). In figure (5) a comparison between the solutions and the velocities corresponding to the model analyzed () in this work and the model of a average component analyzed in the previous

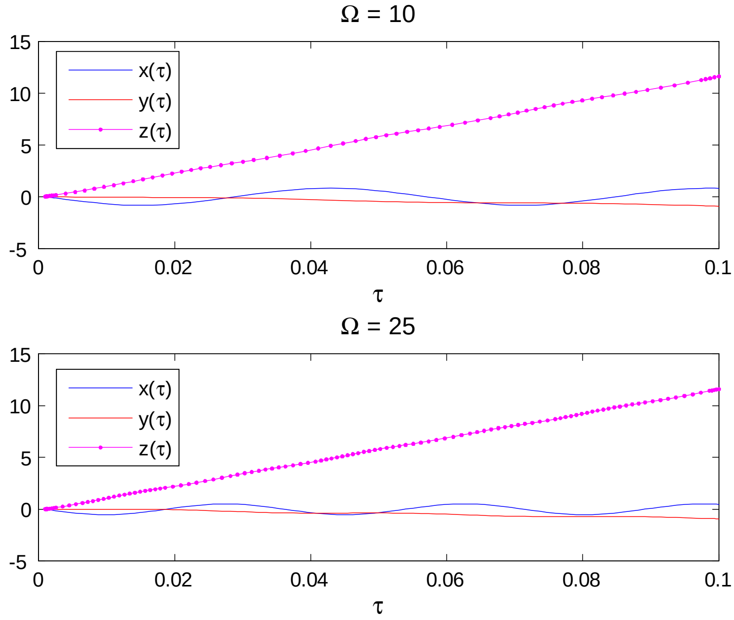

Figure 1: The solutions for the dimensionless Larmor frequency $\Omega = 10$ and $\Omega = 25$ and a fixed shear parameter $K_{sh} = 5.5$.

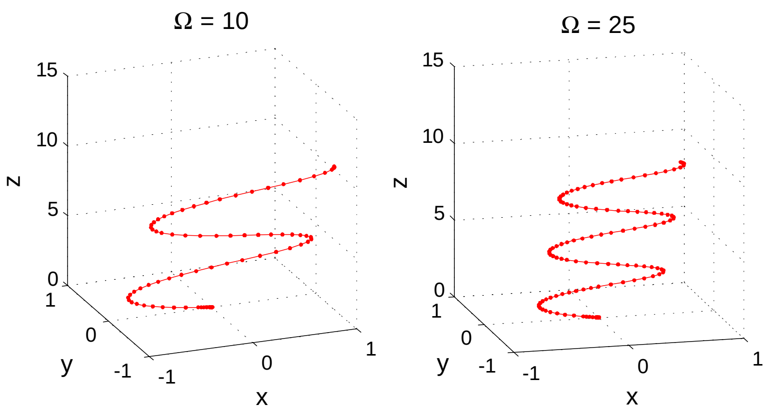

Figure 2: The trajectories for the dimensionless Larmor frequency $\Omega = 10$ and $\Omega = 25$ and a fixed shear parameter $K_{sh} = 5.5$.

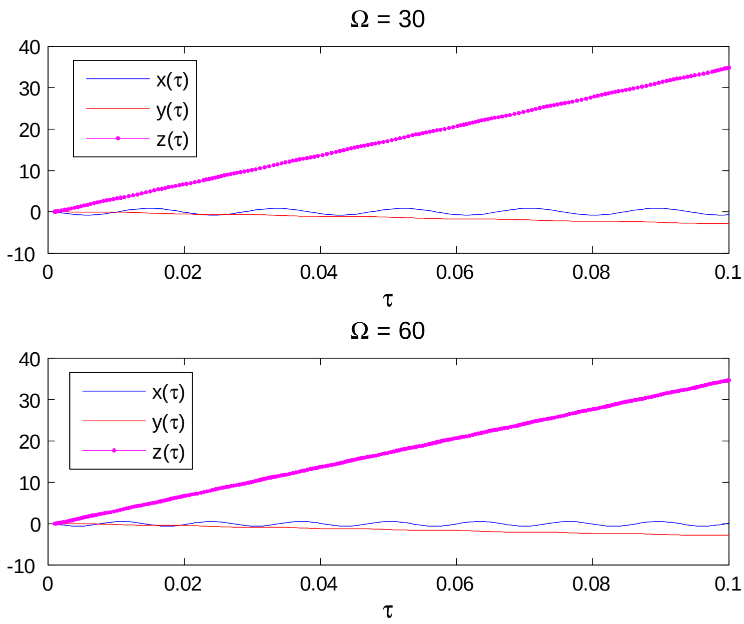

Figure 3: The solutions for the dimensionless Larmor frequency $\Omega = 30$ and $\Omega = 60$ and a fixed shear parameter $K_{sh} = 10$.

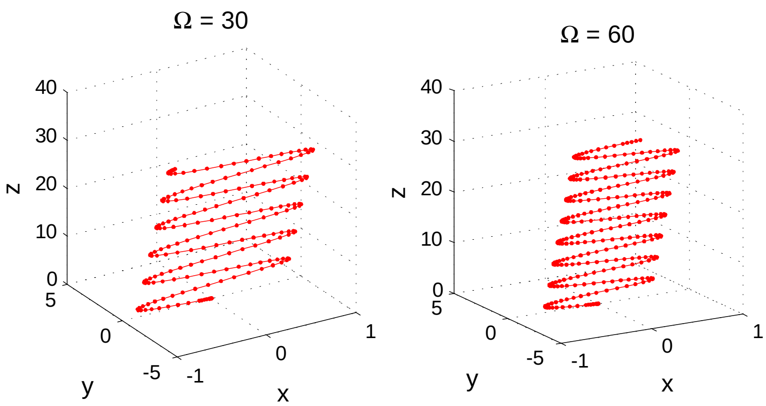

Figure 4: The trajectories for the dimensionless Larmor frequency $\Omega = 30$ and $\Omega = 60$ and a fixed shear parameter $K_{sh} = 10$.

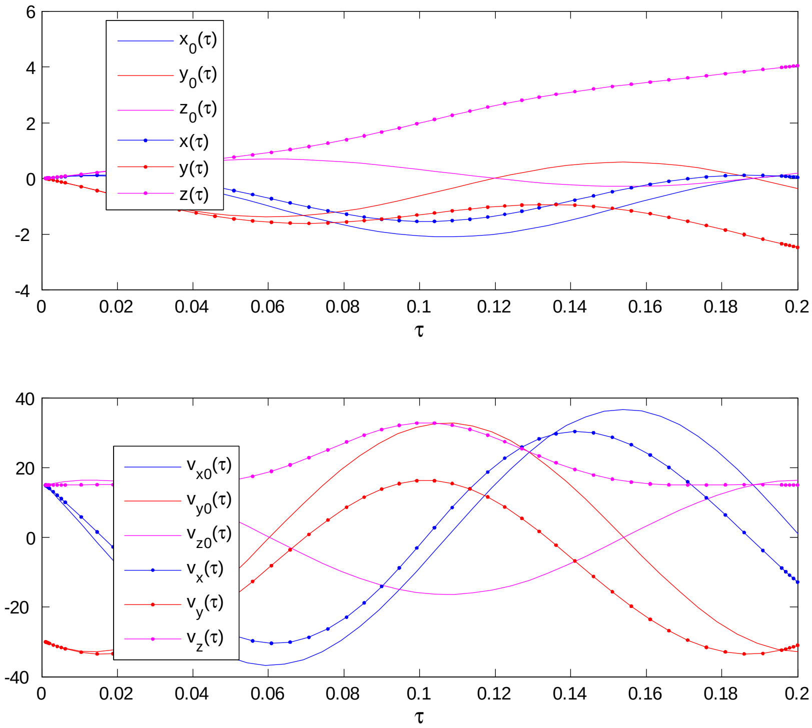

Figure 5: Comparison between the solutions and the velocities corresponding to the model analyzed in this work and the model of a constant shear analyzed in the previous paper.

paper [1] is shown (the subscript "0" is for the model studied in [1]). We note from the figure 5 that $x(\tau)$ and $y(\tau)$ have similar behaviour for both studied models, but $z(\tau)$ display and strong escape in comparison with $z_0(\tau)$, because the ecuation of acceleration is: $\frac{dv_z}{d\tau} = \frac{qB_0t_0}{m}xLL_s^{-1}v_x$ instead of $\frac{dv_{z0}}{d\tau} = \frac{qB_0t_0}{m}b_{sh}v_{x0}$, (in the model [1]); practically there is a $x-$deppendence that imposes such kind of behaviour.

4 Conclusions

The dust particles motion was studied and their trajectories were calculated for a class of Larmor frequency and shear parameter. The increase of the Larmor frequency and of the shear parameter produce closed trajectories. More informations on the dynamics will be obtained from the analysis of the running and asymptotic diffusion coefficients but this issue is left for a future paper.

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