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No cross-couplings of a mixed symmetry $(k, 1)$ tensor field to matter fields of spin 0 and/or ½

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Abstract

In this paper we analyze all consistent and nontrivial couplings that can be introduced between a massless tensor field with the mixed symmetry $(k, 1)$ for $k \geq 4$ and a matter theory of spin 0 and/or ½ in the context of the antifield-BRST deformation method under some standard "selection rules" from Quantum Field Theory.

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1 Introduction

Irreducible exotic representations of the group $GL(D, \mathbb{R})$ determined the introduction of tensor fields with mixed symmetry. There is a complex motivation for the study of this class of fields due to the fact that they are involved in many important physical theories such as superstrings, supergravity, or supersymmetric high-spin theories. The analysis of gauge theories containing bosonic tensor fields with mixed symmetries brought into attention many interesting problems, like for instance the dual formulation of spin-two or higher-spin theories [1–6]. Gauge theories including in their field spectrum exotic representations of the Lorentz group are appropriately approached within the algebraic-differential setting based on N-complexes. The notion of N-complex [7–9] emerged in relation with irreducible tensor fields with mixed Young symmetries in view of generalizing at some extent the differential calculus. This generalized differential framework of N-complexes offers an elegant formulation for symmetric tensor gauge fields and their Hodge duals [10, 11], resembling to the formulation of electrodynamics in differential form language.

In what follows we focus on the class of massless real tensor fields transforming according to the irreducible representations of $GL(D, \mathbb{R})$ corresponding to some "spin 2" (two-column) Young diagrams with $(k + 1)$ cells and $k \geq 4$ rows, also known as "hook diagrams" or, in other words, fields with the mixed symmetry $(k, 1)$. For arbitrary values of k, such tensor fields (massless and massive) have initially been investigated more than two decades ago [12–16] and more recently (including within the BRST method) for instance in [17, 18]. One of the main reasons of their study is that in the free limit such theories produce one of the dual formulations of linearized gravity in $D = k+3$ spacetime dimensions. The construction of gravity-like dual theories benefits of a raised interest in

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the context of some new results, such as the reformulation of nonlinear Einstein gravity in terms of a dual graviton in the presence of a usual metric and of a shift gauge field [19].

The construction of couplings between gauge and matter fields represents a topic of special interest in both theoretical and phenomenological high energy physics, which led to essential results related to the build-up of the Standard Model. In this sense we recall the construction of Scalar Electrodynamics and, in this context, the Abelian Higgs mechanism (Englert, Brout, Higgs, Guralnik, Hagen, Kibble — 1964) [20–22] in the presence of a "Mexican hat" potential, the Abelian sector of Quantum Electrodynamics [23], and respectively the non-Abelian sector brought by Quantum Chromodynamics, endowed with the crucial property of asymptotic freedom (Gross, Politzer, Wilczek — 1973) [24, 25].

The purpose of this paper is to analyze the consistent and nontrivial couplings between a massless tensor field with the mixed symmetry $(k, 1)$ $(D \geq k+2,\ k \geq 4)$ and a matter theory (of spin 0 and/or ½). We use one of the main applications of the antibracket-antifield BRST symmetry [26–29] related to the construction of consistent interactions in gauge field theories [30–33] via the deformation of the canonical generator of the antifield-BRST differential (the solution to the classical master equation). This deformation procedure requires the computation of the local cohomology of the BRST differential in ghost number 0 and in maximum form degree by specific techniques [34–37]. The whole construction is done in the presence of some clearly defined selection rules (working hypotheses) imposed on the deformations of the solution to the master equation: analyticity in the coupling constant, spacetime locality, Lorentz covariance, Poincaré invariance, and conservation of the differential order of the free field equations at the level of the coupled theories. The last hypothesis is strengthened by asking that the interacting vertices display the maximum derivative order of the free Lagrangian density at any order in the coupling constant. We adopt the Standard Model vision according to which by matter theory we understand any field theory that describes particles of spin 0 (scalar bosons) and/or ½ (quarks or leptons), so it possesses no nontrivial gauge symmetries. The results obtained here complete various findings from the literature [38–57] related to massless tensor fields with mixed symmetries. In particular, the cohomological approach to consistent interactions between gauge and matter theories via the deformation of the canonical generator of the BRST symmetry has been implemented in both Lagrangian and Hamiltonian versions for various classes of gauge fields, like massless tensors with the mixed symmetry (3,1), Abelian vector fields, Non-Abelian gauge fields, or topological BF models [39, 58–61].

Under the working hypotheses specified in the above we obtained the following results:

1. The first-order deformation responsible for the cross-couplings between the mixed symmetry $(k, 1)$ tensor field and matter fields of spin 0 and/or ½ may contain nonvanishing terms only in antighost numbers 0 and 1. Their existence and nontriviality is equivalent to the requirement that the Lagrangian action of the matter theory possesses a nontrivial rigid (global) symmetry whose parameters are the components of a constant, bosonic, and antisymmetric tensor of order $(k + 1)$;

2. Assuming this is the case, the antighost number 1 component is linear in the antifields of the matter fields, which couple to the first-order derivatives of the antisymmetric ghosts of pure ghost number equal to 1 from the $(k, 1)$ sector precisely through the generators of the above mentioned rigid symmetry. The piece of antighost number 0 is simultaneously linear in the first-order derivatives of the field $(k, 1)$ and in the (nontrivially) conserved current following from the rigid invariance of the matter action via Noether's Theorem. This means that at the first order of perturbation

theory there appears a standard "conserved current – gauge field" coupling and the matter fields gain gauge transformations obtained by gauging the global ones;

3. Examining the cases under consideration, of matter theories with spin 0 and/or ½ particles, we conclude that there are no nontrivial rigid symmetries with the required properties and thus no nontrivial consistent couplings with the tensor field $(k,1)$;

4. The only terms allowed in the deformed solution to the classical master equation reduce to the sum between the selfinteractions of the tensor field with the mixed symmetry $(k,1)$ and all nontrivial couplings among the matter fields. In this context the matter fields gain no nontrivial gauge transformations.

2 Free theory. BRST symmetry

The starting point is given by the Lagrangian action describing a free massless tensor field with the mixed symmetry $(k,1)$ for $k \geq 4$ and a matter theory with spin 0 and/or ½ fields

$$S_0\left[t_{\mu_1\ldots\mu_k|\alpha},\phi^{\Delta}\right] = S_0^{\mathrm{t}}[t_{\mu_1\ldots\mu_k|\alpha}] + S_0^{\mathrm{mat}}\left[\phi^{\Delta}\right]. \tag{1}$$

We work on a Minkowski spacetime of dimension $D \geq k+2 \geq 6$ endowed with a mostly positive metric $\sigma_{\mu\nu} = \sigma^{\mu\nu} = (-+\ldots+)$ and define the Levi-Civita symbol in D dimensions $\varepsilon^{\mu_1\ldots\mu_D}$ by $\varepsilon^{01\ldots D-1} = -1$. The field $t_{\mu_1\ldots\mu_k|\alpha}$ is antisymmetric in its first k (Lorentz) indices and satisfies the identities $t_{[\mu_1\ldots\mu_k|\alpha]} \equiv 0$, while its trace, $t_{\mu_1\ldots\mu_{k-1}} = t_{\mu_1\ldots\mu_k|\alpha}\sigma^{\mu_k\alpha}$, is a completely antisymmetric tensor of order $(k-1)$. Each matter field ϕ^{Δ} displays a well defined Grassmann parity, ε_{Δ} (equal to 0 for the scalar bosons and respectively 1 for the spin ½ fields — quarks and/or leptons). The components S_0^{t} and S_0^{mat} are given by

$$S_0^{\mathrm{t}}[t_{\mu_1\ldots\mu_k|\alpha}] = -\tfrac{1}{2\cdot(k+1)!}\int\left[F_{\mu_1\ldots\mu_{k+1}|\alpha}F^{\mu_1\ldots\mu_{k+1}|\alpha} - (k+1)F_{\mu_1\ldots\mu_k}F^{\mu_1\ldots\mu_k}\right]d^Dx, \tag{2}$$

$$S_0^{\mathrm{mat}}\left[\phi^{\Delta}\right] = \int \mathcal{L}^{\mathrm{mat}}\left(\left[\phi^{\Delta}\right]\right)d^Dx, \qquad \varepsilon\left(\phi^{\Delta}\right) \equiv \varepsilon_{\Delta}. \tag{3}$$

In the above

$$F_{\mu_1\ldots\mu_{k+1}|\alpha} = \partial_{[\mu_1}t_{\mu_2\ldots\mu_{k+1}]|\alpha}, \tag{4}$$

$$F_{\mu_1\ldots\mu_k} \equiv F_{\mu_1\ldots\mu_{k+1}|\alpha}\sigma^{\mu_{k+1}\alpha} = \partial_{[\mu_1}t_{\mu_2\ldots\mu_k]} + (-)^k\partial^{\alpha}t_{\mu_1\ldots\mu_k|\alpha}, \tag{5}$$

so the tensor $F_{\mu_1\ldots\mu_{k+1}|\alpha}$ displays the mixed symmetry $(k+1,1)$ and its trace, $F_{\mu_1\ldots\mu_k}$, is completely antisymmetric. Everywhere in this paper the notation $[\mu\ldots\nu]$ signifies complete antisymmetry with respect to the (Lorentz) indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. The matter Lagrangian density $\mathcal{L}^{\mathrm{mat}}$ is assumed to be linear (in the case of quarks and/or leptons) or at most quadratic (for scalar bosons) in the first-order spacetime derivatives of the matter fields, so it generates field equations with the derivative order equal to one (in the case of spin ½ particles) and respectively maximum two (for spin 0 particles). Since the action $S_0^{\mathrm{mat}}\left[\phi^{\Delta}\right]$ possesses no nontrivial gauge symmetries, it follows that a generating set of (infinitesimal) gauge transformations of action (3) is given by the set corresponding to the $(k,1)$ sector

$$\delta_{\overset{(1)}{\theta},\overset{(1)}{\epsilon}}t_{\mu_1\ldots\mu_k|\alpha} = \partial_{[\mu_1}\overset{(1)}{\theta}_{\mu_2\ldots\mu_k]|\alpha} + \partial_{[\mu_1}\overset{(1)}{\epsilon}_{\mu_2\ldots\mu_k\alpha]} + (-)^{k+1}(k+1)\partial_{\alpha}\overset{(1)}{\epsilon}_{\mu_1\ldots\mu_k}, \tag{6}$$

supplemented by

$$\delta_{\overset{(1)}{\theta},\overset{(1)}{\epsilon}}\phi^{\Delta}=0 \tag{7}$$

in the matter sector. Both types of gauge parameters are bosonic arbitrary tensors on the spacetime manifold up to the requirements that $\overset{(1)}{\theta}_{\mu_1\ldots\mu_{k-1}|\alpha}$ possesses the mixed symmetry $(k-1,1)$ and $\overset{(1)}{\epsilon}_{\mu_1\ldots\mu_k}$ is completely antisymmetric. Consequently, all the gauge features of the free theory governed by formulas (1), (6), and (7) reduce to those from the $(k,1)$ sector [50, 56]: off-shell reducibility of order $(k-1)$, an Abelian gauge algebra, and a well defined Cauchy order, equal to $(k+1)$. The separate Cauchy order of the matter theory is equal to 1.

Although it has no true gauge symmetries, action S_0^{mat} possesses some rigid symmetries, written in a standard manner in infinitesimal form like

$$\delta_{\xi}\phi^{\Delta}=Z^{\Delta}{}_{\bar{\Delta}}([\phi],x^{\mu})\xi^{\bar{\Delta}}. \tag{8}$$

We assume the rigid parameters $\xi^{\bar{\Delta}}$ display definite Grassmann parities

$$\varepsilon(\xi^{\bar{\Delta}})\equiv\varepsilon_{\bar{\Delta}},\qquad \varepsilon(Z^{\Delta}{}_{\bar{\Delta}})=(\varepsilon_{\Delta}+\varepsilon_{\bar{\Delta}}) \bmod 2 \tag{9}$$

and consider only rigid symmetries whose generators are local, so $Z^{\Delta}{}_{\bar{\Delta}}$ may depend on the matter fields and their spacetime derivatives up to a finite order. Noether's Theorem ensures the existence of some conserved currents $j^{\mu}{}_{\bar{\Delta}}$

$$\frac{\delta^{\mathrm{R}}\mathcal{L}^{\mathrm{mat}}}{\delta\phi^{\Delta}}Z^{\Delta}{}_{\bar{\Delta}}([\phi],x^{\mu})+\partial_{\mu}j^{\mu}{}_{\bar{\Delta}}([\phi],x^{\mu})=0,\qquad \varepsilon(j^{\mu}{}_{\bar{\Delta}})=\varepsilon_{\bar{\Delta}}, \tag{10}$$

where $\delta^{\mathrm{R}}/\delta\phi^{\Delta}$ signify the right Euler–Lagrange (EL) derivatives with respect to ϕ^{Δ}. We mention in this context the notion of trivial rigid symmetry, defined via some generators that vanish on the stationary surface of the matter theory

$$\delta_{\xi}\phi^{\Delta}=\frac{\delta^{\mathrm{R}}\mathcal{L}^{\mathrm{mat}}}{\delta\phi^{\Delta'}}Z^{\Delta'\Delta}{}_{\bar{\Delta}}([\phi],x^{\mu})\xi^{\bar{\Delta}},\qquad \varepsilon(Z^{\Delta'\Delta}{}_{\bar{\Delta}})=(\varepsilon_{\Delta'}+\varepsilon_{\Delta}+\varepsilon_{\bar{\Delta}}) \bmod 2 \tag{11}$$

and mandatorily satisfy the generalized symmetry properties

$$Z^{\Delta'\Delta}{}_{\bar{\Delta}}=(-)^{1+\varepsilon_{\Delta'}\varepsilon_{\Delta}}Z^{\Delta\Delta'}{}_{\bar{\Delta}}. \tag{12}$$

This means that they are antisymmetric in their upper indices for any pair of purely bosonic fields and symmetric in the purely fermionic case. The global invariance of the matter action under (11) takes place automatically due to properties (12), irrespective of the Lagrangian density expression

$$\frac{\delta^{\mathrm{R}}\mathcal{L}^{\mathrm{mat}}}{\delta\phi^{\Delta}}\frac{\delta^{\mathrm{R}}\mathcal{L}^{\mathrm{mat}}}{\delta\phi^{\Delta'}}Z^{\Delta'\Delta}{}_{\bar{\Delta}}([\phi],x^{\mu})\equiv 0. \tag{13}$$

Under these circumstances, Noether's Theorem

$$\frac{\delta^{\mathrm{R}}\mathcal{L}^{\mathrm{mat}}}{\delta\phi^{\Delta}}\frac{\delta^{\mathrm{R}}\mathcal{L}^{\mathrm{mat}}}{\delta\phi^{\Delta'}}Z^{\Delta'\Delta}{}_{\bar{\Delta}}([\phi],x^{\mu})+\partial_{\mu}\bar{j}^{\mu}{}_{\bar{\Delta}}([\phi],x^{\mu})=0 \tag{14}$$

emphasizes some trivially conserved currents

$$\partial_{\mu}\bar{j}^{\mu}{}_{\bar{\Delta}}([\phi],x^{\mu})\equiv 0\Rightarrow \bar{j}^{\mu}{}_{\bar{\Delta}}([\phi],x^{\mu})=\partial_{\nu}\bar{j}^{\nu\mu}{}_{\bar{\Delta}}([\phi],x^{\mu}),\qquad \bar{j}^{\nu\mu}{}_{\bar{\Delta}}=-\bar{j}^{\mu\nu}{}_{\bar{\Delta}}. \tag{15}$$

The attribute "trivial" related to the rigid transformations (11) and associated conserved currents (15) underlines the lack of physical significance of these objects with respect to any matter theory.

Next, we pass to the construction of the antifield-BRST symmetry for the theory under study. Regarding the $(k,1)$ sector, we maintain all the notations, conventions, formulas, and results from [50, 56, 56]. Consequently, the BRST differential algebra is constructed starting with the $(k,1)$ generators

$$\Phi^A \equiv \Big\{ t_{\mu_1\ldots\mu_k|\alpha},\ \big\{ \overset{(m)}{C}_{\mu_1\ldots\mu_{k-m}|\alpha},\ \overset{(m)}{\eta}_{\mu_1\ldots\mu_{k-m+1}} \big\}_{m=\overline{1,k-1}},\ \overset{(k)}{\eta}_{\mu} \Big\}, \tag{16}$$

$$\Phi^*_A \equiv \Big\{ t^{*\mu_1\ldots\mu_k|\alpha},\ \big\{ \overset{(m)}{C}{}^{*\mu_1\ldots\mu_{k-m}|\alpha},\ \overset{(m)}{\eta}{}^{*\mu_1\ldots\mu_{k-m+1}} \big\}_{m=\overline{1,k-1}},\ \overset{(k)}{\eta}{}^{*\mu} \Big\}, \tag{17}$$

whose properties are detailed in [50, 56] (a synthetic view is given in Table 1 from [56]), supplemented by the matter fields and their antifields

$$\phi^\Delta : \mathrm{agh}\big(\phi^\Delta\big) = 0 = \mathrm{pgh}\big(\phi^\Delta\big), \tag{18}$$
$$\phi^*_\Delta : \mathrm{agh}\big(\phi^*_\Delta\big) = 1,\ \ \mathrm{pgh}\big(\phi^*_\Delta\big) = 0,\ \ \varepsilon\big(\phi^*_\Delta\big) = (\varepsilon_\Delta + 1) \bmod 2. \tag{19}$$

The BRST differential simply decomposes like

$$s = \delta + \gamma, \quad s^2 = 0 \Leftrightarrow (\delta^2 = 0,\, \gamma^2 = 0,\, \delta\gamma + \gamma\delta = 0) \tag{20}$$

into the sum between the Koszul–Tate differential δ ($\mathbb{N}$-graded in terms of the antighost number agh, $\mathrm{agh}(\delta) = -1$) and the longitudinal exterior derivative γ (a true differential in this case, which anticommutes with δ and is $\mathbb{N}$-graded along the pure ghost number pgh, $\mathrm{pgh}(\gamma) = 1$). The BRST differential is $\mathbb{Z}$-graded in terms of the ghost number gh defined like pgh − agh, such that $\mathrm{gh}(s) = \mathrm{gh}(\delta) = \mathrm{gh}(\gamma) = 1$. The actions of the operators δ and γ on the BRST generators from the $(k,1)$ sector are given for instance in [56] (see formulas (15)–(23) therein), while on those from the matter sector read as

$$\gamma\phi^\Delta = 0, \quad \gamma\phi^*_\Delta = 0, \quad \delta\phi^\Delta = 0, \quad \delta\phi^*_\Delta = -\frac{\delta^{\mathrm{L}}\mathcal{L}^{\mathrm{mat}}}{\delta\phi^\Delta}, \tag{21}$$

where $\delta^{\mathrm{L}}/\delta\phi^\Delta$ denote the left EL derivatives of the matter Lagrangian density with respect to ϕ^Δ. It is useful to recall the relationship between the left ant the right EL derivatives of the matter Lagrangian density with respect to ϕ^Δ

$$\frac{\delta^{\mathrm{L}}\mathcal{L}^{\mathrm{mat}}}{\delta\phi^\Delta} = (-)^{\varepsilon_\Delta}\frac{\delta^{\mathrm{R}}\mathcal{L}^{\mathrm{mat}}}{\delta\phi^\Delta}. \tag{22}$$

The solution to the classical master equation reduces to the sum between that corresponding to the free massless tensor field with the mixed symmetry $(k,1)$, S^{t}, and the one associated with the matter theory, which, due to definitions (21), coincides with the matter Lagrangian action (3)

$$S = S^{\mathrm{t}} + S_0^{\mathrm{mat}}\big[\phi^\Delta\big], \tag{23}$$

where

$$S^{\mathrm{t}} = S_0^{\mathrm{t}}[t_{\mu_1\ldots\mu_k|\alpha}] + \int \Big\{ t^{*\mu_1\ldots\mu_k|\alpha}\Big[\partial_{[\mu_1}\overset{(1)}{C}_{\mu_2\ldots\mu_k]|\alpha} + \partial_{[\mu_1}\overset{(1)}{\eta}_{\mu_2\ldots\mu_k\alpha]}$$

$$
\begin{aligned}
&+ (-)^{k+1}(k+1)\partial_\alpha \overset{(1)}{\eta}_{\mu_1\ldots\mu_k}\Big] + \overset{(k-1)}{C}{}^{*\mu_1|\alpha} \partial_{(\mu_1} \overset{(k)}{\eta}_{\alpha)} \\
&+ \sum_{m=1}^{k-2} \overset{(m)}{C}{}^{*\mu_1\ldots\mu_{k-m}|\alpha} \Big[\partial_{[\mu_1} \overset{(m+1)}{C}_{\mu_2\ldots\mu_{k-m}]|\alpha} + \partial_{[\mu_1} \overset{(m+1)}{\eta}_{\mu_2\ldots\mu_{k-m}\alpha]} \\
&+ (-)^{k-m+1}(k-m+1)\partial_\alpha \overset{(m+1)}{\eta}_{\mu_1\ldots\mu_{k-m}}\Big] \\
&+ \sum_{m=1}^{k-1} \tfrac{k-m}{k-m+2} \overset{(m)}{\eta}{}^{*\mu_1\ldots\mu_{k-m+1}} \partial_{[\mu_1} \overset{(m+1)}{\eta}_{\mu_2\ldots\mu_{k-m+1}]}\Big\} d^D x.
\end{aligned} \tag{24}
$$

3 Interacting gauge field theories from local BRST cohomology

The reformulation of the problem of constructing consistent interactions in gauge field theories within the antifield-BRST formalism [30–33] is based on the fact that if consistent couplings can be introduced, then the solution to the classical master equation of the initial gauge theory, S, may be deformed into a solution to the classical master equation for the interacting gauge theory

$$\bar{S} = S + \lambda S_1 + \lambda^2 S_2 + \lambda^3 S_3 + \cdots, \quad \tfrac{1}{2}(\bar{S}, \bar{S}) = 0. \tag{25}$$

Related to the coupled theory, we maintain the field, ghost, and antifield spectra of the original gauge theory in order to preserve the number of physical degrees of freedom. Also, we do not deform either the antibracket or the general properties $\bar{S}$ compared to those of the starting theory, but only the canonical generator itself, so $\bar{S}$ remains a bosonic functional of fields, ghosts, and antifields with the ghost number equal to 0. The projection of the equation $\frac{1}{2}(\bar{S}, \bar{S}) = 0$ on the various powers in the coupling constant λ is equivalent to the tower of equations

$$\lambda^0 : \tfrac{1}{2}(S,S) = 0, \lambda^1 : (S_1, S) = 0, \lambda^2 : (S_2, S) + \tfrac{1}{2}(S_1, S_1) = 0, \lambda^3 : (S_3, S) + (S_1, S_2) = 0, \cdots$$

known as the equation of the antifield-BRST deformation method. In this context the functionals S_i, $i \geq 1$, are called deformations of order i of the solution to the master equation. The first equation is fulfilled by assumption, while the others may be written (due to the canonical action $s\cdot = (\cdot, S)$) as

$$\lambda^1 : sS_1 = 0, \quad \lambda^2 : sS_2 + \tfrac{1}{2}(S_1, S_1) = 0, \quad \lambda^3 : sS_3 + (S_1, S_2) = 0, \quad \cdots \tag{26}$$

The solutions to the first-order deformation equation $sS_1 = 0$ always exist since they belong to the cohomology of the BRST differential s in ghost number 0 computed in the space of all functionals (local and nonlocal) of fields, ghosts, and antifields, $H^0(s)$, which is nonempty due to its isomorphism to the algebra of physical observables of the initial gauge theory. Moreover, trivial first-order deformations, defined as trivial elements of $H^0(s)$ (s-exact functionals), should be ruled out due to the fact that they provoke trivial interactions in the sense of field theory (can be eliminated by some possibly nonlinear field redefinitions). The existence of solutions to the remaining higher-order equations from (26) has been shown in [31] by means of the triviality of the antibracket map in the BRST cohomology $H(s)$ computed in the space of all functionals. In conclusion, if we impose no restrictions on the interactions (spacetime locality, etc.), then the antifield-BRST deformation procedure can be developed without obstructions.

Nevertheless, if we work with local functionals, then the procedure goes as follows. We make the notations

$$S_i = \int a_i d^D x \equiv \int \overset{[D]}{a_i}, \quad i \geq 1, \tag{27}$$

where the nonintegrated densities of the deformations of order $i \geq 1$, a_i, are now elements of the BRST algebra of local "functions", namely, polynomials in ghosts, antifields, and their derivatives, smooth in the original fields, and polynomials in their derivatives up to a finite order, with or respectively without an explicit dependence on the spacetime coordinates x^μ. The overscript between brackets represents the form degree deg. (If we require the Poincaré invariance of the deformed solution to the master equation, then we work without an explicit dependence on x^μ.) In form language, $\overset{[D]}{a_i}$ are elements of the algebra of local forms with or without an explicit dependence on x^μ. The general properties of S_i are transferred to a_i and $\overset{[D]}{a_i}$

$$\varepsilon(a_i) = 0, \quad \mathrm{gh}(a_i) = 0, \quad \deg(\overset{[D]}{a_i}) = D, \quad \mathrm{gh}(\overset{[D]}{a_i}) = 0. \tag{28}$$

The equation satisfied by the first-order deformation (the first equation from (26)) takes the local form

$$s\overset{[D]}{a_1} + d\overset{[D-1]}{b_1} = 0, \quad \deg(\overset{[D-1]}{b_1}) = D - 1, \quad \mathrm{gh}(\overset{[D-1]}{b_1}) = 1, \tag{29}$$

or, equivalently, in dual language

$$sa_1 + \partial_\mu b_1^\mu = 0, \quad \varepsilon(b_1^\mu) = 1, \quad \mathrm{gh}(b_1^\mu) = 1, \tag{30}$$

where the $(D-1)$ form $\overset{[D-1]}{b_1} = 0$ and the current b_1^μ should be local. In other words, the first-order deformation defines precisely a class from the local BRST cohomology in maximum form degree and in ghost number equal to zero, $H^{0,D}(s|d)$, computed in the algebra of local forms with or without an explicit dependence on x^μ, where d symbolizes the exterior differential in spacetime. From now on, the procedure is model-dependent via the properties of $H^{0,D}(s|d)$. Supposing equation (29) (or (30)) possesses local solutions, the resulting first-order deformations are then filtered (if necessary) according to the "selection rules" associated with other working hypotheses than the spacetime locality (such as Lorentz covariance, PT invariance, maximum derivative order of the interaction vertices, etc.). Meanwhile, all purely trivial contributions from $H^{0,D}(s|d)$ computed in the selected algebra of local forms

$$\overset{[D]}{a}{}_1^{\mathrm{triv}} = s\overset{[D]}{c} + d\overset{[D-1]}{e}, \tag{31}$$

$$\deg\left(\overset{[D]}{c}\right) = D, \quad \deg\left(\overset{[D-1]}{e}\right) = D - 1, \quad \mathrm{gh}\left(\overset{[D]}{c}\right) = -1, \quad \mathrm{gh}\left(\overset{[D-1]}{e}\right) = 0, \tag{32}$$

should be discarded since they generate only trivial interactions. By trivial first-order deformations in the context of equation (30) we understand any s-exact object modulo a divergence

$$a_1^{\mathrm{triv}} = sc + \partial_\mu e^\mu, \tag{33}$$

$$\varepsilon(c) = 1, \quad \varepsilon(e^\mu) = 0, \quad \mathrm{gh}(c) = -1, \quad \mathrm{gh}(e^\mu) = 0, \tag{34}$$

with both c and e^μ local. Assuming there exist nontrivial first-order deformations a_1, the next step is represented by their consistency at order two in the coupling constant, viz.

the existence of second-order deformations as solutions to the second equation from (26). Notations (27) together with

$$\tfrac{1}{2}(S_1, S_1) \equiv \int \Gamma_2 d^D x, \quad \varepsilon(\Gamma_2) = 1, \quad \mathrm{gh}(\Gamma_2) = 1, \tag{35}$$

allow us to express equivalently the second equation from (26) in local from as

$$sa_2 + \Gamma_2 + \partial_\mu b_2^\mu = 0, \quad \varepsilon(b_2^\mu) = 1, \quad \mathrm{gh}(b_2^\mu) = 1, \tag{36}$$

where the current b_2^μ should be local. Because S_1 is local by assumption and the antibracket preserves the spacetime locality, it follows that the quantity Γ_2 defined by (35) is also local. Equation (36) requires that Γ_2 is s-exact modulo some divergences, but in this context the existence of both local solutions a_2 and currents b_2^μ is no longer granted. Supposing there exist local nontrivial second-order deformations that also satisfy the additional working hypotheses (if any), we recall (27) and add the notation

$$(S_1, S_2) \equiv \int \Gamma_3 d^D x, \quad \varepsilon(\Gamma_3) = 1, \quad \mathrm{gh}(\Gamma_3) = 1, \tag{37}$$

such that the third-order deformation equation (the third equation from (26)) takes the local form

$$sa_3 + \Gamma_3 + \partial_\mu b_3^\mu = 0, \quad \varepsilon(b_3^\mu) = 1, \quad \mathrm{gh}(b_3^\mu) = 1. \tag{38}$$

The object Γ_3 is local by construction. However, the existence of some local elements a_3 and b_3^μ (with a_3 also nontrivial) fulfilling (38) is not a priori ensured. This procedure may end after a finite number of steps (either if the antibrackets among all the deformations starting from a certain order vanish or whenever there appear obstructions at finding solutions to the deformation of say order "i") or may go on indefinitely (as it happens for instance during the construction of graviton selfinteractions starting from the Pauli–Fierz model that outputs the Einstein–Hilbert action).

4 Cohomological ingredients

The main aim of this paper is to construct all nontrivial, consistent interactions that can be added to the free model described by formulas (1), (6), and (7) with the help of the antifield-BRST deformation method outlined in the previous section. We require that the deformation of the solution to the master equation, (25), is analytical in the coupling constant, local in spacetime, Lorentz covariant, Poincaré invariant, and conserves the differential order of the free field equations at the level of the coupled theories. The last hypothesis is strengthened by asking that the interacting vertices display the maximum derivative order of the free Lagrangian density at any order in the coupling constant. Due to the locality hypothesis, we introduce notations (27) and obtain in dual language that the nonintegrated density of the first-order deformation, a_1, is solution to equation (30), and thus, as argued in the previous section, should be a nontrivial element of the local BRST cohomology $H^{0,D}(s|d)$. The last cohomology will be computed in the BRST algebra of local forms $\bar{\Lambda}$ whose coefficients are elements of the BRST algebra of local "functions" $\bar{\mathcal{A}}$, namely polynomials in the ghosts, antifields (including ϕ^*_Δ) and their spacetime derivatives up to a finite order, smooth in the undifferentiated fields (with the mixed symmetry $(k, 1)$ and matter, ϕ^Δ), polynomials in the field derivatives up to a finite order, and without an explicit dependence on the spacetime coordinates (due to the Poincaré invariance). It is useful to write compactly all the BRST generators via the notations

$$\bar{\Phi}^{\bar{A}} = \{\Phi^{A}, \phi^{\Delta}\}, \qquad \bar{\Phi}^{*}_{\bar{A}} = \{\Phi^{*}_{A}, \phi^{*}_{\Delta}\}, \tag{39}$$

where Φ^{A} and Φ^{*}_{A} are given by (16) and (17). All the BRST cohomological results exposed in Refs. [56, 57] related to the case a single massless tensor field with the mixed symmetry $(k,1)$ remain valid. The only modifications appear in the concrete expressions of the representatives belonging to the various cohomologies needed in the BRST context, which may depend now on the matter fields and their derivatives and/or the matter antifields and their derivatives.

More precisely, related to the cohomology of the longitudinal exterior differential $H(\gamma)$ and of its local version, $H(\gamma|d)$, both computed in $\bar{\mathcal{A}}$, all the results from Ref. [56] still hold up to the following specifications. First, the general representatives of the algebra of invariant "polynomials" (the cohomology $H^{0}(\gamma)$ — in pure ghost number 0 — computed in $\bar{\mathcal{A}}$) read as

$$H^{0}(\gamma) \text{ in } \bar{\mathcal{A}} = \{\text{algebra of invariant "polynomials"}\} \equiv \left\{\bar{\alpha}\left(\left[\bar{\Phi}^{*}_{\bar{A}}\right], [K], \left[\phi^{\Delta}\right]\right)\right\}, \tag{40}$$

where K denotes the components of the curvature tensor

$$K_{\mu_1\ldots\mu_{k+1}|\alpha\beta} = \partial_{\alpha} F_{\mu_1\ldots\mu_{k+1}|\beta} - F_{\mu_1\ldots\mu_{k+1}|\alpha} \equiv \partial_{[\mu_1} t_{\mu_2\ldots\mu_{k+1}]|[\beta,\alpha]}, \tag{41}$$

and the notation $f([y])$ means that f depends on y and its derivatives up to a finite order. The quotes around the term polynomial indicate that the elements of $H^{0}(\gamma)$ computed in $\bar{\mathcal{A}}$ are true polynomials in all the arguments of $\bar{\alpha}$ excepting the bosonic matter fields (scalar fields), if any, in which they may be smooth functions. Consequently, the elements of the cohomology $H^{0}(\gamma)$ computed in the algebra of local forms $\bar{\Lambda}$ will change accordingly

$$H^{0}(\gamma) = \bigoplus_{p=0}^{D} H^{0,p}(\gamma), \quad H^{0,p}(\gamma) \ni \overset{[p]}{\bar{\alpha}} = \tfrac{1}{p!}\bar{\alpha}_{\mu_1\ldots\mu_p}\left(\left[\bar{\Phi}^{*}_{\bar{A}}\right], [K], \left[\phi^{\Delta}\right]\right) dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_p}. \tag{42}$$

Second, Table 2 from Ref. [56] is replaced with Table 1 below, where the quantity $\overset{(1)}{\mathcal{F}}_{\mu_1\ldots\mu_{k+1}}$ is specific to the $(k,1)$ sector, being defined by

$$\partial_{[\mu_1} \overset{(1)}{\eta}_{\mu_2\ldots\mu_{k+1}]} \equiv \overset{(1)}{\mathcal{F}}_{\mu_1\ldots\mu_{k+1}}, \quad \varepsilon\left(\overset{(1)}{\mathcal{F}}_{\mu_1\ldots\mu_{k+1}}\right) = 1, \quad \mathrm{pgh}\left(\overset{(1)}{\mathcal{F}}_{\mu_1\ldots\mu_{k+1}}\right) = 1. \tag{43}$$

From Table 1 we find that the general, nontrivial elements a from the cohomology $H(\gamma)$ computed in the algebra $\bar{\mathcal{A}}$ with the properties

$$\gamma a = 0, \quad a \in \bar{\mathcal{A}}, \quad \mathrm{pgh}(a) = l \geq 0, \quad \mathrm{agh}(a) = j \geq 0, \tag{44}$$

take the form

$$a = \sum_{J} \bar{\alpha}_{J}\left(\left[\bar{\Phi}^{*}_{\bar{A}}\right], [K], \left[\phi^{\Delta}\right]\right) e^{J}\left(\overset{(1)}{\mathcal{F}}, \overset{(k)}{\eta}\right), \quad \mathrm{agh}(\bar{\alpha}_{J}) = j \geq 0, \quad \mathrm{pgh}(e^{J}) = l \geq 0. \tag{45}$$

Similarly, the general, nontrivial elements ϖ of $H(\gamma)$ computed in $\bar{\Lambda}$ with the properties

$$\gamma\varpi = 0, \quad \varpi \in \bar{\Lambda}, \quad \deg(\varpi) = p \leq D, \quad \mathrm{pgh}(\varpi) = l \geq 0, \quad \mathrm{agh}(\varpi) = j \geq 0, \tag{46}$$

BRST generator	Nontrivial representatives	pgh
$[t_{\mu_1\ldots\mu_k\vert\alpha}]$	$[K_{\mu_1\ldots\mu_{k+1}\vert\alpha\beta}]$	0
$[\phi^\Delta]$	$[\phi^\Delta]$	0
$[\bar{\Phi}^*_{\bar{A}}]$	$[\bar{\Phi}^*_{\bar{A}}]$	0
$[\overset{(1)}{\eta}_{\mu_1\ldots\mu_k}], [\overset{(1)}{C}_{\mu_1\ldots\mu_{k-1}\vert\alpha}]$	$\overset{(1)}{\mathcal{F}}_{\mu_1\ldots\mu_{k+1}}$	1
$[\overset{(m)}{\eta}_{\mu_1\ldots\mu_{k-m+1}}], [\overset{(m)}{C}_{\mu_1\ldots\mu_{k-m}\vert\alpha}]$	—	$m,\ m=\overline{2,k-1}$
$[\overset{(k)}{\eta}_\alpha]$	$\overset{(k)}{\eta}_\alpha$	k

Table 1: Nontrivial representatives of the cohomology $H(\gamma)$ computed in the algebra $\bar{\mathcal{A}}$.

are expressed by

$$\varpi = \sum_J \overset{[p]}{\bar{\alpha}}_J\left([\bar{\Phi}^*_{\bar{A}}], [K], [\phi^\Delta]\right) e^J\left(\overset{(1)}{\mathcal{F}}, \overset{(k)}{\eta}\right), \tag{47}$$

$$\deg(\overset{[p]}{\bar{\alpha}}_J) = p \leq D, \quad \mathrm{agh}(\overset{[p]}{\bar{\alpha}}_J) = j \geq 0, \quad \mathrm{pgh}(e^J) = l \geq 0. \tag{48}$$

Obviously, the elements of the basis in the ghosts $\overset{(1)}{\mathcal{F}}$ and $\overset{(k)}{\eta}$, e^J, are those from the $(k,1)$ sector discussed in Ref. [56] since the matter fields display no true gauge symmetries. Third, Corollary 3 from Ref. [56], which will be useful in what follows, still holds in the presence of the matter fields, up to replacing the algebra Λ specific to the field $(k,1)$ alone with the algebra of local forms corresponding to the overall free model, $\bar{\Lambda}$.

At the level of the local cohomology of the Koszul–Tate differential (in pure ghost number 0) $H(\delta|d)$ and of the local cohomology of the Koszul–Tate differential computed in the algebra of invariant "polynomials" $H^{\mathrm{inv}}(\delta|d)$ for the free model under consideration, all the results given in Ref. [57] are applicable modulo the next mentions. The statements of Corollary 5, Lemma 6, Theorem 7, and Corollary 8 from Ref. [57] should be completed by mentioning the presence of the matter theory. The nontrivial representatives that span the invariant characteristic cohomology spaces of agh ranging between 2 and $(k+1)$, $\left(H_j^{\mathrm{inv}D}(\delta|d)\right)_{j=\overline{2,k+1}}$, gain no contributions from the matter sector, and coincide with those from Table 1 in [57]. The only space affected by the presence of the matter theory is the characteristic cohomology in agh $=1$ (and pgh $=0$), $H_1^D(\delta|d)$, which is isomorphic to the space of inequivalent, nontrivial rigid symmetries of the starting free theory, defined like the space of equivalence classes of rigid symmetries of action (1) modulo trivial ones. A general result [34, 35] is given by the isomorphism $H_0^{D-1}(d|\delta) \simeq H_1^D(\delta|d)$. As $H_0^{D-1}(d|\delta)$ (in pgh $=0$) is the space of inequivalent, nontrivial conserved currents, defined like the space of equivalence classes of conserved currents of theory (1) modulo the trivial ones, the last isomorphism expresses nothing but the one-to-one correspondence between the inequivalent nontrivial rigid symmetries of action (1) and the associated, inequivalent nontrivial conserved currents, i.e., the cohomological reformulation of Noether's Theorem. Applying the previous result to the matter sector, we infer that any nontrivial global symmetry of the form (8) produces a nontrivial element of the characteristic cohomology $H_1^D(\delta|d)$

$$(-)^{\varepsilon_{\bar{\Delta}}} \phi^*_\Delta Z^\Delta{}_{\bar{\Delta}} d^D x \in H_1^D(\delta|d). \tag{49}$$

Indeed, taking into account properties (9) and (19), relations (21) with respect to the operator δ, and formula (22), it follows that (49) is established (in dual language) by

Noether's Theorem (10), expressed in cohomological form like

$$\delta\Big((-)^{\varepsilon_{\bar{\Delta}}}\phi^{*}_{\Delta}Z^{\Delta}{}_{\bar{\Delta}}\Big) = \partial_{\mu}j^{\mu}{}_{\bar{\Delta}}. \tag{50}$$

Any trivial rigid symmetry of the matter theory, of the type (11)–(12), leads to a trivial element from $H_1^D(\delta|d)$ and must be eliminated from the perspective of constructing nontrivial interactions. This observation is motivated by the fact that the conserved currents corresponding to trivial rigid symmetries are also trivial (see the expression of $\bar{j}^{\mu}{}_{\bar{\Delta}}$ from (15)), such that the right-hand side of the analogue of relation (50) vanishes identically. In this way, the corresponding element belongs to the cohomology space $H_1^D(\delta)$, so the acyclicity of the differential δ in strictly positive values of the antighost number ensures that this element is actually δ-exact, and therefore trivial also in $H_1^D(\delta|d)$.

Finally, regarding the local BRST cohomology for theory (1) in maximum form degree evaluated in the algebra $\bar{\Lambda}$ Proposition 10 from Ref. [57] is still valid up to its proper reformulation such as to include the matter sector.

5 Computation of the deformed solution to the master equation. Obstructions

We have at hand all the necessary ingredients for generating the deformed solutions to the classical master equation that are consistent, nontrivial, and fulfill all the working hypotheses starting from a massless tensor field with the mixed symmetry $(k,1)$ and a set of matter fields of spin 0 and/or ½. In agreement with the general method exposed in Section 3, we develop the deformed solution along the coupling constant like in (25), where the first piece, S, reads now as in (23) and the deformations in various orders of perturbation theory are subject to equations (26).

In view of the spacetime locality assumption, we maintain notations (27), such that the nonintegrated density of the first-order deformation, a_1, should satisfy equation (30), and therefore should define a nontrivial element of the local BRST cohomology in maximum form degree and in ghost number equal to 0. We decompose S_1 as a sum among three local pieces

$$S_1 = S_1^{\mathrm{t}} + S_1^{\mathrm{mat}} + S_1^{\mathrm{t-mat}}, \tag{51}$$

$$S_1^{\mathrm{t}} = \int a_1^{\mathrm{t}} d^D x, \quad S_1^{\mathrm{mat}} = \int a_1^{\mathrm{mat}} d^D x, \quad S_1^{\mathrm{t-mat}} = \int a_1^{\mathrm{t-mat}} d^D x, \tag{52}$$

where S_1^{t} describes the selfinteractions of the tensor $(k,1)$, S_1^{mat} those among the matter fields, and $S_1^{\mathrm{t-mat}}$ the cross-couplings between these two field sectors, so a_1 inherits a similar decomposition

$$a_1 = a_1^{\mathrm{t}} + a_1^{\mathrm{mat}} + a_1^{\mathrm{t-mat}}. \tag{53}$$

Because a_1^{t} may depend only on the BRST generators from the $(k,1)$ sector and a_1^{mat} only on the matter ones, while each term from $a_1^{\mathrm{t-mat}}$ should contain at least one generator from each sector, equation (30) becomes equivalent with three independent equations

$$sa_1^{\mathrm{t}} + \partial_{\mu}b_1^{\mathrm{t}\mu} = 0, \quad sa_1^{\mathrm{mat}} + \partial_{\mu}b_1^{\mathrm{mat}\mu} = 0, \quad sa_1^{\mathrm{t-mat}} + \partial_{\mu}b_1^{\mathrm{t-mat}\mu} = 0. \tag{54}$$

The first equation has been analyzed in Ref. [55] in the context of the same deformation method and general assumptions employed here, where it has been shown that we can stop the first-order deformation a_1^{t} in antighost number 1 and the current $b_1^{\mathrm{t}\mu}$ in antighost number 0

$$a_1^{\mathrm{t}} = a_{1,1}^{\mathrm{t}} + a_{1,0}^{\mathrm{t}}, \qquad b_1^{\mathrm{t}\mu} = b_{1,0}^{\mathrm{t}\mu}, \tag{55}$$

$$\gamma a_{1,1}^{\mathrm{t}} = 0, \qquad \delta a_{1,1}^{\mathrm{t}} + \gamma a_{1,0}^{\mathrm{t}} + \partial_\mu b_{1,0}^{\mathrm{t}\mu} = 0, \tag{56}$$

$$a_{1,1}^{\mathrm{t}} = c\delta_{2\bar{k}}^{k}\delta_{4\bar{k}}^{D}\varepsilon_{\mu_1\ldots\mu_{4\bar{k}}} t^{*\mu_1\ldots\mu_{2\bar{k}-1}} \overset{(1)}{\mathcal{F}}{}^{\mu_{2\bar{k}}\cdots\mu_{4\bar{k}}}, \tag{57}$$

$$a_{1,0}^{\mathrm{t}} = -c\delta_{2\bar{k}}^{k}\delta_{4\bar{k}}^{D}\tfrac{(2\bar{k}-1)(2\bar{k}+1)}{(2\bar{k})!8\bar{k}^2}\varepsilon_{\mu_1\ldots\mu_{4\bar{k}}} F^{\mu_1\cdots\mu_{2\bar{k}}} F^{\mu_{2\bar{k}+1}\cdots\mu_{4\bar{k}}}, \tag{58}$$

$$b_{1,0}^{\mathrm{t}\mu} = c\delta_{2\bar{k}}^{k}\delta_{4\bar{k}}^{D}\tfrac{2\bar{k}-1}{(2\bar{k})!}\varepsilon_{\nu_1\ldots\nu_{4\bar{k}}} F^{\mu\nu_1\ldots\nu_{2\bar{k}-1}} \overset{(1)}{\mathcal{F}}{}^{\nu_{2\bar{k}}\cdots\nu_{4\bar{k}}} \tag{59}$$

$$= -c\delta_{2\bar{k}}^{k}\delta_{4\bar{k}}^{D}\tfrac{2\bar{k}-1}{(2\bar{k})!}\tfrac{2\bar{k}+1}{2\bar{k}}\varepsilon_{\nu_1\ldots\nu_{4\bar{k}}} F^{\nu_1\ldots\nu_{2\bar{k}}} \overset{(1)}{\mathcal{F}}{}^{\mu\nu_{2\bar{k}+1}\cdots\nu_{4\bar{k}}}. \tag{60}$$

Starting with formula (55), the second lower index of the quantities involved in the various orders of perturbation theory signifies their antighost number. In the above c signifies an arbitrary real constant and the supplementary factors $\delta_{2\bar{k}}^{k}$ and $\delta_{4\bar{k}}^{D}$ were introduced in order to highlight that relations (57)–(60) are valid solely for even values of k, equal to $2\bar{k}$, and only in $D = 4\bar{k}$ spacetime dimensions. We mention that in a previous step, the general expression of the component in agh $= 1$, $a_{1,1}^{\mathrm{t}}$, can be shown to read as [55]

$$a_{1,1}^{\mathrm{t}} = c\delta_{2k}^{D}\varepsilon_{\mu_1\ldots\mu_{2k}} t^{*\mu_1\ldots\mu_{k-1}} \overset{(1)}{\mathcal{F}}{}^{\mu_k\cdots\mu_{2k}}, \tag{61}$$

but the consistency of the first-order deformation a_1^{t} in agh $= 0$, i.e., the existence of $a_{1,0}^{\mathrm{t}}$ as solution to the latter equation from (56), finally requires that $k = 2\bar{k}$ (and hence $D = 4\bar{k}$) and leads to expressions (57) and (58).

Related to the second equation from (54), we proceed standardly by developing the nonintegrated density of the first-order deformation and the associated current along the antighost number. Since by assumption the matter theory has no nontrivial gauge symmetry, it follows that the matter sector contains no BRST generator of strictly positive pure ghost number, such that a_1^{mat} coincides with its component of antighost number 0, which automatically depends only on the matter fields and their spacetime derivatives up to a finite order

$$a_1^{\mathrm{mat}} \equiv a_{1,0}^{\mathrm{mat}}\left(\left[\phi^\Delta\right]\right). \tag{62}$$

Definitions (21) with respect to the matter fields show that a_1^{mat} satisfies the homogeneous equation ($b_1^{\mathrm{mat}\mu} = 0$)

$$s a_{1,0}^{\mathrm{mat}}\left(\left[\phi^\Delta\right]\right) = 0. \tag{63}$$

The only restrictions imposed on the matter Lagrangian density in order one of perturbation theory, $a_{1,0}^{\mathrm{mat}}\left(\left[\phi^\Delta\right]\right)$, are given by the working hypotheses combined with the nontriviality of the resulting action

$$S_1^{\mathrm{mat}}\left[\phi^\Delta\right] \equiv \int a_{1,0}^{\mathrm{mat}}\left(\left[\phi^\Delta\right]\right) d^D x, \qquad a_{1,0}^{\mathrm{mat}} \neq \frac{\delta^{\mathrm{R}}\mathcal{L}^{\mathrm{mat}}}{\delta\phi^\Delta} F^\Delta\left(\phi^{\Delta'}\right) + \partial_\mu m^\mu. \tag{64}$$

Next, we pass to the construction of solutions to the equation checked by the cross-coupling first-order deformation

$$s a_1^{\mathrm{t-mat}} + \partial_\mu b_1^{\mathrm{t-mat}\mu} = 0 \tag{65}$$

by means of the cohomological analysis exposed in Section 4. Due to the fact that the matter theory is not involved either in the elements of ghost basis from the cohomology $H(\gamma)$ for pgh > 0 or in the invariant characteristic cohomology for agh ≥ 2, while each term from $a_1^{\mathrm{t-mat}}$ mandatorily depends on at least one BRST generator from each sector, from the analogue of Proposition 10 given in [57] we conclude that this deformation may stop earliest in agh $= 1$. Moreover, the validity of Corollary 3 from [56] adapted to the present context allows us to choose the current $b_1^{\mathrm{t-mat}\mu}$ to display nonvanishing components only in agh $= 0$. Also taking into account decomposition (20) of the BRST differential, we immediately find the equations satisfied by the pieces of fixed antighost number of the cross-coupling first-order deformation. The last statements are translated into the formulas

$$a_1^{\mathrm{t-mat}} = a_{1,0}^{\mathrm{t-mat}} + a_{1,1}^{\mathrm{t-mat}}, \qquad b_1^{\mathrm{t-mat}\mu} = b_{1,0}^{\mathrm{t-mat}\mu}, \tag{66}$$

$$\gamma a_{1,1}^{\mathrm{t-mat}} = 0, \qquad \delta a_{1,1}^{\mathrm{t-mat}} + \gamma a_{1,0}^{\mathrm{t-mat}} + \partial_\mu b_{1,0}^{\mathrm{t-mat}\mu} = 0. \tag{67}$$

The solution to the former equation in (66) results from formula (45) for $j = l = 1$

$$a_{1,1}^{\mathrm{t-mat}} = \bar{\alpha}_1\big([t^*], [\phi^*_\Delta], [K], [\phi^\Delta]\big) e^1\Big(\overset{(1)}{\mathcal{F}}\Big), \tag{68}$$

where the invariant polynomial $\bar{\alpha}_1$ displays agh $= 1$, such that it is constrained to be a monomial of order one in both types of antifields and their derivatives up to a finite order, and e^1 coincides with $\overset{(1)}{\mathcal{F}}$ given in (43). The derivative order hypothesis at the level of $a_{1,0}^{\mathrm{t-mat}}$ forbids the dependence of $\bar{\alpha}_1$ on the curvature tensor or on its derivatives as well as on the derivatives of the antifield t^*. In addition, the piece linear in t^* cannot involve the derivatives of the matter fields, but is forced to depend on the undifferentiated matter fields in order to render cross-couplings. Regarding the terms linear in the derivatives of the matter antifields, we can always move the derivatives on the matter fields (by some integrations by parts) and eliminate the terms containing the derivatives of $\overset{(1)}{\mathcal{F}}$ together with the accompanying divergences as all these quantities are trivial in $H(\gamma)$ due to the relation

$$\partial_{\rho_1} \overset{(1)}{\mathcal{F}}_{\mu_1\ldots\mu_{k+1}} = \gamma\Big(\tfrac{(-)^{k+1}}{k} F_{\mu_1\ldots\mu_{k+1}|\rho_1}\Big) \tag{69}$$

completed with the γ-invariance of both the matter fields and their antifields (see the first two definitions from (21)). In other words, so far we argued that $a_{1,1}^{\mathrm{t-mat}}$ may contain just two nontrivial classes of representatives, both linear in the undifferentiated antifields of the original fields

$$a_{1,1}^{\mathrm{t-mat}} : \Big\{ t^* \overset{(\phi)}{\rightleftharpoons} \overset{(1)}{\mathcal{F}}, \quad \phi^*_\Delta \overset{([\phi])}{\rightleftharpoons} \overset{(1)}{\mathcal{F}} \Big\}, \tag{70}$$

where by (ϕ) and respectively $([\phi])$ we symbolized the allowed dependence on the matter fields of the functions that "glue" the two kinds of antifields to the element $\overset{(1)}{\mathcal{F}}$. Related to the second class of cross-couplings, the allowed dependence is only of the undifferentiated matter fields in the case of scalar bosons and respectively at most linear in the first-order derivatives of the matter fields for leptons and quarks. Lorentz covariance and Poincaré invariance arguments, completed by the mixed symmetry $(k,1)$ of the antifield t^* as well as by the requirements $D \geq k+2$, $k \geq 4$, further lead to two types of eligible terms

$$a_{1,1}^{\mathrm{t-mat}} = \delta^D_{2k} \varepsilon_{\mu_1\ldots\mu_{2k}} f(\phi) t^{*\mu_1\ldots\mu_{k-1}} \overset{(1)}{\mathcal{F}}{}^{\mu_k\ldots\mu_{2k}} + \phi^*_\Delta Z^\Delta{}_{\mu_1\ldots\mu_{k+1}}(\phi, \partial\phi) \overset{(1)}{\mathcal{F}}{}^{\mu_1\ldots\mu_{k+1}} \tag{71}$$

$$\equiv a_{1,1}^{\mathrm{t-mat}}(f) + a_{1,1}^{\mathrm{t-mat}}(Z). \tag{72}$$

The bosonic character of $a_{1,1}^{\mathrm{t-mat}}$ is ensured provided that

$$\varepsilon(f(\phi)) = 0, \qquad \varepsilon\big(Z^{\Delta}{}_{\mu_1 \ldots \mu_{k+1}}(\phi, \partial\phi)\big) = \varepsilon_{\Delta}. \tag{73}$$

Starting from (71), we investigate the solutions $a_{1,0}^{\mathrm{t-mat}}$ to the latter equation from (67). In order to simplify the arguments, it is useful to remark that the first piece from (71) contains two BRST generators from the $(k,1)$ sector and the second a single one. The action of the operator δ on t^* is linear in the second-order derivatives of t, that on ϕ_{Δ}^* contains maximum two derivatives of the matter fields, while the first-order derivatives of $\overset{(1)}{\mathcal{F}}$ read as γ acting on quantities linear in the first-order derivatives of t. Under these circumstances, it follows that, if consistent in agh $= 0$, then the two terms from $a_{1,1}^{\mathrm{t-mat}}$ produce in $a_{1,0}^{\mathrm{t-mat}}$ two kinds of functionally independent vertices (irrespective of the matter field content and without taking into consideration the additional fact that the former class of terms breaks the PT invariance and is nonvanishing only for $D = 2k$): the first type contains two spacetime derivatives and two fields $(k,1)$, while the latter comprises maximum two spacetime derivatives and a single tensor field t. As a consequence, the consistency of the cross-coupling first-order deformation in agh $= 0$ must take place separately for each component

$$\delta a_{1,1}^{\mathrm{t-mat}}(f) + \gamma a_{1,0}^{\mathrm{t-mat}}(f) + \partial_{\mu} b_{1,0}^{\mathrm{t-mat}\mu}(f) = 0, \tag{74}$$

$$\delta a_{1,1}^{\mathrm{t-mat}}(Z) + \gamma a_{1,0}^{\mathrm{t-mat}}(Z) + \partial_{\mu} b_{1,0}^{\mathrm{t-mat}\mu}(Z) = 0. \tag{75}$$

Comparing the first term from the right-hand side of (71), denoted by $a_{1,1}^{\mathrm{t-mat}}(f)$, with (61), we observe that it is obtained by multiplying the piece of agh $= 1$ that describes the selfinteractions of the tensor field t for $c = 1$, $a_{1,1}^{\mathrm{t}}|_{c=1}$, with the function $f(\phi)$

$$a_{1,1}^{\mathrm{t-mat}}(f) = f(\phi) a_{1,1}^{\mathrm{t}}|_{c=1}. \tag{76}$$

On the other hand, in [55] it was showed that (61) is consistent in agh $= 0$ if and only if

$$k = 2\bar{k}, \qquad D = 4\bar{k}. \tag{77}$$

Implementing the previous conditions in (76) by means of formula (57), we conclude that

$$a_{1,1}^{\mathrm{t-mat}}(f) = f(\phi) a_{1,1}^{\mathrm{t}}|_{c=1} \equiv f(\phi) \delta_{2\bar{k}}^{k} \delta_{4\bar{k}}^{D} \varepsilon_{\mu_1 \ldots \mu_{4\bar{k}}} t^{*\mu_1 \ldots \mu_{2\bar{k}-1}} \overset{(1)}{\mathcal{F}}{}^{\mu_{2\bar{k}} \cdots \mu_{4\bar{k}}}. \tag{78}$$

At this stage, we act with δ on $a_{1,1}^{\mathrm{t-mat}}(f)$ and employ the latter equation from (56), where $a_{1,0}^{\mathrm{t}}$ is given in (58) and the current $b_{1,0}^{\mathrm{t}\mu}$ is considered like in (59) or (60), which further yields

$$\delta a_{1,1}^{\mathrm{t-mat}}(f) = -\gamma\big[f(\phi) a_{1,0}^{\mathrm{t}}|_{c=1}\big] - \partial_{\mu}\big[f(\phi) b_{1,0}^{\mathrm{t}\mu}|_{c=1}\big] + \big[\partial_{\mu} f(\phi)\big] b_{1,0}^{\mathrm{t}\mu}|_{c=1}, \tag{79}$$

also due to the fact that $\gamma f(\phi) = 0$. By means of (79) and (74) we get the necessary condition (which in this case turns out to be also sufficient) for the existence of $a_{1,0}^{\mathrm{t-mat}}(f)$

$$b_{1,0}^{\mathrm{t}\mu}|_{c=1} = -\gamma a_{1,0}^{\mathrm{t}\mu} - \partial_{\nu} b_{1,0}^{\mathrm{t}\nu\mu}, \qquad b_{1,0}^{\mathrm{t}\nu\mu} = -b_{1,0}^{\mathrm{t}\mu\nu}, \tag{80}$$

$$\varepsilon(a_{1,0}^{\mathrm{t}\mu}) = 0, \qquad \mathrm{agh}(a_{1,0}^{\mathrm{t}\mu}) = 0 = \mathrm{pgh}(a_{1,0}^{\mathrm{t}\mu}), \tag{81}$$

$$\varepsilon(b_{1,0}^{\mathrm{t}\nu\mu}) = 1, \qquad \mathrm{pgh}(b_{1,0}^{\mathrm{t}\nu\mu}) = 1, \qquad \mathrm{agh}(b_{1,0}^{\mathrm{t}\nu\mu}) = 0. \tag{82}$$

Indeed, supposing that (80) takes place, where the antisymmetry of the 'current' $b_{1,0}^{\mathrm{t}\nu\mu}$ is essential, relation (79) becomes

$$\delta a_{1,1}^{\mathrm{t-mat}}(f) = -\gamma\Big[f(\phi)a_{1,0}^{\mathrm{t}}|_{c=1} + \big(\partial_\mu f(\phi)\big)a_{1,0}^{\mathrm{t}\mu}\Big] - \partial_\mu\Big[f(\phi)b_{1,0}^{\mathrm{t}\mu}|_{c=1} + \big(\partial_\nu f(\phi)\big)b_{1,0}^{\mathrm{t}\mu\nu}\Big], \quad (83)$$

such that the solution to equation (74) is given by

$$a_{1,0}^{\mathrm{t-mat}}(f) = f(\phi)a_{1,0}^{\mathrm{t}}|_{c=1} + \big(\partial_\mu f(\phi)\big)a_{1,0}^{\mathrm{t}\mu}. \quad (84)$$

It can be shown by direct computation, starting from any of the expressions (59) or (60), that equation (80) cannot hold, such that (79) is compatible with the consistency equation of the cross-coupling first-order deformation in agh = 0, (74), iff

$$\partial_\mu f(\phi) = 0 \Rightarrow f = \tilde{c} \in \mathbb{R}. \quad (85)$$

Inserting the constant solution (85) back in (78) and (79) we find no cross-couplings between the matter fields and the field with the mixed symmetry $(k,1)$, but instead simply revert to the first-order deformation responsible for the selfinteractions of the tensor $(k,1)$

$$a_{1,1}^{\mathrm{t-mat}}(f) \to \tilde{c}\delta_{2\bar{k}}^{k}\delta_{4\bar{k}}^{D}\varepsilon_{\mu_1\ldots\mu_{4\bar{k}}}t^{*\mu_1\ldots\mu_{2\bar{k}-1}}\overset{(1)}{\mathcal{F}}{}^{\mu_{2\bar{k}}\ldots\mu_{4\bar{k}}}, \quad (86)$$

$$a_{1,0}^{\mathrm{t-mat}}(f) \to -\tilde{c}\delta_{2\bar{k}}^{k}\delta_{4\bar{k}}^{D}\frac{(2\bar{k}-1)(2\bar{k}+1)}{(2\bar{k})!8\bar{k}^2}\varepsilon_{\mu_1\ldots\mu_{4\bar{k}}}F^{\mu_1\ldots\mu_{2\bar{k}}}F^{\mu_{2\bar{k}+1}\ldots\mu_{4\bar{k}}}. \quad (87)$$

This result is not eligible here, in the context of computing $a_1^{\mathrm{t-mat}}$, and must therefore be eliminated by setting $\tilde{c} = 0$, which annihilates the component $a_{1,1}^{\mathrm{t-mat}}(f)$ from (72).

As a result of the previous discussion, it follows that the only eligible term from (71) is that linear in the antifields of the matter fields

$$a_{1,1}^{\mathrm{t-mat}} = a_{1,1}^{\mathrm{t-mat}}(Z) \equiv \phi_\Delta^* Z^\Delta{}_{\mu_1\ldots\mu_{k+1}}(\phi,\partial\phi)\overset{(1)}{\mathcal{F}}{}^{\mu_1\ldots\mu_{k+1}}. \quad (88)$$

Let us analyze now the piece of agh = 0 of the nonintegrated density $a_1^{\mathrm{t-mat}}$ as solution to the latter equation from (67). We may represent the general solution $a_{1,0}^{\mathrm{t-mat}}$ under the form

$$a_{1,0}^{\mathrm{t-mat}} = a_{1,0}^{\mathrm{t-mat}}(Z) + \bar{a}_{1,0}^{\mathrm{t-mat}}, \quad (89)$$

where $a_{1,0}^{\mathrm{t-mat}}(Z)$ signifies the solution to equation (75) and $\bar{a}_{1,0}^{\mathrm{t-mat}}$ the general solution to the "homogeneous" equation

$$\gamma\bar{a}_{1,0}^{\mathrm{t-mat}} + \partial_\mu\bar{b}_{1,0}^{\mathrm{t-mat}\mu} = 0. \quad (90)$$

The inquire of a similar class of solutions, which does not modify the initial gauge transformations, has been approached for instance in the framework of computing the consistent deformations for a single massless tensor field $(k,1)$ [55]. Compared with the analysis therein, here we require that $\bar{a}_{1,0}^{\mathrm{t-mat}}$ describes cross-couplings, so each term includes the field t as well as the matter fields. We act with δ on (88) and employ definition (21) together with property (22), which lead to

$$\delta a_{1,1}^{\mathrm{t-mat}}(Z) = \frac{\delta^{\mathrm{R}}\mathcal{L}^{\mathrm{mat}}}{\delta\phi^\Delta}Z^\Delta{}_{\mu_1\ldots\mu_{k+1}}(\phi,\partial\phi)\overset{(1)}{\mathcal{F}}{}^{\mu_1\ldots\mu_{k+1}}. \quad (91)$$

Since the object $\overset{(1)}{\mathcal{F}}$ is not γ-exact, but its derivatives are so (see formula (69)), from (91) we notice that equation (75) possesses solutions if the necessary condition (in this setting also sufficient as the matter fields are γ-invariant) holds

$$\frac{\delta^{\mathrm{R}}\mathcal{L}^{\mathrm{mat}}}{\delta\phi^{\Delta}} Z^{\Delta}{}_{\mu_1\ldots\mu_{k+1}}(\phi,\partial\phi) + \partial_\mu j^\mu{}_{\mu_1\ldots\mu_{k+1}}(\phi,\partial\phi) = 0, \qquad \varepsilon(j^\mu{}_{\mu_1\ldots\mu_{k+1}}) = 0. \tag{92}$$

From the last relation and taking into account formula (10), we infer that the previous condition is nothing but Noether's Theorem corresponding to the invariance of the matter action under the global symmetry

$$\delta_\xi \phi^\Delta = Z^\Delta{}_{\mu_1\ldots\mu_{k+1}}(\phi,\partial\phi)\xi^{\mu_1\cdots\mu_{k+1}} \tag{93}$$

of the type (8), where the rigid parameters $\xi^{\mu_1\cdots\mu_{k+1}}$ stand for the components of a constant, bosonic, antisymmetric tensor of order $(k+1)$, such that the associated conserved current, $j^\mu{}_{\mu_1\ldots\mu_{k+1}}$, is also real and antisymmetric with respect to all its lower indices. The main difference with respect to the general context where we introduced relations (8) and (10) is that here we are constrained to enforce the Poincaré invariance, such that neither the generators Z nor the conserved currents are allowed to depend explicitly on the spacetime coordinates x^μ. The maximum derivative order of the associated conserved current is equal to one as the generators Z may depend at most linearly on the derivatives of the matter fields $\partial\phi$ only for spin ½ (quarks/leptons). We recall that each generator of this rigid symmetry displays the Grassmann parity of the corresponding matter field ϕ^Δ (see the latter formula from (73)). As we have highlighted before (see the paragraph containing formula (49)), condition (92) is equivalent to the requirement that the invariant polynomial from (88)

$$\bar{\alpha}_{\mu_1\ldots\mu_{k+1}} \equiv \phi^*_\Delta Z^\Delta{}_{\mu_1\ldots\mu_{k+1}}(\phi,\partial\phi), \quad \bar{\alpha}_{\mu_1\ldots\mu_{k+1}} = \bar{\alpha}_{[\mu_1\ldots\mu_{k+1}]}, \tag{94}$$

$$\varepsilon(\bar{\alpha}_{\mu_1\ldots\mu_{k+1}}) = 1, \quad \mathrm{agh}(\bar{\alpha}_{\mu_1\ldots\mu_{k+1}}) = 1, \quad \mathrm{pgh}(\bar{\alpha}_{\mu_1\ldots\mu_{k+1}}) = 0, \tag{95}$$

for each set of fixed indices $\{\mu_1,\cdots,\mu_{k+1}\}$, constitutes an element of the characteristic cohomology in agh $= 1$, $H_1^D(\delta|d)$, specific to the matter theory

$$\delta\bar{\alpha}_{\mu_1\ldots\mu_{k+1}} = \partial_\mu j^\mu{}_{\mu_1\ldots\mu_{k+1}}. \tag{96}$$

The nontriviality of the invariant polynomial $\bar{\alpha}_{\mu_1\ldots\mu_{k+1}} d^D x$ in $H_1^D(\delta|d)$ is translated into the nontriviality of the global symmetry (93)

$$Z^\Delta{}_{\mu_1\ldots\mu_{k+1}} \neq \frac{\delta^{\mathrm{R}}\mathcal{L}^{\mathrm{mat}}}{\delta\phi^{\Delta'}} Z^{\Delta'\Delta}{}_{\mu_1\ldots\mu_{k+1}}([\phi]), \qquad Z^{\Delta'\Delta}{}_{\mu_1\ldots\mu_{k+1}} = (-)^{1+\varepsilon_{\Delta'}\varepsilon_\Delta} Z^{\Delta\Delta'}{}_{\mu_1\ldots\mu_{k+1}} \tag{97}$$

and therefore also of the associated conserved current. On behalf of relations (91) and (69) we are able to generate the cross-coupling Lagrangian density in order one of perturbation theory as solution to equation (75) in the form

$$a_{1,0}^{\mathrm{t-mat}}(Z) = \tfrac{(-)^k}{k} j^\mu{}_{\mu_1\ldots\mu_{k+1}}(\phi,\partial\phi) F^{\mu_1\ldots\mu_{k+1}|}{}_\mu, \tag{98}$$

where F is linear in the first-order derivatives of t (formula (4)). Regarding the solutions to the "homogenous" equation (90) that does not deform the gauge symmetries of the original Lagrangian action, if we invoke all the hypotheses that must be fulfilled by $\bar{a}_{1,0}^{\mathrm{t-mat}}$, like the maximum derivative order, the Poincaré invariance, or the cross-coupling condition, then we find no acceptable solutions in $D \geq k+2 \geq 6$

$$\bar{a}_{1,0}^{\mathrm{t-mat}} = 0. \tag{99}$$

By means of results (88), (89), (98), and (99) we can state that the most general expression of the cross-coupling first-order deformation that verifies all the imposed restrictions reduces to

$$a_1^{\mathrm{t-mat}} = \phi_\Delta^* Z^\Delta{}_{\mu_1\ldots\mu_{k+1}}(\phi,\partial\phi)\overset{(1)}{\mathcal{F}}{}^{\mu_1\ldots\mu_{k+1}} + \tfrac{(-)^k}{k} j^\mu{}_{\mu_1\ldots\mu_{k+1}}(\phi,\partial\phi) F^{\mu_1\ldots\mu_{k+1}|}{}_\mu. \tag{100}$$

According to the relationship between the pieces of fixed antighost number from the deformed solution to the master equation and the specific features of the accompanying interacting gauge theory [31], we observe that (100) on the one hand generates the cross-coupling Lagrangian density at order one of perturbation theory in the standard conserved current–gauge field form

$$S_{0(1)}\left[t_{\mu_1\ldots\mu_k|\alpha}, \phi^\Delta\right] = \tfrac{(-)^k}{k} \int j^\mu{}_{\mu_1\ldots\mu_{k+1}}(\phi,\partial\phi) F^{\mu_1\ldots\mu_{k+1}|}{}_\mu d^D x \tag{101}$$

and, on the other hand, endows the matter fields (at the same perturbation order) with nontrivial gauge transformations obtained by gauging the rigid ones, (93), where the rigid parameters are replaced by the (antisymmetric) first-order derivatives of the antisymmetric gauge parameters $\overset{(1)}{\epsilon}$ from the $(k,1)$ sector

$$\delta^{(1)}_{\overset{(1)}{\theta},\overset{(1)}{\epsilon}} \phi^\Delta = Z^\Delta{}_{\mu_1\ldots\mu_{k+1}}(\phi,\partial\phi)\partial^{[\mu_1}\overset{(1)}{\epsilon}{}^{\mu_2\ldots\mu_{k+1}]}. \tag{102}$$

So far, we argued that it is possible to couple a massless tensor field with the mixed symmetry $(k,1)$ to a matter theory if and only if the latter one possesses a bosonic, nontrivial rigid symmetry of the form (93), with the rigid parameters identified with the components of a constant, fully antisymmetric tensor or order $(k+1)$. Nevertheless, the matter field spectrum considered here (of spin 0 and/or ½) does not allow the existence of such rigid symmetries for any possible value $k \geq 4$, which renders

$$Z^\Delta{}_{\mu_1\ldots\mu_{k+1}} = 0, \qquad j^\mu{}_{\mu_1\ldots\mu_{k+1}} = 0. \tag{103}$$

As a consequence, the cross-coupling first-order deformation (100) as well as the cross-coupling Lagrangian action together with the gauge transformations (102) of the matter fields are annihilated, so the overall first-order deformation of the solution to the master equation (51) simply reduces to the sum between its first two pieces

$$S_1 = S_1^{\mathrm{t}} + S_1^{\mathrm{mat}}, \tag{104}$$

with S_1^{mat} like in (64). Due to the fact that all the antibrackets between the BRST matter generators and those from the $(k,1)$ sector vanish, by means of solving the higher-order deformation equations present in (26), it follows that the fully deformed solution to the classical master equation for the model under consideration, (25), that is consistent to all orders in the coupling constant and satisfies all the working hypotheses, reduces to

$$\bar{S} = \bar{S}^{\mathrm{t}} + S_0^{\mathrm{mat}}\left[\phi^\Delta\right] + \lambda S_1^{\mathrm{mat}}\left[\phi^\Delta\right]. \tag{105}$$

In the above $\bar{S}^{\mathrm{t}}$ governs the selfinteractions of the massless tensor field with the mixed symmetry $(k,1)$ [55], S_0^{mat} is the original action of the matter theory, and S_1^{mat} is subject to condition (64), so it gathers all the allowed cross-couplings among the matter fields that comply with the working hypotheses and are not included in S_0^{mat}. The deformation procedure developed here does not allow the matter fields to be endowed with nontrivial gauge transformations, as happens for instance in the case of couplings between a matter theory and a single vector field, a collection of such fields, or a BF theory [58–61].

6 Conclusion

The final conclusion of this paper is that, under the standard hypotheses on interacting gauge field theories, no trivial cross-couplings between a massless tensor field with the mixed symmetry $(k,1)$ and a matter theory with spin 0 and/or ½ fields can be introduced, irrespective of $k \geq 4$. The sole consistent deformations of the solution to the classical master equation are those leading to selfinteractions within the $(k,1)$ sector, accompanied by possible nontrivial couplings among the matter fields considered here. Unfortunately, the matter fields cannot be empowered with (nontrivial) gauge transformations. It is nevertheless possible that the extension of the matter field spectrum might generate nontrivial cross-couplings with the tensor $(k,1)$ and might introduce nontrivial gauge symmetries in the matter sector.

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