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Collisionless particle diffusion in mixed electrostatic and magnetic stochastic fields

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Abstract

Guiding centers moving in a combination of a shearless stochastically perturbed confining magnetic field and a fluctuating two-dimensional electrostatic field is considered. The motion in such a type of stochastic field is of general interest (e.g., particle and energy transport in magnetically confined plasmas) and various approximations of it have been considered previously. Our interest lies in two limiting cases, that of high values of the magnetic and electrostatic Kubo numbers in which case the charged particles explore (statistically) the topology including the trapping regions of both stochastic fields before they have time to change, and that of the opposite case of weak Kubo numbers where the fields are varying sufficiently fast so that almost no trapping of trajectory occurs. The scope of the study is to determine the change of behavior of DCT trajectories while varying the main parameters of the problem.

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1 Introduction

The motion of charged particles, driven by stochastic velocity fields, is representative of a large class of physical processes (i.e., particle and energy transport in plasma) and various approximations or geometrical configurations of the problem have been studied previously ([1]-[4]). A model is proposed similar to a Langevin type system of equations describing the motion of a guiding center through the highest significant order in the drift parameter [2]. By assuming that the time necessary for the particle to traverse the parallel correlation length is much longer than the transverse correlation time it is possible by scale separation to consider to first order the statistical properties of projected trajectories onto a two-dimensional plane transverse to the constant longitudinal magnetic field. The method for analyzing the dynamics which will be used is the "decorrelation trajectory method", which accounts in a statistical way for the trapping effects of the trajectories in the structures of the fluctuating electric and magnetic fields. The strongest trapping effects appear in the limiting case of large electrostatic and magnetic Kubo numbers. Results will be compared to the other limiting case of small Kubo numbers where trapping effects are weak. Because two Kubo numbers are in action, an interesting class of behaviors can be analyzed: the decorrelation trajectories, the velocities, the solutions and the hodographs.

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2 The properties of the electric and magnetic fluctuating fields

The geometry considered is that of a shearless slab for the confining magnetic field perturbed by fluctuating perpendicular components

$$\mathbf{B}(\mathbf{X},t) = B_0 \left\{ \mathbf{e}_z + \beta b_x \left(\mathbf{X},t \right) \mathbf{e}_x + \beta b_y \left(\mathbf{X},t \right) \mathbf{e}_y \right\} \tag{1}$$

Here β is a dimensionless parameter measuring the amplitude of the magnetic fluctuations in the plane $\mathbf{X} \equiv (X,\ Y)$ relative to the main magnetic field $\mathbf{B}_0$=$B_0\mathbf{e}_z$. The magnetic field fluctuations are described by the dimensionless Gaussian statistical processes $b_i \left(\mathbf{X},t \right)$ for $i = (X,Y)$ [5]. The stochastic electric field is considered to have the form:

$$\mathbf{E}(X,Y,t) = E_x \left(X,Y,t \right) \mathbf{e}_x + E_y \left(X,Y,t \right) \mathbf{e}_y + E_z \left(X,Y,t \right) \mathbf{e}_z \tag{2}$$

3 The system of equations

The system of equations for the guiding center coordinates $\mathbf{X}$ are (neglecting all the magnetic gradients) [2]:

$$\dot{\mathbf{X}} \simeq U\frac{\mathbf{B}}{B}+\frac{c}{B^2} \left(\mathbf{E} \times \mathbf{B} \right) \tag{3}$$

Using the eqs.(1, 2, 3) and the corresponding electric drift which has the expression obtained using eqs. (1, 2):

$$\frac{c \left(\mathbf{E} \times \mathbf{B} \right)}{B^2} = \frac{c}{B_0} \left\{ \mathbf{e}_x \left(-E_z \beta b_y + E_y \right) - \mathbf{e}_y \left(E_x - E_z \beta b_x \right) + \mathbf{e}_z \left(E_x \beta b_y - E_y \beta b_x \right) \right\} \tag{4}$$

the dimensional guiding center equations takes the form:

$$\frac{dX \left(t \right)}{dt} = U \beta b_x \left(X,Y,t \right) + \frac{c}{B_0} E_y \left(X,Y,t \right) \tag{5}$$

$$\frac{dY \left(t \right)}{dt} = U \beta b_y \left(X,Y,t \right) - \frac{c}{B_0} E_x \left(X,Y,t \right) \tag{6}$$

This form of the system is justified by the following reasonable approximation: $\beta b_i E_j \ll 1$, $i \neq j$ where $i = x, y$ and $j = x,\ y, z$ that is valid because both the electrostatic and the perpendicular magnetic field are fluctuating. We introduce the coordinates $\mathbf{x} = (x,\ y)\ ,\tau$ which are dimensionless quantities related to the dimensional ones $\mathbf{X} = (X,\ Y)\ ,t$ by the relations:

$$\frac{\mathbf{X}}{\lambda_{el}} = \mathbf{x}; \quad ; \quad \frac{t}{\tau_{el}} = \tau_{el} \tag{7}$$

where $\lambda_{el},\ \lambda_m$ are the electrostatic and magnetic correlation lengths in the perpendicular direction to the main magnetic field $B_0\mathbf{e}_z$, T is the ratio between the correlation times and is defined in eq.(8) (τ_m is the correlation time for the purely magnetic fluctuations and τ_{el} is corresponding to the electrostatic fluctuations) and ε is a parameter measuring the electrostatic fluctuations. The ratio of the correlation times is:

$$T = \frac{\tau_{el}}{\tau_m} \tag{8}$$

where τ_m is the correlation time for the purely magnetic fluctuations and τ_{el} is corresponding to the electrostatic fluctuations. The parallel guiding center velocity U is considered constant :

$$U = const. \equiv U_0 \tag{9}$$

Taking into account (7), the dimensionless system of equations corresponding to the dimensional system (5)-(6) becomes:

$$\frac{dx(\tau)}{d\tau} = \frac{U_0 \beta \tau_{el}}{\lambda_{el}} b_x(x,y,\tau) + \frac{c\tau_{el}}{\lambda_{el} B_0} E_y(x,y,\tau) \tag{10}$$

$$\frac{dy(\tau)}{d\tau} = \frac{U_0 \beta \tau_{el}}{\lambda_{el}} b_y(x,y,\tau) - \frac{c\tau_{el}}{\lambda_{el} B_0} E_x(x,y,\tau) \tag{11}$$

The fluctuating components of the electrostatic and the magnetic field can be obtained from appropriate stochastic potentials:

$$E_x(x,y,\tau) = -\frac{\varepsilon \partial \Phi(x,y,\tau)}{\lambda_{el} \partial x}; \quad E_y(x,y,\tau) = -\frac{\varepsilon \partial \Phi(x,y,\tau)}{\lambda_{el} \partial y} \tag{12}$$

where Φ is the dimensionless electrostatic potential and ε is a dimensional quantity measuring the electrostatic fluctuations $[\Phi(X,Y,t) \rightarrow \varepsilon \Phi\left(\frac{\mathbf{X}}{\lambda_{el}}, \frac{t}{\tau_{el}}\right)]$ and

$$b_x(x,y,\tau) = \frac{\partial \psi(x,y,\tau)}{\partial y}; \quad b_y(x,y,\tau) = -\frac{\partial \psi(x,y,\tau)}{\partial x} \tag{13}$$

where ψ is the magnetic dimensionless potential. Introducing eqs. (12) and (13) in the system (10) - (11) one obtain:

$$\frac{dx(\tau)}{d\tau} = \frac{U_0 \beta \tau_{el}}{\lambda_{el}} \frac{\partial \psi(x,y,\tau)}{\partial y} - \frac{\varepsilon c \tau_{el}}{\lambda_{el}^2 B_0} \frac{\partial \Phi(x,y,\tau)}{\partial y} \tag{14}$$

$$\frac{dy(\tau)}{d\tau} = -\frac{U_0 \beta \tau_{el}}{\lambda_{el}} \frac{\partial \psi(x,y,\tau)}{\partial x} + \frac{c \varepsilon \tau_{el}}{\lambda_{el}^2 B_0} \frac{\partial \Phi(x,y,\tau)}{\partial x} \tag{15}$$

The coefficients from the r.h.s. of the system (14-15) can be expressed as:

$$\frac{U_0 \beta \tau_c}{\lambda_{el}} = \frac{U_0 \beta \tau_m}{\lambda_m} \cdot \frac{\tau_{el}}{\tau_m} \cdot \frac{\lambda_m}{\lambda_{el}} \equiv T \Lambda K_m; \quad T = \frac{\tau_{el}}{\tau_m} \leq 1; \quad \Lambda = \frac{\lambda_m}{\lambda_{el}} \geq 1 \tag{16}$$

where we have defined the magnetic Kubo number:

$$K_m = \frac{U_0 \beta \tau_m}{\lambda_m} \tag{17}$$

and

$$K_{el} = \frac{\varepsilon c \tau_{el}}{\lambda_{el}^2 B_0} \tag{18}$$

which is the electrostatic Kubo number. Taking into account on eqs.(16- 18) the system (14-15) becomes:

$$\frac{dx(\tau)}{d\tau} = T \Lambda K_m \frac{\partial \psi(x,y,\tau)}{\partial y} - K_{el} \frac{\partial \Phi(x,y,\tau)}{\partial y} \tag{19}$$

$$\frac{dy(\tau)}{d\tau} = -T \Lambda K_m \frac{\partial \psi(x,y,\tau)}{\partial x} + K_{el} \frac{\partial \Phi(x,y,\tau)}{\partial x} \tag{20}$$

The system will be solved with the initial condition $\mathbf{x}(0) = \mathbf{0}$. This is the Langevin system of equations for the collisionless particle in the electrostatic and magnetic stochastic configuration and will be the starting point of our analysis.

4 The DCT method for the combined electrostatic and magnetic turbulence

The system (19) - (20) becomes in the absence of the magnetic fluctuations ($K_m \equiv 0$) the system analyzed in [1] for electrostatic turbulence. We deal in our case with two fluctuating stochastic potentials ψ and Φ. We will suppose that these two potentials are stochastically independent (i.e., $\langle \psi\Phi \rangle = 0$) and the fluctuations (both the electrostatic and the magnetic) are characterized by different correlation lengths λ_{el}, λ_m and different correlation times τ_{el}, τ_m such that: $\langle \Phi(\mathbf{0},0)\,\Phi(\mathbf{x},t)\rangle \approx 0$, $\langle \psi(\mathbf{0},0)\,\psi(\mathbf{x},t)\rangle \approx 0$ when $|\mathbf{x}| \gg \lambda_{el}, \lambda_m$ and/or $t \gg \tau_{el}$ or correspondingly $t \gg \tau_m$.

The two dimensionless parameters defined in (16) and (18) characterize the combined electrostatic and magnetic stochastic fields. and in the forthcoming analysis we will consider both the small ($K_m,\ K_{el} < 1$) and high ($K_m,\ K_{el} > 1$) Kubo numbers regimes. In this way the trapping process which plays an essential role in the strongly turbulent phenomena will be analyzed.

The study of the transport problem implies the evaluation of the Lagrangian correlations. The system (19) - (20) can be written as:

$$\frac{dx(\tau)}{d\tau} = V_x(\mathbf{x}(\tau), \tau)$$

$$\frac{dy(\tau)}{d\tau} = V_y(\mathbf{x}(\tau), \tau) \tag{21}$$

where

$$V_x(\mathbf{x}(\tau), \tau) = T\Lambda K_m \frac{\partial \psi(x,y,\tau)}{\partial y} - K_{el}\frac{\partial \Phi(x,y,\tau)}{\partial y} \equiv T\Lambda K_m b_x(x,y,\tau) + K_{el} v_x(x,y,\tau) \tag{22}$$

$$V_y(\mathbf{x}(\tau), \tau) = -T\Lambda K_m \frac{\partial \psi(x,y,\tau)}{\partial x} + K_{el}\frac{\partial \Phi(x,y,\tau)}{\partial x} \equiv T\Lambda K_m b_y(x,y,\tau) + K_{el} v_y(x,y,\tau) \tag{23}$$

are the fluctuating velocities. We can easily verify that the $\nabla \cdot \mathbf{V} = 0$ and as a consequence it can be derived from the following "mixed" potential:

$$\Phi_{em}(\mathbf{x}(\tau), \tau) = -T\Lambda K_m \psi(\mathbf{x}(\tau), \tau) + K_{el}\Phi(\mathbf{x}(\tau), \tau) \tag{24}$$

The velocities defined in eqs.(22,23) can then be written as:

$$V_x(\mathbf{x}(\tau), \tau) = -\frac{\partial \Phi_{em}(x,y,\tau)}{\partial y}, \qquad V_y(\mathbf{x}(\tau), \tau) = \frac{\partial \Phi_{em}(x,y,\tau)}{\partial x} \tag{25}$$

Assuming stationary turbulence, the Lagrangian correlations depend only on the difference of two times and the following Lagrangian correlation tensor can be defined:

$$L_{ij}(\tau) = \int d\mathbf{x}\, \langle V_i(\mathbf{0};0) V_j[\mathbf{x}(\tau);\tau]\, \delta(\mathbf{x} - \mathbf{x}(\tau))\rangle \tag{26}$$

where $\langle ... \rangle$ denotes the average over the realizations of the fluctuating velocities V_i ($i = x,\ y$).

In our paper we use the the DCT approximation, a significant step beyond the well-known Corrsin approximation [7] which includes the quasilinear and the Bohm approximations. In the framework of DCT method general expressions of the running (and consequently asymptotic) diffusion coefficients can be derived. We briefly recall the main ideas of the DCT approximation (see Refs. [1], [4]). The space of realizations of the potential fluctuations is subdivided into *subensembles* S, characterized by given values of the "mixed" potential and of the fluctuating velocities at the starting point of the trajectories. The exact expression of the Lagrangian correlation can be written as a superposition of Lagrangian correlations in the various subensembles. In each subensemble a deterministic decorrelation trajectory is defined by the following criterion: the *Eulerian average* of the potential calculated along this deterministic trajectory should equal the *Lagrangian* average of the potential in S:

$$\Phi^S[\mathbf{x}^S(t),t] = \langle \Phi[\mathbf{x}(t),t] \rangle^S ; \quad \psi^S[\mathbf{x}^S(t),t] = \langle \psi[\mathbf{x}(t),t] \rangle^S ; \quad \Phi^S_{em}[\mathbf{x}^S(t),t] = \langle \Phi_{em}[\mathbf{x}(t),t] \rangle^S \tag{27}$$

Implementing this approximation in the exact formulae for the Lagrangian fields correlations yields an approximation that is valid, in principle, for arbitrarily large values of K_m and K_{el}. The main reason for this statement is that the DCT method takes into account the trapping processes, which are neglected in previous theories based on the Corrsin approximation. These processes are an essential ingredient of strong turbulence theories. The validity of the approximation involved in DCT method can be assessed by *a posteriori* comparison with experiment and simulations, as is done in all theories of strong turbulence.

In order to apply the DCT method we note that the fluctuating quantities b_i, v_i are derived from the potentials ψ and Φ. In all theories based on Langevin equations the Eulerian potential autocorrelation represents the starting point and is defined *a priori*. We assume that the Eulerian autocorrelations of the dimensionless potentials have the following factorized form:

$$\langle \psi(\mathbf{0},0)\,\psi(\mathbf{x},\tau) \rangle = E^m(\mathbf{x})\,T^m(\tau) \equiv E^m(\mathbf{x},\tau) \tag{28}$$

$$\langle \Phi(\mathbf{0},0)\,\Phi(\mathbf{x},\tau) \rangle = E^{el}(\mathbf{x})\,T^{el}(\tau) \equiv E^{el}(\mathbf{x},\tau) \tag{29}$$

where $E^m(\mathbf{x})$ and $E^{el}(\mathbf{x})$ are dimensionless functions of position and $T^m(\tau)$ and $T^{el}(\tau)$ are functions of time.We stress again that we consider that the two potentials are stochastically independent, i.e. $\langle \psi\Phi \rangle = 0$.

We define the following notations for the Eulerian correlations [see Refs. [1], [4]]:

$$\langle \psi(\mathbf{0},0)\,b_j(\mathbf{x},\tau) \rangle = E^m_{\psi j}(\mathbf{x})\,T^m(\tau) \tag{30}$$

$$\langle b_i(\mathbf{0},0)\,b_j(\mathbf{x},\tau) \rangle = E^m_{ij}(\mathbf{x})\,T^m(\tau) \tag{31}$$

$$\langle \Phi(\mathbf{0},0)\,v_j(\mathbf{x},\tau) \rangle = E^{el}_{\Phi j}(\mathbf{x})\,T^{el}(\tau) \tag{32}$$

$$\langle v_i(\mathbf{0},0)\,v_j(\mathbf{x},\tau) \rangle = E^{el}_{ij}(\mathbf{x})\,T^{el}(\tau) \tag{33}$$

All these Eulerian correlations can be derived by appropriate derivatives from the primary potential autocorrelations given in (28) and (29):

$$E^m_{\psi x}(\mathbf{x}) = -E^m_{x\psi}(\mathbf{x}) = \frac{\partial E^m(\mathbf{x})}{\partial y} \tag{34}$$

$$E^m_{\psi y}(\mathbf{x}) = -E^m_{y\psi}(\mathbf{x}) = -\frac{\partial E^m(\mathbf{x})}{\partial x} \tag{35}$$

$$E_{xx}^{m}(\mathbf{x}) = -\frac{\partial^2 E^m(\mathbf{x})}{\partial y^2};\quad E_{yy}^{m}(\mathbf{x}) = -\frac{\partial^2 E^m(\mathbf{x})}{\partial x^2};\quad E_{xy}^{m}(\mathbf{x}) = E_{yx}^{m}(\mathbf{x}) = \frac{\partial^2 E^m(\mathbf{x})}{\partial x \partial y} \tag{36}$$

$$E_{\Phi x}^{el}(\mathbf{x}) = -E_{x\Phi}^{el}(\mathbf{x}) = -\frac{\partial E^{el}(\mathbf{x})}{\partial y} \tag{37}$$

$$E_{\Phi y}^{el}(\mathbf{x}) = -E_{y\Phi}^{el}(\mathbf{x}) = \frac{\partial E^{el}(\mathbf{x})}{\partial x} \tag{38}$$

$$E_{xx}^{el}(\mathbf{x}) = -\frac{\partial^2 E^{el}(\mathbf{x})}{\partial y^2};\quad E_{yy}^{el}(\mathbf{x}) = -\frac{\partial^2 E^{el}(\mathbf{x})}{\partial x^2};\quad E_{xy}^{el}(\mathbf{x}) = E_{yx}^{el}(\mathbf{x}) = \frac{\partial^2 E^{el}(\mathbf{x})}{\partial x \partial y} \tag{39}$$

The correlations defined in Eqs. (34-39) can be written in a compact form using the antisymmetric tensor ε_{ij} ($\varepsilon_{12} = -\varepsilon_{21} = 1, \varepsilon_{11} = \varepsilon_{22} = 0$) as:

$$E_{ij}^{m}(\mathbf{x}) = -\varepsilon_{in}\varepsilon_{jp}\frac{\partial^2 E^m(\mathbf{x})}{\partial x_n \partial x_p};\quad E_{\psi i}^{m}(\mathbf{x}) = -E_{i\psi}^{m}(\mathbf{x}) = \varepsilon_{in}\frac{\partial E^m(\mathbf{x})}{\partial x_n};\quad i,j,p,n = x,y \tag{40}$$

and

$$E_{ij}^{el}(\mathbf{x}) = -\varepsilon_{in}\varepsilon_{jp}\frac{\partial^2 E^{el}(\mathbf{x})}{\partial x_n \partial x_p};\quad E_{\Phi i}^{el}(\mathbf{x}) = -E_{i\Phi}^{el}(\mathbf{x}) = -\varepsilon_{in}\frac{\partial E^{el}(\mathbf{x})}{\partial x_n};\quad i,j,p,n = x,y \tag{41}$$

The Eulerian correlations corresponding to the "mixed" potential are calculated using the eqs.(30 - 41) and the definitions (22-25) and their expressions are:

$$\langle \Phi_{em}(\mathbf{0},0)\,\Phi_{em}(\mathbf{x},\tau)\rangle = (T\Lambda)^2 K_m^2 E^m(\mathbf{x})\,T^m(\tau) + K_{el}^2 E^{el}(\mathbf{x})\,T^{el}(\tau) \equiv E^{em}(\mathbf{x},\tau) \tag{42}$$

$$\langle \Phi_{em}(\mathbf{0},0)\,V_j(\mathbf{x},\tau)\rangle = -(T\Lambda)^2 K_m^2 E_{\psi j}^m(\mathbf{x})\,T^m(\tau) + K_{el}^2 E_{\Phi j}^{el}(\mathbf{x})\,T^{el}(\tau) \equiv E_{\Phi^{em} j}^{em}(\mathbf{x},\tau) \tag{43}$$

$$\langle V_j(\mathbf{0},0)\,\Phi_{em}(\mathbf{x},\tau)\rangle = -(T\Lambda)^2 K_m^2 E_{j\psi}^m(\mathbf{x})\,T^m(\tau) + K_{el}^2 E_{j\Phi}^{el}(\mathbf{x})\,T^{el}(\tau) \equiv E_{j\Phi^{em}}^{em}(\mathbf{x},\tau) \tag{44}$$

$$\langle V_i(\mathbf{0},0)\,V_j(\mathbf{x},\tau)\rangle = (T\Lambda)^2 K_m^2 E_{ij}^m(\mathbf{x})\,T^m(\tau) + K_{el}^2 E_{ij}^{el}(\mathbf{x})\,T^{el}(\tau) \equiv E_{ij}^{em}(\mathbf{x},\tau) \tag{45}$$

We now develop the DCT method closely folowing the main line given in [1]. The idea consists in the decomposition of the ensemble of realizations of the turbulent ensemble into subensembles. This decomposition is slightly different from that used in [1] because of the existence of two stochastic potentials instead one. We define the subensemble S as the set of realizations in which Φ_{em} and V_i ($i = x,\ y$) have given values at time 0:

$$S : \Phi_{em}(\mathbf{0},0) = \Phi_{em}^0,\quad V_i(\mathbf{0},0) = V_i^0;\quad i = x,\ y \tag{46}$$

In this case the probability distribution of the initial values can be defined as:

$$P_0^{em}(\mathbf{V}^0, \Phi_{em}^0) = P(V_x^0)P(V_y^0)P(\Phi_{em}^0) =$$

$$= (2\pi)^{-3/2}\left(E^{em}(\mathbf{0},0)\,E_{xx}^{em}(\mathbf{0},0)\,E_{yy}^{em}(\mathbf{0},0)\right)^{-1/2}$$
$$exp\left[-\frac{(\Phi_{em}^0)^2}{2E^{em}(\mathbf{0},0)} - \frac{(V_x^0)^2}{2E_{xx}^{em}(\mathbf{0},0)} - \frac{(V_y^0)^2}{2E_{yy}^{em}(\mathbf{0},0)}\right] \tag{47}$$

where:

$$\begin{aligned} E^{em}(\mathbf{0},0) &\equiv \left[(T\Lambda)^2 K_m^2 E^m(\mathbf{0})\,T^m(0) + K_{el}^2 E^{el}(\mathbf{0})\,T^{el}(0)\right]^{1/2} \\ E_{xx}^{em}(\mathbf{0},0) &\equiv \left[(T\Lambda)^2 K_m^2 E_{xx}^m(\mathbf{0})\,T^m(0) + K_{el}^2 E_{xx}^{el}(\mathbf{0})\,T^{el}(0)\right]^{1/2} \\ E_{yy}^{em}(\mathbf{0},0) &\equiv \left[(T\Lambda)^2 K_m^2 E_{yy}^m(\mathbf{0})\,T^m(0) + K_{el}^2 E_{yy}^{el}(\mathbf{0})\,T^{el}(0)\right]^{1/2} \end{aligned} \tag{48}$$

The Lagrangian correlation tensor is:

$$L_{ij}^{em}(\tau)=\int d\Phi_{em}^{0}d\mathbf{V}^{0}P_{0}^{em}(\mathbf{V}^{0},\Phi_{em}^{0})V_{i}^{0}(0,0;0)\left\langle V_{j}\left[\mathbf{x}(\tau);\tau\right]\right\rangle^{S} \tag{49}$$

where $\langle ...\rangle^{S}$ denotes the average in the subensemble. The averaged velocity from the integrand can be written (see Refs. [3], [4], [1]) as:

$$\left\langle V_{j}\left[\mathbf{x}(\tau);\tau\right]\right\rangle^{S}=\frac{\Phi_{em}^{0}}{E^{em}\left(\mathbf{0},0\right)}E_{\Phi^{em}j}^{em}\left(\mathbf{x},\tau\right)+\frac{V_{x}^{0}}{E_{xx}^{em}\left(\mathbf{0},0\right)}E_{xj}^{em}\left(\mathbf{x},\tau\right)+\frac{V_{y}^{0}}{E_{yy}^{em}\left(\mathbf{0},0\right)}E_{yj}^{em}\left(\mathbf{x},\tau\right) \tag{50}$$

We define next in a subensemble S a deterministic trajectory by the following equations of motion:

$$\frac{dx^{S}\left(\tau\right)}{d\tau}=V_{x}^{S}\left(\mathbf{x}^{S}\left(\tau\right),\tau\right)\equiv\left\langle V_{x}\left[\mathbf{x}(\tau);\tau\right]\right\rangle^{S} \tag{51}$$

$$\frac{dy^{S}\left(\tau\right)}{d\tau}=V_{y}^{S}\left(\mathbf{x}^{S}\left(\tau\right),\tau\right)\equiv\left\langle V_{y}\left[\mathbf{x}(\tau);\tau\right]\right\rangle^{S} \tag{52}$$

$$\mathbf{x}^{S}\left(0\right)=\mathbf{0} \tag{53}$$

where the velocities are defined in eq.(50). The system (51-53) determines the motion of the fictitious quasiparticle characteristic to the DCT method. The deterministic DCT are introduced in the Lagrangian correlations tensor (49) and these quantities evaluated in the DCT approximation are:

$$\begin{aligned}L_{ij}^{em}(\tau) &= \int d\Phi_{em}^{0}d\mathbf{V}^{0}P_{0}^{em}(\mathbf{V}^{0},\Phi_{em}^{0})V_{i}^{0}(\mathbf{0};0)\left\langle V_{j}\left[\mathbf{x}(\tau);\tau\right]\right\rangle^{S}= \\ &= \int d\Phi_{em}^{0}d\mathbf{V}^{0}P_{0}^{em}(\mathbf{V}^{0},\Phi_{em}^{0})V_{i}^{0}(\mathbf{0};0)\times \\ &\times\left\{\frac{\Phi_{em}^{0}}{E^{em}\left(\mathbf{0},0\right)}E_{\Phi^{em}j}^{em}\left(\mathbf{x},\tau\right)+\frac{V_{x}^{0}}{E_{xx}^{em}\left(\mathbf{0},0\right)}E_{xj}^{em}\left(\mathbf{x},\tau\right)+\frac{V_{y}^{0}}{E_{yy}^{em}\left(\mathbf{0},0\right)}E_{yj}^{em}\left(\mathbf{x},\tau\right)\right\}\end{aligned} \tag{54}$$

In order to make explicit calculations we need to choose the dimensionless functions $E^{m}\left(\mathbf{x}\right)$ and $E^{el}\left(\mathbf{x}\right)$. We consider the following functions (see Refs. [1], [4], [6]):

$$E^{m}\left(\mathbf{x}\right)=\exp\left(-\frac{\mathbf{x}^{2}}{2}\right) \tag{55}$$

and

$$E^{el}\left(\mathbf{x}\right)=\left(1+\mathbf{x}^{2}\right)^{-\alpha}\quad;\alpha\in[0.5\ ,\ 2] \tag{56}$$

The magnetic correlation is a localized space-function while the electrostatic one is an extended function with algebraic behaviour at large distances; these have a strong impact on the scaling of the diffusion coefficient. This effect is left for a future paper. For now the role of α will be studied based on DCT trajectories. The time dependence is considered to be different for the correlations of the magnetic and electric fields:

$$T^{m}\left(\frac{t}{\tau_{m}}\right)=\exp\left(-T\tau\right)\equiv T^{m}\left(\tau,T\right) \tag{57}$$

$$T^{el}\left(\frac{t}{\tau_{el}}\right)=\exp\left(-\tau\right)\equiv T^{el}\left(\tau\right) \tag{58}$$

where the parameter T was defined in eq.(16). Using (55, 56, 43, 45), we obtain for the components of the fluctuating velocities (see Ref. [1]) the following expressions:

$$V_x^S\left(\mathbf{x}^S,\tau\right)=\frac{\Phi_{em}^0}{E^{em}\left(\mathbf{0},0\right)}E_{\Phi^{em}x}^{em}\left(\mathbf{x}^S,\tau\right)+\frac{V_x^0}{E_{xx}^{em}\left(\mathbf{0},0\right)}E_{xx}^{em}\left(\mathbf{x}^S,\tau\right)+\frac{V_y^0}{E_{yy}^{em}\left(\mathbf{0},0\right)}E_{yx}^{em}\left(\mathbf{x}^S,\tau\right)=$$

$$=\frac{\Phi_{em}^0}{E^{em}\left(\mathbf{0},0\right)}\left[-\left(T\Lambda\right)^2K_m^2E_{\psi x}^m\left(\mathbf{x}^S\right)T^m\left(\tau\right)+K_{el}^2E_{\Phi x}^{el}\left(\mathbf{x}^S\right)T^{el}\left(\tau\right)\right]+$$

$$+\frac{V_x^0}{E_{xx}^{em}\left(\mathbf{0},0\right)}\left[\left(T\Lambda\right)^2K_m^2E_{xx}^m\left(\mathbf{x}^S\right)T^m\left(\tau\right)+K_{el}^2E_{xx}^{el}\left(\mathbf{x}^S\right)T^{el}\left(\tau\right)\right]+$$

$$+\frac{V_y^0}{E_{yy}^{em}\left(\mathbf{0},0\right)}\left[\left(T\Lambda\right)^2K_m^2E_{yy}^m\left(\mathbf{x}^S\right)T^m\left(\tau\right)+K_{el}^2E_{yy}^{el}\left(\mathbf{x}^S\right)T^{el}\left(\tau\right)\right]\tag{59}$$

$$V_y^S\left(\mathbf{x}^S,\tau\right)=\frac{\Phi_{em}^0}{E^{em}\left(\mathbf{0},0\right)}E_{\Phi^{em}y}^{em}\left(\mathbf{x}^S,\tau\right)+\frac{V_x^0}{E_{xx}^{em}\left(\mathbf{0},0\right)}E_{xy}^{em}\left(\mathbf{x}^S,\tau\right)+\frac{V_y^0}{E_{yy}^{em}\left(\mathbf{0},0\right)}E_{yy}^{em}\left(\mathbf{x}^S,\tau\right)=$$

$$=\frac{\Phi_{em}^0}{E^{em}\left(\mathbf{0},0\right)}\left[-\left(T\Lambda\right)^2K_m^2E_{\psi y}^m\left(\mathbf{x}^S\right)T^m\left(\tau\right)+K_{el}^2E_{\Phi y}^{el}\left(\mathbf{x}^S\right)T^{el}\left(\tau\right)\right]+$$

$$+\frac{V_x^0}{E_{xx}^{em}\left(\mathbf{0},0\right)}\left[\left(T\Lambda\right)^2K_m^2E_{xy}^m\left(\mathbf{x}^S\right)T^m\left(\tau\right)+K_{el}^2E_{xy}^{el}\left(\mathbf{x}^S\right)T^{el}\left(\tau\right)\right]+$$

$$+\frac{V_y^0}{E_{yy}^{em}\left(\mathbf{0},0\right)}\left[\left(T\Lambda\right)^2K_m^2E_{yy}^m\left(\mathbf{x}^S\right)T^m\left(\tau\right)+K_{el}^2E_{yy}^{el}\left(\mathbf{x}^S\right)T^{el}\left(\tau\right)\right]\tag{60}$$

The specific magnetic and electrostatic Eulerian correlations are calculated using (55, 56) and the general definitions given in (40, 41) and they have the following expressions:

$$\begin{aligned}
E_{\psi x}^m\left(\mathbf{x}^S\right)&=-y^SE^m\left(\mathbf{x}^S\right)\\
E_{\Phi x}^{el}\left(\mathbf{x}^S\right)&=A^Sy^SE^{el}\left(\mathbf{x}^S\right)\\
E_{xx}^m\left(\mathbf{x}^S\right)&=\left(1-\left(y^S\right)^2\right)E^m\left(\mathbf{x}^S\right)\\
E_{xx}^{el}\left(\mathbf{x}^S\right)&=\left[A^S-B^S\left(y^S\right)^2\right]E^{el}\left(\mathbf{x}^S\right)\\
E_{yy}^m\left(\mathbf{x}^S\right)&=\left(1-\left(x^S\right)^2\right)E^m\left(\mathbf{x}^S\right)\\
E_{yy}^{el}\left(\mathbf{x}^S\right)&=\left[A^S-B^S\left(x^S\right)^2\right]E^{el}\left(\mathbf{x}^S\right)\\
E_{\psi y}^m\left(\mathbf{x}^S\right)&=x^SE^m\left(\mathbf{x}^S\right)\\
E_{\Phi y}^{el}\left(\mathbf{x}^S\right)&=-A^Sx^SE^{el}\left(\mathbf{x}^S\right)\\
E_{xy}^m\left(\mathbf{x}^S\right)&=x^Sy^SE^m\left(\mathbf{x}^S\right)\\
E_{xy}^{el}\left(\mathbf{x}^S\right)&=x^Sy^SB^SE^{el}\left(\mathbf{x}^S\right)
\end{aligned}\tag{61}$$

where we have introduced the functions A^S and B^S as:

$$A^S=\frac{2\alpha}{1+\left(\mathbf{x}^S\right)^2}\tag{62}$$

and

$$B^S = \frac{4\alpha\,(\alpha+1)}{\left(1+(\mathbf{x}^S)^2\right)^2} \tag{63}$$

Using the eqs. (54) and (61-63) the explicit components of the Lagrangian correlations tensor are:

$$L_{xx}^{em}(\tau) = \int d\Phi_{em}^0 d\mathbf{V}^0 P_0^{em}(\mathbf{V}^0, \Phi_{em}^0) V_x^0(\mathbf{0};0)$$

$$\times \left\{ \frac{\Phi_{em}^0}{E^{em}(\mathbf{0},0)} E_{\Phi^{em}x}^{em}(\mathbf{x},\tau) + \frac{V_x^0}{E_{xx}^{em}(\mathbf{0},0)} E_{xx}^{em}(\mathbf{x},\tau) + \frac{V_y^0}{E_{yy}^{em}(\mathbf{0},0)} E_{yx}^{em}(\mathbf{x},\tau) \right\} \tag{64}$$

$$L_{xy}^{em}(\tau) = \int d\Phi_{em}^0 d\mathbf{V}^0 P_0^{em}(\mathbf{V}^0, \Phi_{em}^0) V_x^0(\mathbf{0};0)$$

$$\times \left\{ \frac{\Phi_{em}^0}{E^{em}(\mathbf{0},0)} E_{\Phi^{em}y}^{em}(\mathbf{x},\tau) + \frac{V_x^0}{E_{xx}^{em}(\mathbf{0},0)} E_{xy}^{em}(\mathbf{x},\tau) + \frac{V_y^0}{E_{yy}^{em}(\mathbf{0},0)} E_{yy}^{em}(\mathbf{x},\tau) \right\} \tag{65}$$

$$L_{yx}^{em}(\tau) = \int d\Phi_{em}^0 d\mathbf{V}^0 P_0^{em}(\mathbf{V}^0, \Phi_{em}^0) V_y^0(\mathbf{0};0)$$

$$\times \left\{ \frac{\Phi_{em}^0}{E^{em}(\mathbf{0},0)} E_{\Phi^{em}x}^{em}(\mathbf{x},\tau) + \frac{V_x^0}{E_{xx}^{em}(\mathbf{0},0)} E_{xx}^{em}(\mathbf{x},\tau) + \frac{V_y^0}{E_{yy}^{em}(\mathbf{0},0)} E_{yx}^{em}(\mathbf{x},\tau) \right\} \tag{66}$$

$$L_{yy}^{em}(\tau) = \int d\Phi_{em}^0 d\mathbf{V}^0 P_0^{em}(\mathbf{V}^0, \Phi_{em}^0) V_y^0(\mathbf{0};0)$$

$$\times \left\{ \frac{\Phi_{em}^0}{E^{em}(\mathbf{0},0)} E_{\Phi^{em}y}^{em}(\mathbf{x},\tau) + \frac{V_x^0}{E_{xx}^{em}(\mathbf{0},0)} E_{xy}^{em}(\mathbf{x},\tau) + \frac{V_y^0}{E_{yy}^{em}(\mathbf{0},0)} E_{yy}^{em}(\mathbf{x},\tau) \right\} \tag{67}$$

Next we choose different subensembles S and study the decorrelation trajectories for different values of the involved parameters, $T,\ \Lambda,\ \alpha,\ K_{el},\ K_m$.

5 Results

We have analyzed the influence of the Kubo numbers, of the exponent of the Lorentzian and, of the specific parameters that characterize the subensemble, on the solutions of the system (21. As a consequence a set of DCT trajectories and hodographs is obtained. We have considered in all the analysis two values of the Lorentzian exponent: in red $\alpha = 0.5$ and in blue $\alpha = 2$. Moreover we have considered in all the cases that $\Lambda = 1,\ T = 1$.

In all the figures, in subplots (a) are represented the radial solutions of the system (21), in subplots (b) the poloidal solutions of the system (21), in subplots (c) are represented the trajectories and in subplots (d) are represented the hodographs.

In Figure 1, the subensemble S is defined by the parameters: $\Phi_{em}(\mathbf{0},0) = \Phi_{em}^0 = 3.5$, $V_x(\mathbf{0},0) = 1$, $V_y(\mathbf{0},0) = 0.1$. The values for the level of the electromagnetic turbulence are given by the following Kubo numbers: $K_m = K_{el} = 0.5$. It is observed that in the case of the higher Lorentzian exponent, i.e. $\alpha = 2$ the trajectory form a closed loop. Keeping the same Kubo numbers, but looking at smaller value for α (here $\alpha = 0.5$) we observe the DCT trajectory comes to a stop before completing a full cycle.

In Figure 2, the same values as in Fig. 1 of the parameters determining the subensemble S are taken except that higher Kubo numbers namely $K_m = K_{el} = 3$ are considered. The increase of the turbulence level given by the higher values of the Kubo numbers (but accordingly slower time evolution) modifies the shapes of the solutions of Eqs. (21),

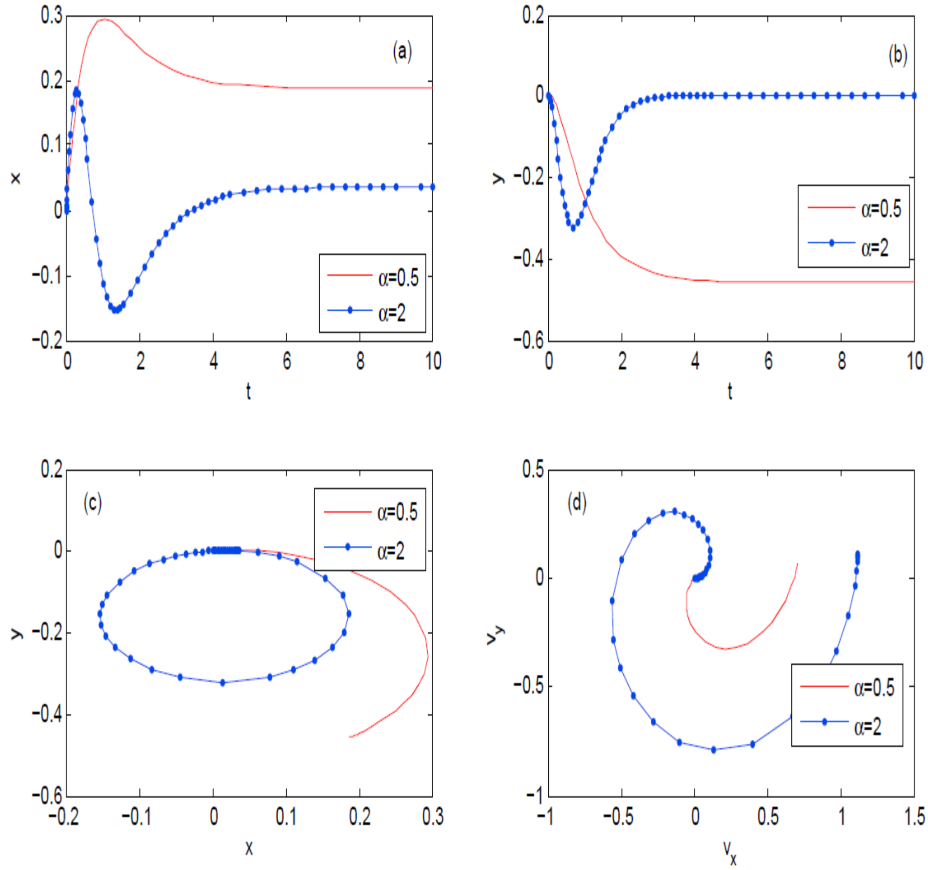

Figure 1: Solutions of the system (5), trajectories and the hodographs for $K_m = K_{el} = 0.5$

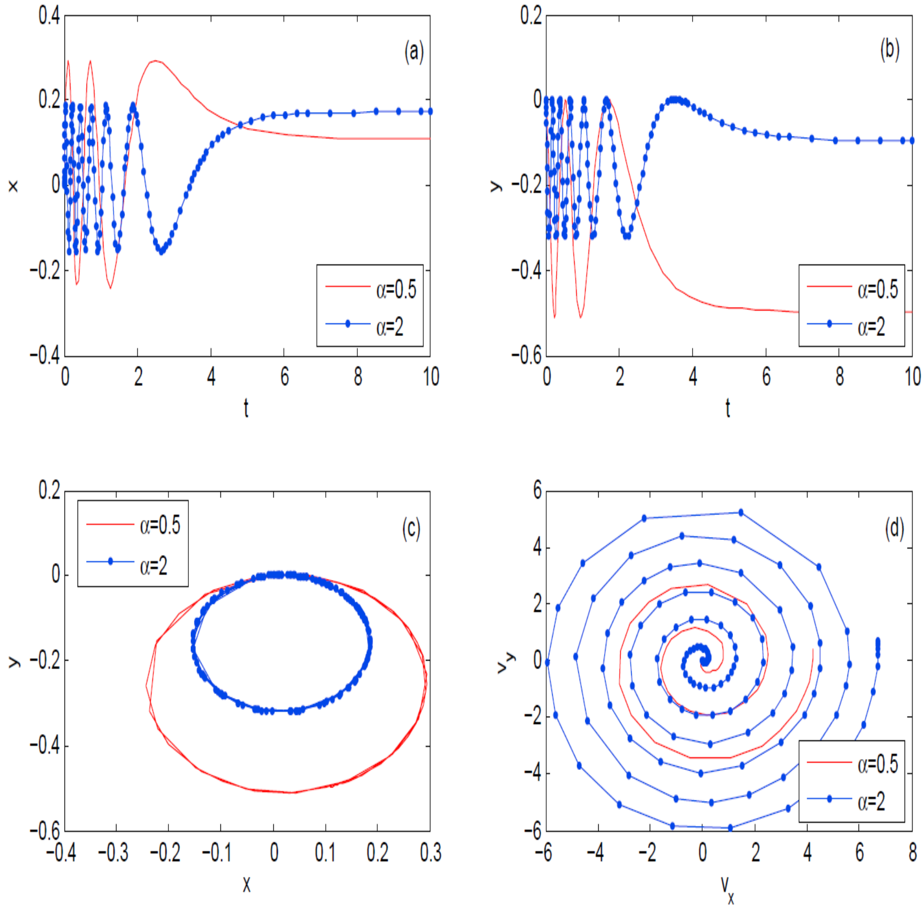

Figure 2: Solutions of the system (5), trajectories and the hodographs for $K_m = K_{el} = 3$

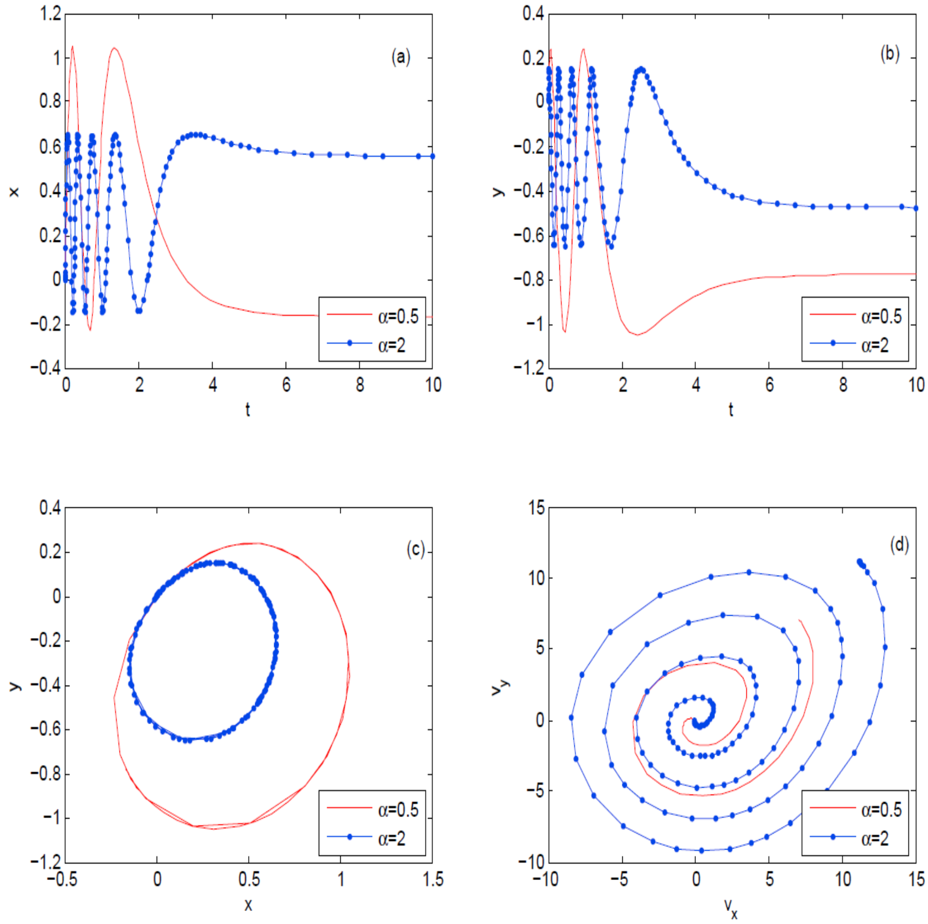

Figure 3: Solutions of the system (5), trajectories and the hodographs for $K_m = K_{el} = 5$

the trajectories and the hodographs. The trajectories form closed loops for any values of the Lorentzian exponent but cover different areas. The greater area the smaller the Lorentzian exponent is.

Figure 3 shows the solution of Eqs. (21), the DCT trajectories, the DCT velocities and the hodographs for the following parameters specific to the subensemble S: $\Phi_{em}(\mathbf{0},0) = \Phi_{em}^{0} = 1.5, \quad V_x(\mathbf{0},0) = V_y(\mathbf{0},0) = 1$. The asymptotic regimes specific to a subensemble (i.e. the stopping of the trajectory of the quasiparticle) are reached in poloidal and radial directions practically at the same asymptotic moment $t_{as} = 7$. It can then be concluded that the level of turbulence, the subensembles parameters and the Lorentzian exponents have no influence on the asymptotic time. As in the previous case, the trajectories are closed for both exponents $\alpha = 0.5$ and $\alpha = 2$ but their radius are smaller the greater exponent is.

Even higher level of turbulence than considered in the previous cases are used here i.e. $K_m = K_{el} = 5$. The trapping phenomena remains but the high level of electromagnetic turbulence leads to an increased number of oscillations in the solutions both in the radial and the poloidal directions. For $\alpha = 2$ (Lorentz exponent) the trapping is more pronounced than in the case $\alpha = 0.5$. We can conclude our analysis by the following statement: for very small values of Kubo numbers the particle motion is very quickly damped, before the end of a cycle and one could say that it is not influenced by the landscape of the stochastic potential. On the contrary, for large values of Kubo numbers the particle goes many times around the cycle before stopping; it thus explores efficiently the correlated region, and fully feels the effect of trapping. This is the interpretation of the set of oscillations in Figures 2 and 3.

6 Conclusions

In the present study it has been established that the stochastic magnetic and electrostatic turbulence provide a decorrelation mechanism for the particle dynamics. It has also been shown that the trapping effect is more pronounced for larger Kubo numbers. The solutions of the DCT system begin with an oscillating part, defining a trapping regime always followed by a stopping asymptotic one. The role of the global trapping is evidenced by the DCT method. The trapping effect is more pronounced the larger Λ, are, (K_m, K_{el}) and an influence of the Lorentz exponent on the trapping effect has also been shown. Physically speaking the trapping of the particles in very slowly fluctuating potentials necessarily leads to a decrease in the diffusionas compared to rapidly varying potentials. The larger the intensity of the fluctuations, the longer is the time spent by the particles around the vortex (trap). An asymptotical vanishing of the diffusion coefficient means that in the limit of very large Kubo numbers and alpha exponent the system generates a real particle dynamics of the subdiffusive type. In the "frozen" state, the majority of particles is trapped in a rigid, static potential and have no possibility of diffusing. Additional work is needed to determine the scaling to the low value of the particle diffusion coefficient from time dependent fields to this kind of "frozen" electric and magnetic fields. A numerical code has been prepared which will allow the calculation of the diffusion coefficients. The system of equations is numerically integrated using a fourth order Runge Kutta, adaptive step-size method.

7 Acknowledgment

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