Balescu Kinetic Equation with Exchange Interaction

V V Belyi [1]*and Yu A Kukharenko[2]

[1]IZMIRAN Russian Academy of Sciences, Troitsk, Moscow,142190, Russia

and Service de Physique Statistique et des Plasmas, CP 231, ULB, 1050 Bruxelles, Belgium

[2]UIPTP Russian Academy of Sciences, Moscow, Russia

Abstract

Starting with the quantum BBGKY-hierarchy for the distribution functions, we have obtained the quantum kinetic equation including the dynamical screening of the interaction potential, which exactly takes into account the exchange scattering in the plasma. The collision integral is expressed in terms of the Green function of the linearized Hartree-Fock equation.The potential energy takes into account the polarization and exchange interaction too.

1 Introduction

The exchange interaction and dynamical screening are important in the cases of degenerate gases and liquids, electrons in metals and in degenerate semiconductors, in quantum dots. The non-linear kinetic equation taking dynamical screening due to plasma polarization into account for weakly coupled plasma was derived the first time by Balescu [1] using Prigogine's diagram techniques and a little bit later by Lenard [2] who solved the Bogoliubov equation for the pair correlation function in the plasma approximation. Before Balescu and Lenard the plasma kinetic description was restricted to the Vlasov equation for distribution function $f_a(\mathbf{p},\mathbf{r},t)$ with self-consistent field $F_a(\mathbf{r},t)$ [3]

$$\frac{\partial}{\partial t}f_a(\mathbf{p},\mathbf{r},t)+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}f_a(\mathbf{p},\mathbf{r},t)+F_a(\mathbf{r},t)\frac{\partial}{\partial \mathbf{p}}f_a(\mathbf{p},\mathbf{r},t)=0 \tag{1}$$

or to the Landau equation [4]

$$\frac{\partial}{\partial t}f_a(\mathbf{p},\mathbf{r},t)+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}f_a(\mathbf{p},\mathbf{r},t)+F_a(\mathbf{r},t)\frac{\partial}{\partial \mathbf{p}}f_a(\mathbf{p},\mathbf{r},t)=I^L$$

$$=\sum_b 2e_a^2e_b^2n_b\frac{\partial}{\partial p_i}\int\frac{k_ik_j}{\mathbf{k}^4}\delta(\mathbf{kv}-\mathbf{kv}')\left(\frac{\partial}{\partial p_j}-\frac{\partial}{\partial p_j'}\right)f_a(\mathbf{p},\mathbf{r},t)f_b(\mathbf{p}',\mathbf{r},t)\mathbf{kp}' \tag{2}$$

with a collision integral, which was derived from the Boltzmann equation by expansion in powers of small momentum transfer. In these conditions, the presence of the long range Coulomb potential interaction $4\pi e_ae_b/k^2$ leads to divergence of the collision integral for

*sbelyi@izmiran.ru

small wave-numbers: $I^L \approx \int_0^\infty dk/k$. To avoid this divergence Landau introduced ad hoc a cut-off at the Debye wave number: $I^L \approx \int_{k_D}^{k_{\max}} dk/k$

By contrast, Balescu and Lenard took into account collective interaction of particles in the collision integral, taking dynamical screening due to plasma polarization into account:

$$I^{B-L} = \sum_b 2e_a^2 e_b^2 n_b \frac{\partial}{\partial p_i} \int \frac{k_i k_j}{\mathbf{k}^4} \frac{\delta(\mathbf{kv} - \mathbf{kv}')}{|\varepsilon(\mathbf{kv}, \mathbf{k})|^2} \left(\frac{\partial}{\partial p_j} - \frac{\partial}{\partial p_j'} \right) f_a(\mathbf{p}, \mathbf{r}, t) f_b(\mathbf{p}', \mathbf{r}, t) \mathbf{kp}'. \quad (3)$$

Due to polarization in the collision integral there appears the screened interaction potential $\frac{4\pi e_a e_b}{k^2} \frac{1}{|\varepsilon(\mathbf{kv},\mathbf{k})|^2}$ which in the static approximation has the following form $\frac{4\pi e_a e_b}{k^2} \frac{r_D^2 k^2}{1+r_D^2 k^2}$. Therefore the divergence in this collision integral for small wave-numbers disappeared. This kinetic equations are fundamental for plasma physics and are comparable with the Boltzmann equation for diluted gas.

These results were generalized to the quantum case by Balescu [5] and one year later by Guernsey [6].

$$\frac{\partial}{\partial t} f_a(\mathbf{p}, t) = 4\pi \sum_b \int \mathbf{kq} \frac{e_a^2 e_b^2}{\mathbf{k}^4} \frac{\delta(\Delta_k E_b(\mathbf{q}) - \Delta_k E_a(\mathbf{p}))}{|\varepsilon(\Delta_k E_a(\mathbf{p})/\hbar, \mathbf{k})|^2}$$
$$\{f_a(\mathbf{p} + \hbar\mathbf{k}/2) f_b(\mathbf{q} - \hbar\mathbf{k}/2)[1 - f_a(\mathbf{p} - \hbar\mathbf{k}/2)][1 - f_b(\mathbf{q} + \hbar\mathbf{k}/2)]$$
$$- f_a(\mathbf{p} - \hbar\mathbf{k}/2) f_b(\mathbf{q} + \hbar\mathbf{k}/2)[1 - f_a(\mathbf{p} + \hbar\mathbf{k}/2)][1 - f_b(\mathbf{q} - \hbar\mathbf{k}/2)], \quad (4)$$

where

$$\Delta_k E_a(\mathbf{p}) = E_a(\mathbf{p} + \hbar\mathbf{k}/2) - E_a(\mathbf{p} - \hbar\mathbf{k}/2), \quad (5)$$

and

$$\varepsilon(z, \mathbf{k}) = 1 + \sum_a \frac{4\pi e_a^2}{\mathbf{k}^2} \int \mathbf{p} \frac{f_a(\mathbf{p} + \hbar\mathbf{k}/2) - f_a(\mathbf{p} - \hbar\mathbf{k}/2)}{\hbar z - \Delta_k E_a(\mathbf{p})}$$

$$= 1 + \sum_a \frac{4\pi e_a^2}{\mathbf{k}^2} \int \mathbf{p} \frac{f_a(\mathbf{p} + \hbar\mathbf{k}/2)[1 - f_a(\mathbf{p} - \hbar\mathbf{k}/2)] - f_a(\mathbf{p} - \hbar\mathbf{k}/2)[1 - f_a(\mathbf{p} + \hbar\mathbf{k}/2)]}{\hbar z - \Delta_k E_a(\mathbf{p})} \quad (6)$$

is the dielectric function of the quantum plasma. In that expression they only added and subtracted the products of the distribution functions, and actually did not take into account the exchange interaction. Thus in the quantum Balescu-Lenard kinetic equations the exchange interaction of particles was retained only in the distribution functions. But it is also necessary to take into account the exchange interaction in the scattering amplitude and in the dielectric function.

The Balescu-Lenard kinetic equations (classical as well as quantum) take into account the polarization of the system only in the collision integral, while the thermodynamics corresponds to the ideal gas; so the dissipative and non-dissipative phenomena are not treated on equal footing. This discrepancy can be avoided if non-Marcov effects are considered. In the plasma approximation we solved the equation for the pair correlation function considering the first non-Markovian correction. In this way, we obtained a nonlinear kinetic equation which generalizes the Balescu equation for spatially uniform [7] and spatially non-uniform [8] weakly non-ideal polarizable plasma. This equation, which included the

dynamical screening of the interaction potential, described correctly the conservation of the total energy in a non-trivial way.

Starting with the BBGKY-hierarchy for the quantum distribution functions, we have obtained the quantum kinetic equation including the dynamical screening of the interaction potential, which exactly takes into account the exchange scattering in a non-trivial way. The potential energy takes into account the polarization and exchange interaction too.

2 The quantum BBGKY-hierarchy

The quantum hierarchy for a multicomponent plasma in the operator techniques takes the form

$$\frac{\partial}{\partial t} f_a(1) = [H_a(1), f_a(1)] + \sum_b Sp_{(2)} \left[U_{ab}(12), f_{ab}(12) \right], \tag{7}$$

$$\frac{\partial}{\partial t} f_{ab}(12) = [H_{ab}(12), f_{ab}(12)] + \sum_c Sp_{(3)} \left[U_{ac}(13) + U_{bc}(23), f_{abc}(123) \right], \tag{8}$$

where $f_a(1)$ and $f_{ab}(12)$ are one- and two- particle density matrices, $[A, B]$ is the Poisson bracket.

$$H_a(1) = \frac{p^2(1)}{2m_a} \tag{9}$$

is the kinetic energy,

$$H_{ab}(12) = \frac{p^2(1)}{2m_a} + \frac{p^2(2)}{2m_b} + U_{ab}(12) \tag{10}$$

is the two-particle Hamiltonian, and $U_{ab}(12)$ is the two-particle interaction potential.

Let us introduce the new operators [9]:

$$f_{ab}(12) = \gamma_{ab}(12) \widehat{f}_{ab}(12), \tag{11}$$

$$f_{abc}(123) = \gamma_{abc}(123) \widehat{f}_{abc}(123), \tag{12}$$

where the symmetrization operators are

$$\gamma_{ab}(12) = 1 + \delta_{ab} \eta_a P(12), \tag{13}$$

$$\gamma_{abc}(123) = \gamma_{ab}(12) \left\{ 1 + \delta_{ac} \eta_a P(13) + \delta_{bc} \eta_b P(23) \right\}, \tag{14}$$

$\eta_a = 1(Bose), -1(Fermi)$; $P(12)$ is the permutation operator. Therefore,

$$\frac{\partial}{\partial t} f_a(1) = [H_a(1), f_a(1)] + \sum_b Sp_{(2)} \left[U_{ab}(12), \gamma_{ab}(12) \widehat{f}_{ab}(12) \right], \tag{15}$$

$$\frac{\partial}{\partial t} \widehat{f}_{ab}(12) = \left[H_{ab}(12), \widehat{f}_{ab}(12) \right] + \sum_c Sp_{(3)} \left[U_{ac}(13) + U_{bc}(23), (1 + \delta_{ac} \eta_a P(13) + \delta_{bc} \eta_b P(23)) \widehat{f}_{abc}(123) \right]. \tag{16}$$

The symmetrization operators (13), (14) are convenient in that they give the possibility to partially transmitting the permutation operator $P(12)$ from the density matrix to the interaction potentials. The density matrices $f_{ab}(12)$ etc possess the quantum symmetry properties: $P(12)f_{ab}(12) = f_{ab}(12)P(12)$ etc, whereas the density matrices $\widehat{f}_{ab}(12)$ etc possess only the classical symmetry properties: $P(12)\widehat{f}_{ab}(12)P(12) = \widehat{f}_{ab}(12)$ etc. For the classically symmetric density matrices the usual conditions for disentanglement of equations hold, which are the same as those in the classical statistics. Specifically, in the plasma approximation [10], when the triple correlation function is neglected,

$$\widehat{f}_{ab}(12) = f_a(1)f_b(2) + \widehat{g}_{ab}(12), \tag{17}$$

$$\widehat{f}_{abc}(123) = f_a(1)f_b(2)f_c(3) + \widehat{g}_{ab}(12)f_c(3) + \widehat{g}_{ac}(13)f_b(2) + \widehat{g}_{bc}(23)f_a(1), \tag{18}$$

where $\widehat{g}_{ab}(12)$ is the pair correlation function. By substituting (11), (12), (17) and (18) into (7) and (8) one obtains a closed set of equations for the one-particle distribution function and two-particle correlation function.

$$\frac{\partial}{\partial t}f_a(1) = [H'_a(1), f_a(1)] + \sum_b Sp_{(2)}\left[U'_{ab}(12), \gamma_{ab}(12)\widehat{g}_{ab}(12)\right], \tag{19}$$

$$\frac{\partial}{\partial t}\widehat{g}_{ab}(12) = [H'_a(1) + H'_b(1), \widehat{g}_{ab}(12)] + A'_{ab}(12)$$

$$+ \sum_c Sp_{(3)}\{[U'_{bc}(23), f_b(2)\widehat{g}_{ac}(13)] + [U'_{ac}(13), f_a(1)\widehat{g}_{bc}(23)]\}, \tag{20}$$

where

$$H'_a(1) = \frac{p^2(1)}{2m_a} + U_a(1), \tag{21}$$

$$U_a(1) = \sum_b Sp_{(2)}\left[U'_{ab}(12), f_b(2)\right] \equiv U_a^H(1) + U_a^F(1), \tag{22}$$

$$U'_{ab}(12) = \gamma_{ab}(12)U_{ab}(12), \tag{23}$$

$$iA'_{ab}(12) = [1 + \eta_a f_a(1)][1 + \eta_b f_b(2)]U_{ab}(12)f_a(1)f_b(2)$$

$$- f_a(1)f_b(2)U_{ab}(12)[1 + \eta_a f_a(1)][1 + \eta_b f_b(2)], \tag{24}$$

$$U_a^H(1) = \sum_b Sp_{(2)}\left[U_{ab}(12), f_b(2)\right] \tag{25}$$

is the Hartree field, i.e. mean self-consistent field [11] and

$$U_a^F(1) = \sum_b Sp_{(2)}\delta_{ab}\eta_a P(12)\left[U_{ab}(12), f_b(2)\right] \tag{26}$$

is the Fock field, mean field, taking into account only exchange interaction (Pauli's principle) [11].

In the plasma approximation in (20) the term $[U'_{ab}(12), \widehat{g}_{ab}(12)]$ which describes the direct interaction of two particles (1,2) is not taken into account.

Let us consider the homogeneous case, $U_a^{ext}(1) = 0$. Assume $f_a(1)$ to be diagonal with respect to spin variables. In the p-representation (19) and (20) take the form

$$\frac{\partial}{\partial t} f_a(p_1) = \frac{2}{\hbar}\ Im \sum_b \int pp_2 U_{ab}(p)\{\langle p_1 + p\sigma_1, p_2 - p\sigma_2 |\, \widehat{g}_{ab}\, | p_1\sigma_1, p_2\sigma_2\rangle$$

$$+\delta_{ab}\eta_a \langle p_2 - p\sigma_2, p_1 + p\sigma_1 |\, \widehat{g}_{ab}\, | p_1\sigma_1, p_2\sigma_2\rangle\} = J_a(p_1) \tag{27}$$

$$\frac{\partial}{\partial t} \langle p_1, p_2\, |\widehat{g}_{ab}|\, p_1', p_2'\rangle = [E_a(p_1) + E_b(p_2) - E_a(p_1') - E_a(p_2')] \langle p_1, p_2\, |\widehat{g}_{ab}|\, p_1', p_2'\rangle$$

$$+\sum_c \int p_3 p_3' \{\langle p_2, p_3\, |U_{bc}'|\, p_2', p_3'\rangle\ f_b(p_2') \langle p_1, p_3'\, |\widehat{g}_{ab})|\, p_1', p_3\rangle$$

$$- \langle p_1, p_3\, |\widehat{g}_{ac}|\, p_1', p_3'\rangle\ f_b(p_2') \langle p_2, p_3'\, |U_{bc}'|\, p_2', p_3\rangle + \langle p_1, p_3\, |U_{ac}'|\, p_1', p_3'\rangle\ f_a(p_1') \langle p_2, p_3'\, |\widehat{g}_{bc}|\, p_2', p_3\rangle$$

$$- \langle p_2, p_3\, |\widehat{g}_{bc}|\, p_2', p_3'\rangle\ f_a(p_1) \langle p_1, p_3'\, |U_{ac}'|\, p_1', p_3\rangle\} + i\hbar \langle p_1, p_2\, |A_{ab}'|\, p_1', p_2'\rangle \tag{28}$$

where $p \equiv (\mathbf{p}, \sigma)$, $\int p \equiv \sum\limits_\sigma \int \mathbf{p}$

$$E_a(p_1) = T_a(p_1) + \eta_a \int p_2 U_{aa}(\frac{\mathbf{p}_1 - \mathbf{p}_2}{\hbar}) f_a(p_2) \tag{29}$$

is the energy of the quasiparticle,

$$\langle p_1, p_2\, |U_{ab}'|\, p_1', p_2'\rangle$$

$$= [U_{ab}(\frac{\mathbf{p}_1 - \mathbf{p}_1'}{\hbar}) + \delta_{ab}\eta_a U_{ab}(\frac{\mathbf{p}_1 - \mathbf{p}_2}{\hbar})](2\pi\hbar)^3 \delta_{\sigma_1\sigma_1'}\delta_{\sigma_2\sigma_2'}\delta(p_1 + p_2 - p_1' - p_2') \tag{30}$$

$$i\hbar \langle p_1, p_2\, |A_{ab}'|\, p_1', p_2'\rangle = \langle p_1, p_2\, |U_{ab}|\, p_1', p_2'\rangle \{f_a(p_1') f_b(p_2')[1 + \eta_a f_a(p_1)][1 + \eta_b f_b(p_2)]$$

$$- f_a(p_1) f_b(p_2)[1 + \eta_a f_a(p_1')][1 + \eta_b f_b(p_2')]\} \tag{31}$$

In the Wigner representation the kinetic equation (27) takes the form:

$$\frac{\partial}{\partial t} f_a(\mathbf{p}) = J_a(\mathbf{p}) = 2\hbar^2 \sum_b \int \mathbf{p}'\mathbf{k}[U_{ab}(\mathbf{k}) + \delta_{ab}\eta_a U_{ab}(\mathbf{p}' - \mathbf{p})]\widehat{g}_{ab}(\mathbf{p}, \mathbf{p}', \mathbf{k}). \tag{32}$$

Here the spin variables are omitted for simplicity.

3 The collision integral for quantum plasma with polarization and exchange interaction

The expressions for the collision integral and the internal energy take the forms

$$J_a(p)$$

$$= -2\hbar Imi \sum_{bc} e_a e_b e_c \int \Phi(\mathbf{k})\mathbf{p}'\mathbf{k} z\mathbf{q}[\Phi(\mathbf{k}) + \delta_{ab}\eta_a \Phi(\frac{\mathbf{p}' - \mathbf{p}}{\hbar})] f_c(\mathbf{q} + \frac{\hbar\mathbf{k}}{2})[1 + \eta_a f_c(\mathbf{q} - \frac{\hbar\mathbf{k}}{2})]$$

$$\{\frac{\Psi_a^{(1)}(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})}{\varepsilon^{HF}(z,\mathbf{k})}[\frac{\Gamma_b(\mathbf{p}'+\frac{\hbar\mathbf{k}}{2},\mathbf{q})\delta_{bc}}{\hbar z-\Delta_k E_c(\mathbf{q})}+\frac{\Phi(\mathbf{k})}{\varepsilon^{HF}(z,\mathbf{k})}\Psi_b^{(1)}(\mathbf{p}'+\frac{\hbar\mathbf{k}}{2})\Psi_c^{(2)}(\mathbf{q})]^*$$

$$-\frac{\Psi_b^{*(1)}(\mathbf{p}'+\frac{\hbar\mathbf{k}}{2})}{\varepsilon^{*HF}(z,\mathbf{k})}[\frac{\Gamma_a(\mathbf{p}+\frac{\hbar\mathbf{k}}{2},\mathbf{q})\delta_{ac}}{\hbar z-\Delta_k E_c(\mathbf{q})}+\frac{\Phi(\mathbf{k})}{\varepsilon^{HF}(z,\mathbf{k})}\Psi_a^{(1)}(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})\Psi_c^{(2)}(\mathbf{q})]\}, \tag{33}$$

$$U$$

$$=2Rei\sum_{abc}\int\frac{e_ae_be_c}{\hbar}\mathbf{p}\ \mathbf{p}'\mathbf{k}z\mathbf{q}\Phi(\mathbf{k})[\Phi(\mathbf{k})+\delta_{ab}\eta_a\Phi(\frac{\mathbf{p}'-\mathbf{p}}{\hbar})]f_c(\mathbf{q}+\frac{\hbar\mathbf{k}}{2})[1+\eta_a f_c(\mathbf{q}-\frac{\hbar\mathbf{k}}{2})]$$

$$\frac{1}{\varepsilon^{*HF}(z,\mathbf{k})}\Psi_b^{*(1)}(\mathbf{p}')[\frac{\Gamma_a(\mathbf{p},\mathbf{q})\delta_{ac}}{\hbar z-\Delta_k E_c(\mathbf{q})}+\frac{\Phi(\mathbf{k})}{\varepsilon^{HF}(z,\mathbf{k})}\Psi_a^{(1)}(\mathbf{p})\Psi_c^{(2)}(\mathbf{q})], \tag{34}$$

where the functions $\Psi_{a'}^{(1)}(\mathbf{p}')$, $\Psi_{a'}^{(2)}(\mathbf{p}')$, $\varepsilon^{HF}(\omega,\mathbf{k})$, $\Gamma_a(\mathbf{p},\mathbf{p}')$ are determined by (53), (54), (55), (56) (see the appendix). The complete description of the exchange interaction of particles is reduced to the solution of the linear integral equation (56)).

The collision integral (33) and the internal energy (34) are expressed by the amplitude of the scattering interaction $\Gamma_a(\mathbf{p},\mathbf{p}')$ which satisfies the linear integral equation (56). The solution of this equation in the case of Coulomb interaction of the particles is difficult and requires an appropriate approximation. The simplest approximation is the replacement of the expression under the integral of (56) by the averaged over the impulse value

$$\Phi(\mathbf{k})G(z,\mathbf{k})=\int\mathbf{p}''\Phi(\frac{\mathbf{p}-\mathbf{p}''}{\hbar})\Gamma_a(\mathbf{p}'',\mathbf{p}'). \tag{35}$$

Then, the dielectric function taking into account exchange interaction particles, takes the form:

$$\varepsilon^{HF}(z,\mathbf{k})=1-P(z,\mathbf{k})[1+P(z,\mathbf{k})G(z,\mathbf{k})]^{-1}, \tag{36}$$

where

$$P(z,\mathbf{k})=\Phi(\mathbf{k})\sum_a e_a^2\int\mathbf{p}\frac{\Delta_k f_a(\mathbf{p})}{\hbar z-\Delta_k E_a(\mathbf{p})} \tag{37}$$

is the polarization.

One form of $G(z,\mathbf{k})$ was found using a variation procedure in the problem of dielectric function of Hhelium [12]

$$G(z,\mathbf{k})=\frac{8\pi^2e^4\hbar^4}{k^2}\frac{1}{P^2(z,\mathbf{k})}\int\mathbf{p}\mathbf{p}'\frac{\Delta_{\mathbf{k}}f(\mathbf{p})\Delta_{\mathbf{k}}f(\mathbf{p}')}{|\mathbf{p}-\mathbf{p}'|^2}$$

$$\times\frac{1}{\hbar z-\Delta_k E_a(\mathbf{p})}[\frac{1}{\hbar z-\Delta_k E_a(\mathbf{p}')}-\frac{1}{\hbar z-\Delta_k E_a(\mathbf{p})}] \tag{38}$$

Using the expression for the pair correlation function, we find the Markovian collision integral

$$J_a(\mathbf{p})=4\pi^2e^4\int\Phi^2(\mathbf{k})[1-G(\Delta_k E(\mathbf{q})/\hbar,\mathbf{k})]\mathbf{k}\mathbf{q}\frac{\delta(\Delta_k E(\mathbf{q})-\Delta_k E(\mathbf{p}))}{\left|\widetilde{\varepsilon}(\Delta_k E(\mathbf{q})/\hbar,\mathbf{k})\right|^2}$$

$$\{f(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})f(\mathbf{q}-\frac{\hbar\mathbf{k}}{2})[1-f(\mathbf{p}-\frac{\hbar\mathbf{k}}{2})][1-f(\mathbf{q}+\frac{\hbar\mathbf{k}}{2})]- \tag{39}$$

$$-f(\mathbf{p}-\frac{\hbar\mathbf{k}}{2})f(\mathbf{q}+\frac{\hbar\mathbf{k}}{2})[1-f(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})][1-f(\mathbf{q}-\frac{\hbar\mathbf{k}}{2})]\}, \tag{40}$$

and the internal energy of particles

$$U = e^4 \int \mathbf{pp'k} \frac{\Phi^2(\mathbf{k})}{\Delta_k E(\mathbf{q}) - \Delta_k E(\mathbf{p})} \frac{1}{\left|\widetilde{\varepsilon}(\Delta_k E(\mathbf{q})/\hbar, \mathbf{k})\right|^2}$$

$$\{f(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})f(\mathbf{p'}-\frac{\hbar\mathbf{k}}{2})[1-f(\mathbf{p}-\frac{\hbar\mathbf{k}}{2})][1-f(\mathbf{p'}+\frac{\hbar\mathbf{k}}{2})]- \tag{41}$$

$$-f(\mathbf{p}-\frac{\hbar\mathbf{k}}{2})f(\mathbf{p'}+\frac{\hbar\mathbf{k}}{2})[1-f(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})][1-f(\mathbf{p'}-\frac{\hbar\mathbf{k}}{2})]\}, \tag{42}$$

where

$$\widetilde{\varepsilon}(z,\mathbf{k}) = 1 - (1 - G(z,\mathbf{k}))P(z,\mathbf{k}). \tag{43}$$

From the expressions for the internal energy of the particles (41) and for the collision integral (39) follows that $\left|\widetilde{\varepsilon}(z,\mathbf{k})\right|^2$ plays the role of the screening of the interaction potential $\Phi(\mathbf{k})$. It is interesting to note that (39) and (41) are different from the corresponding Balescu expressions in taking into account the exchange interaction in this screening. Moreover the collision integral (39) contains the additional renormalization of the interaction $(1 - G(z,\mathbf{k}))$. However $\widetilde{\varepsilon}(z,\mathbf{k})$ does not serve as linear response function, in contrast to the Hartree-Fock dielectric function $\varepsilon^{HF}(z,\mathbf{k})$ in (36).

In the equilibrium state, expression (41) satisfies the fluctuation-dissipation theorem since

$$Im\frac{P(z,\mathbf{k})}{\left|\widetilde{\varepsilon}(z,\mathbf{k})\right|^2} = Im\frac{P(z,\mathbf{k})}{\varepsilon^{HF}(z,\mathbf{k})}. \tag{44}$$

4 Conclusion

Using the operator technique within BBGKY hierarchy we obtained a closed set of equations for the one- and two- particle density matrices, referring to the plasma approximation which considers also the exchange interaction. The equation for the pair correlation function is solved with the help of the resolvent of the Hartree-Fock equation. The expression obtained for the pair correlation function takes into account the exchange interaction. The latter is described in the terms of the scattering amplitude which is subject to the integral equation formulated above. The expression for the collision integral and the internal energy are obtained with the exchange interaction and polarization taken into account. The kinetic equation obtained is free of divergencies both at small and large wave numbers. It completely incorporates the exchange interaction between particles both in the renormalization of the potential and of the dielectric permeability. The latter is particularly important in systems with finite number of electrons.

Appendix 1

The solution of the equation for the pair correlation function $\widehat{g}_{ab}(\mathbf{p},\mathbf{p}',\mathbf{k})$ in the Wigner form can be expressed in the spatially homogeneous case in terms of the resolvent (28) and its source (31):

$$\widehat{g}_{ab}(p,p',\mathbf{k},t)$$

$$= \sum_{a'b'} \int \mathbf{q}\mathbf{q}' R_{ab,a'b'}(\mathbf{p},\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},z,\mu t) A'_{a'b'}(\mathbf{q},\mathbf{q}',\mathbf{k},\mu t)\rfloor_{z=0}, \tag{45}$$

$z = \omega + i\Delta,$

$$i\hbar A'_{ab}(\mathbf{p},\mathbf{p}',\mathbf{k},\mu t) = U_{ab}(\mathbf{k})\{f_a(\mathbf{p})f_b(\mathbf{p}')[1+\eta_a f_a(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})][1+\eta_b f_b(\mathbf{p}'-\frac{\hbar\mathbf{k}}{2})]$$

$$-f_a(\mathbf{p}+\frac{\hbar\mathbf{k}}{2})f_b(\mathbf{p}'-\frac{\hbar\mathbf{k}}{2})[1+\eta_a f_a(\mathbf{p})][1+\eta_b f_b(\mathbf{p}')]\} \tag{46}$$

with the resolvent $R_{ab,a'b'}(\mathbf{p},\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},z,\mu t)$ in (45) being a product of two resolvents

$$R_{ab,a'b'}(\mathbf{p},\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},t-t',\mu t') = R_{aa'}(\mathbf{p},\mathbf{q},\mathbf{k},t-t',\mu t')R_{bb'}(\mathbf{p}',\mathbf{q}',\mathbf{k},t-t',\mu t'), \tag{47}$$

which satisfy the linearized Hartree-Fock equation

$$[\hbar z + \Delta_k E_a(\mathbf{p})]R_{aa'}(\mathbf{p},\mathbf{q},\mathbf{k},z,t) = \delta_{aa'}\delta(\mathbf{p}-\mathbf{q})$$

$$+e_a\Delta_k f_a(\mathbf{p})\sum_c e_c \int \mathbf{p}'[\Phi(\mathbf{k}) + \delta_{ac}\eta_a\Phi(\frac{\mathbf{p}-\mathbf{p}'}{\hbar})]R_{ca'}(\mathbf{p}',\mathbf{q},\mathbf{k},z,t), \tag{48}$$

where

$$U_{ab}(\mathbf{k}) = e_a e_b \Phi(\mathbf{k}) \tag{49}$$

$$\Delta_k f_a(\mathbf{p}) = f_a(\mathbf{p}+\frac{\hbar\mathbf{k}}{2}) - f_a(\mathbf{p}-\frac{\hbar\mathbf{k}}{2}), \tag{50}$$

$$E_a(\mathbf{p}) = \frac{\mathbf{p}^2}{2m_a} + \eta_a \int d\mathbf{p}' U_{aa}(\frac{\mathbf{p}-\mathbf{p}'}{\hbar}) f_a(\mathbf{p}'). \tag{51}$$

The solution of (48) takes the form

$$R_{aa'}(\mathbf{p},\mathbf{p}',\mathbf{k},z,t) = \frac{\Gamma_a(\mathbf{p},\mathbf{p}')\delta_{aa'}}{\hbar z - \Delta_k E_{a'}(\mathbf{p}')} + \frac{\Phi(\mathbf{k})}{\varepsilon^{HF}(\omega,\mathbf{k})}\Psi_a^{(1)}(\mathbf{p})\Psi_{a'}^{(2)}(\mathbf{p}'), \tag{52}$$

where we introduced the notations

$$\Psi_a^{(1)}(\mathbf{p}) = e_a \int \mathbf{p}'' \frac{\Gamma_a(\mathbf{p},\mathbf{p}'')\Delta_k f_a(\mathbf{p}'')}{\hbar z - \Delta_k E_a(\mathbf{p}'')}, \tag{53}$$

$$\Psi_{a'}^{(2)}(\mathbf{p}') = e_{a'} \int \mathbf{p}'' \frac{\Gamma_{a'}(\mathbf{p}'',\mathbf{p}')}{\hbar z - \Delta_k E_{a'}(\mathbf{p}')}, \tag{54}$$

and

$$\varepsilon^{HF}(\omega,\mathbf{k}) = 1 - \Phi(\mathbf{k}) \sum_a e_a^2 \int \mathbf{p}\mathbf{p}' \frac{\Gamma_a(\mathbf{p},\mathbf{p}')\Delta_k f_a(\mathbf{p}')}{\hbar z - \Delta_k E_a(\mathbf{p}')} \tag{55}$$

is the dielectric function with exchange interaction.

The exchange scattering amplitude $\Gamma_a(\mathbf{p},\mathbf{p}')$ for (52-55) satisfies an integral equation, which contains only the exchange interaction potential:

$$\Gamma_a(\mathbf{p},\mathbf{p}') = \delta(\mathbf{p}-\mathbf{p}') + e_a^2\eta_a \frac{\Delta_k f_a(\mathbf{p})}{\hbar z - \Delta_k E_a(\mathbf{p})} \int d\mathbf{p}''\Phi(\frac{\mathbf{p}-\mathbf{p}''}{\hbar})\Gamma_a(\mathbf{p}'',\mathbf{p}'). \tag{56}$$

$\Gamma_a(\mathbf{p},\mathbf{p}')$ depends on $\mathbf{k}$ and z as on parameters and is similar to the vertex-function, well-known in many-particle perturbation theory.

The formulae (52-56) yield the general expression for the pair correlation function with complete description of the polarization and the exchange interaction of the particles:

$$g'_{ab}(\mathbf{p},\mathbf{p}',\mathbf{k},t) = -\frac{i}{\hbar}\Phi(\mathbf{k}) \sum_c e_c \int z\mathbf{q} f_c(\mathbf{q}+\frac{\hbar\mathbf{k}}{2})[1+\eta_a f_c(\mathbf{q}-\frac{\hbar\mathbf{k}}{2})]$$

$$\{\frac{\Psi_a^{(1)}(\mathbf{p})}{\varepsilon^{HF}(z,\mathbf{k})}[\frac{\Gamma_b(\mathbf{p}',\mathbf{q})\delta_{bc}}{\hbar z - \Delta_k E_c(\mathbf{q})} + \frac{\Phi(\mathbf{k})}{\varepsilon^{HF}(z,\mathbf{k})}\Psi_b^{(1)}(\mathbf{p}')\Psi_c^{(2)}(\mathbf{q})]^*$$

$$-\frac{1}{\varepsilon^{*HF}(z,\mathbf{k})}\Psi_b^{*(1)}(\mathbf{p}')[\frac{\Gamma_a(\mathbf{p},\mathbf{q})\delta_{ac}}{\hbar z - \Delta_k E_c(\mathbf{q})} + \frac{\Phi(\mathbf{k})}{\varepsilon^{HF}(z,\mathbf{k})}\Psi_a^{(1)}(\mathbf{p})\Psi_c^{(2)}(\mathbf{q})]\}. \tag{57}$$

References

[1] Balescu R 1960 *Phys. of Fluids* **3** 52

[2] Lenard A 1960 *Ann. Phys. (NY)* **10** 390

[3] Vlasov A A 1938 *Sov. Phys. - JETP* **8** 291

[4] Landau L D 1936 *Phys. Zs. Sow. Union* **10** 154

[5] Balescu R 1961 *Phys. of Fluids* **4** 94

[6] Guernsey R L 1962 *Phys. Rev. Lett.* **127** 1446

[7] Belyi V V, Kukharenko Yu A and Wallenborn J 1996 *Phys. Rev. Lett.***76** 3554

[8] Belyi V V, Kukharenko Yu A and Wallenborn J 2002 *Contrib. Plasma Phys* **42** 3

[9] Bogoliubov N 1947 *J. Phys. (Moscow)* **11** 23

[10] Balescu R 1963 *Statistical Mechanics of Charged Particles* (Wiley - Interscience, New York)

[11] Kadanoff L P and Baym G 1964 *Quantum statistical mechanics* (Benjamin, New York)

[12] Brosens F, Lemmens L M and Devrees J T 1977 *Phys. Stat. Sol. (b)* **81** 551