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Dispersion equation in kinetic model for multispecies plasma with radiofrequency waves

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1 Introduction

The issue of transport in tokamak plasmas due to micro and macroinstabilities rests a major problem in physics of plasma fusion with many complex difficulties to solve. Besides turbulence, plasma interact with radio-frequency waves, largely used to heating plasma, which infuence the transport. Electron cyclotron resonance heating (ECRH) as well as ion cyclotron resonance heating (ICRH) power modulation have shown the effect of heating on the anomalous pinch but also on the impurity transport as shown for example in [1] and [2] respectively.

The quasi-neutrality condition represents one of the fundamental equations used to describe the transport processes in plasma and permit to obtain the dispersion relation, see for example [3], [4], [5]. The goal of this paper is to analyse the quasi-neutrality equation for plasma with radiofrequency heating.

The paper use the quasilinear transport theory, largely used to study the turbulent transport in plasmas. Quasilinear refer here to using a quasilinear gyrokinetic equation in describing fluctuating distribution function.

2 The kinetic equation

We consider a non-ohmic multi-component plasma heated at the ion cyclotron resonance for species i and weakly turbulent due to a fluctuating electrostatic field $\delta\mathbf{E}$.

The kinetic equation for the particles of species i that applies in such conditions is written as (see for example [6]) :

$$\partial_t f^{\ i} + \mathbf{v}\cdot\boldsymbol{\nabla} f^{\ i} + (L_0^i + \delta L^i) f^{\ i} = C^\alpha\left(f^{\ i}, f^{\ j}\right) + Q\left(f^{\ i}\right) \tag{1}$$

where the force operator has been split into two contributions: one, L_0^i, due to the equilibrium electric and magnetic field

$$L_0^i \equiv \frac{e_i}{m_i}\left(\mathbf{E}_0 + \frac{1}{c}\mathbf{v}\times\mathbf{B}_0\right)\cdot\frac{\partial}{\partial\mathbf{v}} \tag{2}$$

and the second one, $\delta L^i \equiv \delta L_t^i + \delta L_{rf}^i$, arising from the fluctuating electric field and RF electromagnetic field

$$\delta L_t^i \equiv \frac{e_i}{m_i}\delta\mathbf{E}\cdot\frac{\partial}{\partial\mathbf{v}} \quad , \quad \delta L_{rf}^i \equiv \frac{e_i}{m_i}\left(\delta\mathbf{E}_{rf} + \frac{1}{c}\mathbf{v}\times\delta\mathbf{B}_{rf}\right)\cdot\frac{\partial}{\partial\mathbf{v}} \tag{3}$$

with e_i and m_i the charge and mass of the particles of species i. The non-canonical phase-space variables are the particle's position $\mathbf{x}$ and velocity $\mathbf{v}$. Here $\delta\mathbf{E} = -\nabla\delta\Phi$ since we assume an electrostatic turbulent field. The operator C^{α} is the collision operator and Q the term which describe dissipation due to rf waves.

The distribution function is splited into two contributions: $f^i = F^i + \delta f^i$; the first one, F^i corresponding to the ensemble averaged part and the second one, δf^i, corresponding to the rapid varying part part of the distribution (due to both turbulence and rf waves). All the calculations will be performed, assuming, for simplicity, an axisymmetric model for equilibrium magnetic field for a large aspect ratio tokamak with circular concentric magnetic surfaces

$$\mathbf{B} = B_0 \frac{R_0}{R} \left(-\frac{r}{q_s R_0} \mathbf{e}_\theta + \mathbf{e}_\zeta \right)$$

where R_0 is the major radius, r the toroidal radial coordinate, $R = R_0 + r\cos\theta$ the cilindrical radial coordinate and $q(r) = |rB_\zeta / R_0 B_\theta|$ the safety factor, see for example [7]. The magnitude of the equilibrium magnetic field, see also [8], is

$$B(r) = \frac{\mathcal{B}_0(r)}{1 + \eta\cos\theta} = \frac{B_0}{1 + \eta\cos\theta} \sqrt{1 + \eta^2 q^{-2}} \tag{4}$$

with $-\epsilon \leq \eta \leq \epsilon$, $\eta = r/R_0$, $\epsilon = a/R_0$ the inverse aspect ratio and $B_0 = B(r = 0)$.

By ensamble averaging the kinetic equation (1) results

$$\partial_t F^i + \mathbf{v} \cdot \boldsymbol{\nabla} F^i + L_0^i F^i + \left\langle \delta L^i \delta f^i \right\rangle_{ens} = C^i \left(F^i, F^j\right) + Q\left(F^i\right) \tag{5}$$

We writte F^i as $F^i = F_0^i + F_1^i$ where F_0^i is assumed Maxwellian and F_1^i is the deviation from Maxwellian which depends on both turbulence and particles - RF waves interaction.

2.1 Kinetic equation for perturbed distribution function

Neglecting the quantity $\delta L^i \delta f^{\ i} - \langle \delta L^i \delta f^{\ i} \rangle_{ens}$ and approximating the collision term as Krook's collision [5], result the kinetic equation for the perturbed distribution function

$$\partial_t \delta f^i + \mathbf{v} \cdot \boldsymbol{\nabla} \delta f^i + L_0^i \delta f^i + \delta L^i F_0^i = -\nu_{eff} \left(\delta f^i + \frac{e_i F_0^i}{T_i} \delta\Phi \right) - \nu_{rf} \delta f^i \tag{6}$$

where ν_{eff} is the effective collision frequency and ν_{rf} the rate of the rf- dissipation. The fluctuating part of the distribution function δf^i is linearly approximated as $\delta f^i = \delta f^{i,t} + \delta g^i$ where $\delta f^{i,t}$ refers to the solution in absence of RF waves and δg^i is the solution in absence of turbulence. We must note that the time scales variation of $\delta f^{i,t}$ is usual more slowly than time scales variation of δg^i because the frequency ω for turbulent instability is usual much smaller than frequency ω_{rf} of RF waves. This justify the linear separation of the two process.

2.1.1 Solution for perturbed distribution function in absence of RF heating

The kinetic equation (6) for ions of species i in absence of RF-heating has the formal solution

$$\delta f^{\ i,t} = -\frac{e_i F_0^i}{T_i} \delta\Phi + \delta h_i \tag{7}$$

where, see for example [4], [5], the first and the second term are named, respectively, the adiabatic and non-adiabatic part. The gyrokinetic equation for the non-adiabatic part is then

$$\partial_t \delta h_i + \mathbf{v} \cdot \boldsymbol{\nabla} \delta h_i + L_0^i \delta h_i + \delta L^i F_0^i = -\nu_{eff} \delta h_i \tag{8}$$

With the balooning-mode representation

$$\delta f_n = \sum_{m=-\infty}^{\infty} \exp\left[i\left(m\theta + k_r r\right)\right] \int_{-\infty}^{\infty} d\theta' \exp\left(-im\theta'\right) \exp\left[-in\left(\zeta - q\theta'\right) - i\omega t\right] \delta f_n\left(\theta'\right) \tag{9}$$

where ζ is the toroidal angle and θ the extended poloidal angle, the gyrokinetic equation (8) leads to, see for example [9]:

$$i\frac{v_{\|}}{qR}\frac{\partial \delta h^i_{\mathbf{k},\omega}}{\partial \theta} + \left(\omega - \omega_D + i\nu_{eff}\right)\delta h^i_{\mathbf{k},\omega} = \left(\omega - \omega_{*T}\right) J_0\left(\alpha\right) F_M \frac{e_i \delta\phi_{\mathbf{k},\omega}}{T_i} \tag{10}$$

Using dimensionless variable for velocities, $u = v/V_{Ti}$, with $V_{Ti} = \sqrt{2T_i/m_i}$ the thermal velocity of ions species i, the Maxwellian distribution function reads as $F^i_M = n_{0i}\left(\pi V^2_{Ti}\right)^{-3/2} \exp\left(-u^2\right)$. We introduce notation

$$\begin{aligned}
\omega_D &= \frac{2}{\tau_i}\epsilon_n \omega_{*e}\left(\cos\theta + \widehat{s}\theta\sin\theta\right)\left(u^2_{\|} + u^2_{\perp}/2\right) \\
\omega_{*T} &= -\frac{1}{\tau_e}\omega_{*e}\left[1 + \eta_i\left(u^2_{\|} + u^2_{\perp} - 3/2\right)\right]
\end{aligned}$$

where $\omega_{*e} = ck_\theta T_e/eBL_n$ is the electron diamagnetic drift frequency, L_n (L_T) the density (temperature) gradient scale length, $\eta_i = L_{ni}/L_{T_i}$, $\epsilon_n = L_{ni}/R$, $\tau_i = T_e/T_i$, $\widehat{s}$ the magnetic shear, Ω_i the ion gyrofrequency and $J_0\left(\alpha\right)$ is the Bessel function of zeroth order with $\alpha = \sqrt{2b_i}u_\perp$, $b_i = V^2_{Ti}k^2_\perp/2\Omega^2_i$.

The solution for the non-adiabatic part of the perturbed distribution function $\delta h^i_{\mathbf{k},\omega}$ (in electrostatic case) reads as

$$\delta h^i_{\mathbf{k},\omega} = -i\int_{-\infty}^{\infty} d\theta' \exp\left[i\left(\beta - \beta'\right) sgn\left(\theta - \theta'\right)\right] \frac{qR}{\left|v_{||}\right|}\left(\omega - \omega_{*T}\right) J_0\left(\alpha'\right) F_M \frac{e_i\delta\phi_{\mathbf{k},\omega}}{T_i} \tag{11}$$

with

$$\beta\left(\theta\right) = \int^{\theta} d\theta'' \frac{qR}{\left|v_{||}\right| V_{T_i}}\left(\omega - \omega_D + i\nu_{eff}\right)$$

Disregarding the dependence on θ , the solution of equation (10) - see [10] - is

$$\delta h^i_{\mathbf{k},\omega} = \frac{\omega - \omega_{*T}}{\omega - \omega_D + i\nu_{eff}} J_0\left(\alpha\right) F_M \frac{e_i\delta\phi_{\mathbf{k},\omega}}{T_i}$$

Hence, the perturbed distribution function in the absence of RF waves reads as

$$\begin{aligned}
\delta f^{\,i,t}_{\mathbf{k},\omega} &= -\frac{e_i\delta\phi_{\mathbf{k},\omega}}{T_i} J_0\left(\alpha\right) F^i \qquad (12)\\
&\quad -i\int_{-\infty}^{\infty} d\theta' \exp\left[i\left(\beta - \beta'\right) sgn\left(\theta - \theta'\right)\right] \frac{qR}{\left|v_{||}\right|}\left(\omega - \omega_{*T}\right) J_0\left(\alpha'\right) F_M \frac{e_i\delta\phi_{\mathbf{k},\omega}}{T_i}
\end{aligned}$$

2.1.2 Solution for perturbed distribution function in presence of RF waves

The solution for δg^i resulting from (6) in first order approximation is given as :

$$\delta g^i = -\frac{e_i}{m_i}\int_{-\infty}^{t} dt' \int d\mathbf{k} \int d\omega_{rf} \delta \mathbf{E}^{rf}_{\mathbf{k},\omega} \cdot \left[\mathbb{I}\left(1 - \frac{\mathbf{v}'\cdot\mathbf{k}_{rf}}{\omega_{rf}}\right) + \frac{\mathbf{v}'\mathbf{k}_{rf}}{\omega_{rf}} \right] \exp\left(i\mathbf{k}_{rf}\cdot\mathbf{r}' - i\omega_{rf}t\right) \cdot \frac{\partial F^i_M}{\partial \mathbf{v}}$$

where Maxwell's induction equation $\delta \mathbf{B}^{rf}_{\mathbf{k},\omega} = (\mathbf{k}_{rf}c/\omega_{rf}) \times \delta \mathbf{E}^{rf}_{\mathbf{k},\omega}$ was used and $\mathbb{I}$ is the unit diadic. With - see [11] pg 252 -

$$U = -\frac{2v_\perp}{m_i V_{Ti}^2} F^i_M \quad , \quad V = 0 \quad , \quad W = -\frac{2v_\parallel}{m_i V_{Ti}^2} F^i_M$$

we have

$$\begin{aligned} \delta g^i &= -e_i \exp\left(i\mathbf{k}_{rf}\cdot\mathbf{r} - i\omega_{rf}t\right) \int_0^\infty d\tau \ \exp\left(i\gamma\right) \left\{ E_x U \cos\left(\varphi + \Omega\tau\right) \right. \\ &\left. + E_y U \sin\left(\varphi + \Omega\tau\right) + E_z \left[\frac{1}{m_i} \frac{\partial F^i_0}{\partial v_\parallel} - V \cos\left(\varphi - \theta + \Omega\tau\right) \right] \right\} \end{aligned}$$

where

$$\gamma = -\frac{k_\perp v_\perp}{\Omega} \left[\sin\left(\varphi - \theta + \Omega\tau\right) - \sin\left(\varphi - \theta\right) \right] + \left(\omega - k_\parallel v_\parallel\right)\tau$$

In the following assume $\theta = 0$ and F^i_0 a Maxwellian; we obtaine

$$\begin{aligned} \delta g^i_{\mathbf{k},\omega} &= -e_i \int_0^\infty d\tau \ \exp\left[i\left(\omega - k_\parallel v_\parallel + n\Omega\right)\tau\right] \exp\left(-i\alpha\left[\sin\left(\varphi + \Omega\tau\right) - \sin\varphi\right]\right) \\ &\times \left\{ -\frac{2v_\perp F^i_M}{m_i V_{Ti}^2} \left[\delta E^{rf}_r \cos\left(\varphi + \Omega\tau\right) + \delta E^{rf}_\theta \sin\left(\varphi + \Omega\tau\right) \right] + \delta E^{rf}_\parallel \left[-\frac{2v_\parallel F^i_M}{m_i V_{Ti}^2} \right] \right\} \end{aligned}$$

where φ is the gyrophase angle.

The gyrokinetic equations must be completed with the Poisson equation, which for long-wave-length fluctuations (whose wave vector is much smaller than the Debye vector) degenerates into the electroneutrality condition

$$-e \ \delta n^e_{\mathbf{k},\omega} + \sum_i e_i \ \delta n^i_{\mathbf{k},\omega} = 0 \qquad (13)$$

where e is the positive elementary charge and, in the η_i mode study, the perturbed electron density $\delta n^e_{\mathbf{k},\omega}$ is taken to be the adiabatic response to the electrostatic perturbation, $\delta n^e_{\mathbf{k},\omega} = \left(e n_{0e}/T_e\right)\delta\phi_{\mathbf{k},\omega}$ and the perturbed ion density of species i in an axisymmetric toroidal geometry is given as $\delta n^i_{\mathbf{k},\omega} = \delta n^{i,t}_{\mathbf{k},\omega} + \delta n^{i,rf}_{\mathbf{k},\omega}$ where

$$\delta n^{i,t}_{\mathbf{k},\omega} = -\frac{e_i n_{0i}}{T_i}\delta\phi_{\mathbf{k},\omega} + 2\pi \int_{-\infty}^{\infty} dv_\parallel \int_0^\infty dv_\perp \ v_\perp J_0\left(\alpha\right) \ \delta h^i_{\mathbf{k},\omega} \qquad (14)$$

$$\delta n^{i,rf}_{\mathbf{k},\omega} = 2\pi \int_{-\infty}^{\infty} dv_\parallel \int_0^\infty dv_\perp \ v_\perp J_0\left(\alpha\right) \ \delta g^i_{\mathbf{k},\omega} \qquad (15)$$

With (11) results

$$\delta n^{i,t}_{\mathbf{k},\omega} = n_{0i}\left[-1+\int_{-\infty}^{\infty} d\theta' K_i\left(\theta,\theta'\right)\right]\frac{e_i\delta\phi_{\mathbf{k},\omega}}{T_i}$$

where

$$K_i\left(\theta,\theta'\right) = -2\pi i\int\limits_{-\infty}^{\infty} dv_{\parallel}\int\limits_{0}^{\infty} dv_{\perp}v_{\perp}J_0\left(\alpha\right)\exp\left[i\left(\beta-\beta'\right)sgn\left(\theta-\theta'\right)\right]\frac{qR}{\left|v_{\parallel}\right|}\left(\omega-\omega_{*T}\right)J_0\left(\alpha'\right)\frac{F_M}{n_{0i}} \tag{16}$$

Then, the electroneutrality condition will read as

$$\left(1+\sum_i z_i\right)\delta\phi_{\mathbf{k},\omega}\left(\theta\right) = \sum_i z_i\int_{-\infty}^{\infty} d\theta' K_i\left(\theta,\theta'\right)\delta\phi_{\mathbf{k},\omega}\left(\theta'\right) \tag{17}$$

with $z_i = Z_i^2\left(n_{0i}T_e/n_{0e}T_i\right)$.

2.2 Density perturbation driven by turbulence

We evaluate the density perturbation driven by turbulence from (14) whith $\delta h^i_{\mathbf{k},\omega}$ given in (11). This implicate evaluation of quantity

$$K_i\left(\theta,\theta'\right) = -i\frac{2qR}{\pi^{1/2}V_{Ti}}\int\limits_{-\infty}^{\infty}\frac{du_{\parallel}}{\left|u_{\parallel}\right|}\exp\left(-u_{\parallel}^2\right)I_{\perp}\left(u_{\parallel}\right)$$

where

$$I_{\perp}\left(u_{\parallel}\right) = \int\limits_{0}^{\infty} du_{\perp}\ u_{\perp}J_0\left(\alpha\right)J_0\left(\alpha'\right)\exp\left(-u_{\perp}^2\right)\exp\left[i\left(\beta-\beta'\right)sgn\left(\theta-\theta'\right)\right]\left(\omega-\omega_{*T}\right)$$

The density perturbation due to turbulence in lectrostatic limit will read as

$$\left[\delta n^{\alpha}_{\mathrm{t,es,na}}\right]_{\mathbf{k}\omega} = -\frac{1}{2}\left(\frac{2T_\alpha}{m_\alpha}\right)^{3/2}\frac{e_\alpha\delta\Phi_{\mathbf{k},\omega}}{T_\alpha}\int\limits_{0}^{\infty}dx\int\limits_{0}^{\lambda_c}d\lambda\frac{B\sqrt{x}}{\sqrt{1-\lambda B}}F^{\alpha}\frac{\omega-\omega^T_{*\alpha}\left(v\right)}{k_{\parallel}v_{\parallel}-\omega}J_0^{\alpha}\left(k_{\perp}v_{\perp}/\Omega_\alpha\right)$$

where $\lambda_c = 1/B_{\max}$.

2.3 Density perturbation driven by rf waves

In this case, the density perturbation is given by

$$\begin{aligned}\delta n^{i,rf}_{\mathbf{k},\omega} &= 4\pi\frac{e_i}{m_i}\sum_{n=-\infty}^{\infty}\int\limits_{-\infty}^{\infty}dv_{\parallel}\int\limits_{0}^{\infty}dv_{\perp}\ v_{\perp}\int\limits_{0}^{\infty}d\tau\ \exp\left[i\left(\omega-k_{\parallel}v_{\parallel}-n\Omega\right)\tau\right]\\ &\times\left\{v_{\perp}\left[\delta E^{rf}_r\frac{n}{\alpha}J_n^2\left(\alpha\right)+\delta E^{rf}_\theta iJ_nJ_n'\right]+\delta E^{rf}_{\parallel}v_{\parallel}J_n^2\left(\alpha\right)\right\}\frac{F^i_M}{V^2_{Ti}}\end{aligned}$$

In the following we approximate

$$\frac{\delta n^{i,rf}_{\mathbf{k},\omega}}{n_{0i}} \approx i\frac{4e_i}{m_i\pi^{1/2}V_{Ti}}\left(A_{-1}+A_0+A_1\right)$$

By straighforward calculation result

$$\begin{aligned} A_1(n &= 1) = \frac{\sqrt{\pi}\exp\left(-b_i\right)}{2k_{rf,\|}V_{T_i}}\left\{-\frac{1}{\sqrt{2b_i}}Z_0\left(\frac{\omega_{rf}-\Omega}{k_{rf,\|}V_{T_i}}\right)I_1\left(b_i\right)\delta E^{rf}_r\right. \\ &\left.-Z_0\left(\frac{\omega_{rf}-\Omega}{k_{rf,\|}V_{T_i}}\right)i\left[b_iI_0\left(b_i\right)-\left(1+b_i\right)I_1\left(b_i\right)\right]\delta E^{rf}_\theta-\left(\zeta Z_0+\sqrt{\pi}\right)I_1\left(b_i\right)\delta E^{rf}_\|\right\} \end{aligned}$$

$$\begin{aligned} A_0\left(n=0\right) &= \frac{\sqrt{\pi}\exp\left(-b_i\right)}{2k_{rf,\|}V_{T_i}}\left\{Z_0\left(\frac{\omega_{rf}}{k_{rf,\|}V_{T_i}}\right)i\left[b_iI_0\left(b_i\right)-b_iI_1\left(b_i\right)\right]\delta E^{rf}_\theta\right. \\ &\left.-\left[\frac{\omega_{rf}}{k_{rf,\|}V_{T_i}}Z_0\left(\frac{\omega_{rf}}{k_{rf,\|}V_{T_i}}\right)+\sqrt{\pi}\right]I_0\left(b_i\right)\delta E^{rf}_\|\right\} \end{aligned}$$

$$\begin{aligned} A_{-1}\left(n=-1\right) &= \frac{\sqrt{\pi}\exp\left(-b_i\right)}{2k_{rf,\|}V_{T_i}}\left\{\frac{1}{\sqrt{2b_i}}Z_0\left(\frac{\omega_{rf}+\Omega}{k_{rf,\|}V_{T_i}}\right)I_1\left(b_i\right)\delta E^{rf}_r-iZ_0\left(\frac{\omega_{rf}+\Omega}{k_{rf,\|}V_{T_i}}\right)\right. \\ &\left.\times\left[b_iI_0\left(b_i\right)-\left(1+b_i\right)I_1\left(b_i\right)\right]\delta E^{rf}_\theta-\left(\frac{\omega_{rf}+\Omega}{k_{rf,\|}V_{T_i}}Z_0+\sqrt{\pi}\right)I_1\left(b_i\right)\delta E^{rf}_\|\right\} \end{aligned}$$

3 Dispersion equation

From the drift kinetic equation we deduced the perturbation density of ions (species α) in plasma with electromagnetic turbulence in the presence of ion cyclotron resonance heating. The resultant density perturbation read as

$$-e\ \delta n^e_{\mathbf{k},\omega}+\sum_i e_i\ \delta n^i_{\mathbf{k},\omega}=0$$

with

$$\delta n^e_{\mathbf{k},\omega}=\frac{en_{0e}}{T_e}\delta\phi_{\mathbf{k},\omega}\quad,\qquad \delta n^i_{\mathbf{k},\omega}=\delta n^{i,t}_{\mathbf{k},\omega}+\delta n^{i,rf}_{\mathbf{k},\omega}$$

$$\delta n^{i,t}_{\mathbf{k},\omega}=n_{0i}\left[-1+\int_{-\infty}^{\infty}d\theta' K_i\left(\theta,\theta'\right)\right]\frac{Z_ie\delta\phi_{\mathbf{k},\omega}}{T_i}$$

where $K_i\left(\theta,\theta'\right)$ is given by eqn.(16). The Re part from the electroneutrality condition leads to

$$\sum_i z_i\left[\int_{-\infty}^{\infty}d\theta' K_i\left(\theta,\theta'\right)\delta\phi_{\mathbf{k},\omega}\left(\theta\right)+\frac{\exp\left(-b^{rf}_i\right)}{k_{rf,\|}}A_\theta\delta E^{rf}_\theta\right]=\left[1+\sum_i z_i\right]\delta\phi_{\mathbf{k},\omega}\left(\theta\right)\quad(18)$$

where

$$z_i=Z_i^2\frac{n_{0i}T_e}{n_{0e}T_i}\quad,\quad b^{rf}_i=\frac{V^2_{Ti}k^2_{rf,\perp}}{2\Omega_i^2}$$

$$A_\theta\ =\ \left[Z_0\left(\frac{\omega_{rf}+\Omega}{k_{rf,\|}V_{T_i}}\right)-Z_0\left(\frac{\omega_{rf}}{k_{rf,\|}V_{T_i}}\right)+Z_0\left(\frac{\omega_{rf}-\Omega}{k_{rf,\|}V_{T_i}}\right)\right]b^{rf}_i\left[I_0\left(b^{rf}_i\right)-I_1\left(b^{rf}_i\right)\right]$$

$$
-\left[Z_0\left(\frac{\omega_{rf}+\Omega}{k_{rf,\|}V_{T_i}}\right)+Z_0\left(\frac{\omega_{rf}-\Omega}{k_{rf,\|}V_{T_i}}\right)\right]I_1\left(b_i^{rf}\right)
$$

The dispersion equation (18) resulting from a quasilinear kinetic model is not simple and needs numerical solutions.

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