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Entropy production in plasma with RF heating

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Abstract

Auxiliary plasma heating by radio-frequency waves is a usual procedure in the modern tokamaks. The entropy production in plasma due to radio-frequency heating is evaluated in kinetic model. The different terms of the entropy production are analyzed to reveal the rate of entropy change due to different process in plasma. The evaluation of the entropy production is very useful in comparing the kinetic description with the fluid models.

1 Introduction

Applied radio-frequency (rf) waves, especially in the ion cyclotron range of frequencies (ICRF) could, in principle, provide a flexible and practical means of external transport barrier control. Experimental results suggest ICRF or ion Bernstein wave-induced transport modifications and/or sheared plasma flows.

Usually short-wavelength modes are required for efficient coupling of wave momentum to the plasma. To treat short-wavelength modes, a finite Larmor radius, nonlinear kinetic theory is required.

The framework of our calculation is that of gyrokinetic theory ($k_\perp \rho_i \sim 1$ where ρ is the gyroradius) and ion-cyclotron frequency waves ($\omega \sim \Omega_i$) with resonant wave–particle interactions ($\omega - n\Omega_i \sim k_\parallel v$) and electromagnetic plasma response. The problem of the transport in magnetically confined plasma in the presence of radio-frequency waves was largely discussed in the literature, see for example [1], [2], [3] (only few of many papers).

The problem of the entropy production was also largely discussed, in particular for plasma turbulence - see for example [4], [5], [6]. The problem of force-flux pairs and the closing condition in the fluid model was also analyzed. In the present paper we evaluate the entropy production for magnetically confined plasma in the presence of radio-frequency waves, but we not discuss here the implications of the turbulence (analyzed previously in [5]). The plan of the paper is as follows: In section I from the kinetic equation is written the solution for the distribution function in first two leading orders: one corresponding to the secular behavior and the second corresponding to more rapid scale time variation determined by the radio-frequency wave. In the section II, from the kinetic definition of the entropy and kinetic equation will be obtained the different terms contributing to the entropy production.

2 The kinetic equation

We consider a non-ohmic multi-component plasma heated at the ion cyclotron resonance for species i .

The kinetic equation for the particles of species α ($\alpha = e, H, i$) that applies in such conditions is written in conservative form as :

$$\partial_t f^{\ \alpha} + \nabla \cdot (\mathbf{v} f^{\ \alpha}) + L_0^\alpha f^{\ \alpha} = C^\alpha \left(f^{\ \alpha}, f^{\ \beta}\right) - L_{rf}^\alpha \ f^{\ \alpha} \tag{1}$$

where the force operator has been split into two contributions: one, L_0^α, due to the equilibrium electric and magnetic field

$$L_0^\alpha \equiv \frac{e_\alpha}{m_\alpha} \left(\mathbf{E}_0 + \frac{1}{c}\mathbf{v} \times \mathbf{B}_0\right) \cdot \frac{\partial}{\partial \mathbf{v}} \tag{2}$$

and the second one, L_{rf}^α, arising from the RF electromagnetic field

$$L_{rf}^\alpha \equiv \frac{e_\alpha}{m_\alpha} \left(\mathbf{E}_{rf} + \frac{1}{c}\mathbf{v} \times \mathbf{B}_{rf}\right) \cdot \frac{\partial}{\partial \mathbf{v}} \tag{3}$$

with e_α and m_α the charge and mass of the particles of species α. The non-canonical phase-space variables are the particle's position $\mathbf{x}$ and velocity $\mathbf{v}$. The operator C^α is the collision operator.

The distribution function is splitted into two contributions: one, F^α corresponding to the time averaged part (secular behavior) and the second one, g^α, corresponding to the rapid varying part of the distribution (due to rf waves):

$$f^\alpha = F^\alpha + g^\alpha \tag{4}$$

2.1 Solution for perturbed distribution function in presence of RF waves

The solution for g^α resulting from (1) in zero order approximation

$$\partial_t F^\alpha + \nabla \cdot [\mathbf{v} F^\alpha] + L_0^\alpha F^\alpha = 0$$

and first order

$$\partial_t g^\alpha + \nabla \cdot [\mathbf{v} g^\alpha] = -L_{rf}^\alpha \ F^\alpha$$

is given as :

$$\begin{aligned} g^\alpha &= -\frac{e_i}{m_i} \int_{-\infty}^{t} dt' \left(\mathbf{E}_{rf} + \frac{1}{c}\mathbf{v} \times \mathbf{B}_{rf}\right) \cdot \frac{\partial F_M^\alpha}{\partial \mathbf{v}} \\ &= -\frac{e_i}{m_i} \int_{-\infty}^{t} dt' \int d\mathbf{k} \\ & \quad \int d\omega_{rf} \mathbf{E}_{\mathbf{k},\omega}^{rf} \cdot \left[\mathbb{I}\left(1 - \frac{\mathbf{v}' \cdot \mathbf{k}_{rf}}{\omega_{rf}}\right) + \frac{\mathbf{v}'\mathbf{k}_{rf}}{\omega_{rf}}\right] \exp\left(i\mathbf{k}_{rf} \cdot \mathbf{r}' - i\omega_{rf} t\right) \cdot \frac{\partial F_M^i}{\partial \mathbf{v}} \end{aligned}$$

where Maxwell's induction equation

$$\mathbf{B}^{rf}_{\mathbf{k},\omega} = (\mathbf{k}_{rf} c/\omega_{rf}) \times \mathbf{E}^{rf}_{\mathbf{k},\omega}$$

was used and $\mathbb{I}$ is the unit dyadic.

We introduce the fixed local reference frame (FLRF)-intrinsically determined by the geometry of the equilibrium magnetic field- and defined by ($\mathbf{e}_2$,$\mathbf{e}_3$,$\mathbf{b}$) with $\mathbf{b} = \mathbf{e}_2 \times \mathbf{e}_3$. Then

$$\mathbf{v}_\perp = v_\perp \mathbf{n}_1 = v_\perp \left(\mathbf{e}_2 \cos\varphi - \mathbf{e}_3 \sin\varphi\right)$$

we have

$$\begin{aligned} g^\alpha &= -e_\alpha \exp\left(i\mathbf{k}_{rf}\cdot\mathbf{r} - i\omega_{rf} t\right) \int_0^\infty d\tau \ \exp\left(i\gamma\right) \left\{ E_x U \cos\left(\varphi + \Omega\tau\right) \right. \\ & \left. + E_y U \sin\left(\varphi + \Omega\tau\right) + E_z \left[\frac{1}{m_\alpha} \frac{\partial F_0^\alpha}{\partial v_\parallel} - V \cos\left(\varphi - \theta + \Omega\tau\right) \right] \right\} \end{aligned}$$

where

$$\begin{aligned} \gamma &= \mathbf{k}_{rf} \cdot (\mathbf{r}' - \mathbf{r}) - \omega_{rf}\left(t' - t\right) \\ &= -\frac{k_\perp v_\perp}{\Omega} \left[\sin\left(\varphi - \theta + \Omega\tau\right) - \sin\left(\varphi - \theta\right)\right] + \left(\omega - k_\parallel v_\parallel\right)\tau \end{aligned}$$

In the following assume $\theta = 0$, and

$$z = \frac{k_\perp v_\perp}{\Omega}$$

so

$$\exp\left(i\gamma\right) = \exp\left(-iz\left[\sin\left(\varphi + \Omega\tau\right) - \sin\varphi\right]\right) \exp\left[i\left(\omega - k_\parallel v_\parallel\right)\tau\right]$$

With F_0^α Maxwellian

$$F_M^\alpha = n_{0\alpha}\left(\pi V_{T\alpha}^2\right)^{-3/2} \exp\left(-\frac{v_\parallel^2}{V_{T\alpha}^2} - \frac{v_\perp^2}{V_{T\alpha}^2}\right) \tag{5}$$

and - see [7] pg 252 -

$$\begin{aligned} U &= \frac{1}{m_\alpha}\left[\frac{\partial F_M^\alpha}{\partial v_\perp} + \frac{k_\parallel}{\omega}\left(v_\perp \frac{\partial F_M^\alpha}{\partial v_\parallel} - v_\parallel \frac{\partial F_M^\alpha}{\partial v_\perp}\right)\right] \\ &= -\frac{2v_\perp}{m_\alpha V_{T\alpha}^2} F_M^\alpha = -\frac{v_\perp}{T_\alpha} F_M^\alpha \end{aligned}$$

$$V = \frac{1}{m_\alpha}\frac{k_\perp}{\omega}\left(v_\perp \frac{\partial F_M^\alpha}{\partial v_\parallel} - v_\parallel \frac{\partial F_M^\alpha}{\partial v_\perp}\right) = 0$$

$$W = \left(1 - \frac{n\Omega_\alpha}{\omega_{rf}}\right)\frac{1}{m_\alpha}\frac{\partial F_0^\alpha}{\partial v_\parallel} + \frac{n\Omega_\alpha}{\omega_{rf}}\frac{v_\parallel}{v_\perp}\frac{1}{m_\alpha}\frac{\partial F_0^\alpha}{\partial v_\perp} = -\frac{2}{m_\alpha}\frac{v_\parallel}{V_{T\alpha}^2} F_M^\alpha$$

we obtain

$$\begin{aligned} g^\alpha_{\mathbf{k},\omega} &= -e_\alpha \frac{F_M^\alpha}{T_\alpha} \int_0^\infty d\tau \ \exp\left[i\left(\omega - k_\parallel v_\parallel\right)\tau\right] \exp\left(-iz\left[\sin\left(\varphi + \Omega\tau\right) - \sin\varphi\right]\right) \\ & \times \left\{-v_\perp \ \left[E_r^{rf}\cos\left(\varphi + \Omega\tau\right) + E_\theta^{rf}\sin\left(\varphi + \Omega\tau\right)\right] - v_\parallel E_\parallel^{rf}\right\} \end{aligned}$$

where φ is the gyrophase angle.

2.2 Gyrophase averaging

The gyrophase averaging is defined as

$$\left\langle g^{\alpha}_{\mathbf{k},\omega} \right\rangle_{\varphi} = \frac{1}{2\pi} \int\limits_{0}^{2\pi} d\varphi \; g^{\alpha}_{\mathbf{k},\omega}$$

that means we must evaluate

$$\begin{aligned}
\left\langle g^{\alpha}_{\mathbf{k},\omega} \right\rangle_{\varphi} \quad = \quad & -\frac{1}{2\pi} \frac{e_{\alpha}}{m_{\alpha} V_{T\alpha}^{2}} F^{\alpha}_{M} \int\limits_{0}^{\infty} d\tau \;\; \exp\left[i \left(\omega - k_{\parallel} v_{\parallel} \right) \tau \right] \\
& \times \left\{ -2 v_{\perp} \; E^{rf}_{r} \int\limits_{0}^{2\pi} d\varphi \exp\left(-iz \left[\sin\left(\varphi + \Omega\tau \right) - \sin\varphi \right] \right) \cos\left(\varphi + \Omega\tau \right) \right. \\
& -2 v_{\perp} E^{rf}_{\theta} \int\limits_{0}^{2\pi} d\varphi \exp\left(-iz \left[\sin\left(\varphi + \Omega\tau \right) - \sin\varphi \right] \right) \sin\left(\varphi + \Omega\tau \right) \\
& \left. -2 v_{\parallel} E^{rf}_{\parallel} \int\limits_{0}^{2\pi} d\varphi \exp\left(-iz \left[\sin\left(\varphi + \Omega\tau \right) - \sin\varphi \right] \right) \right\}
\end{aligned}$$

Using the relations, see for example [8]

$$\int\limits_{0}^{2\pi} d\varphi \;\; \exp\left(-iz \left[\sin\left(\varphi + \Omega\tau \right) - \sin\varphi \right] \right) \cos\left(\varphi + \Omega\tau \right) = 2\pi \sum_{n=-\infty}^{\infty} \exp\left(-in\Omega\tau \right) \frac{n}{z} J_{n}^{2}$$

$$\int\limits_{0}^{2\pi} d\varphi \;\; \exp\left(-iz \left[\sin\left(\varphi + \Omega\tau \right) - \sin\varphi \right] \right) \sin\left(\varphi + \Omega\tau \right) = 2\pi \sum_{n=-\infty}^{\infty} \exp\left(-in\Omega\tau \right) i {J_{n}}' J_{n}$$

$$\int\limits_{0}^{2\pi} d\varphi \;\; \exp\left(-iz \left[\sin\left(\varphi + \Omega\tau \right) - \sin\varphi \right] \right) = 2\pi \sum_{n=-\infty}^{\infty} \exp\left(-in\Omega\tau \right) J_{n}^{2}$$

results

$$\begin{aligned}
\left\langle g^{\alpha}_{\mathbf{k},\omega} \right\rangle_{\varphi} \quad = \quad & \frac{2 e_{\alpha}}{m_{\alpha} V_{T\alpha}^{2}} F^{\alpha}_{M} \sum_{n=-\infty}^{\infty} \left\{ v_{\perp} \; E^{rf}_{r} \frac{n}{z} J_{n}^{2} + v_{\perp} E^{rf}_{\theta} i {J_{n}}' J_{n} + v_{\parallel} E^{rf}_{\parallel} J_{n}^{2} \right\} \cdot \\
& \cdot \int\limits_{0}^{\infty} d\tau \;\; \exp\left[i \left(\omega - k_{\parallel} v_{\parallel} - n\Omega_{\alpha} \right) \tau \right]
\end{aligned}$$

Here

$$J_{n} = J_{n}(z) = J_{n} \left(\frac{k_{\perp} v_{\perp}}{\Omega} \right)$$

With

$$\int_0^\infty d\tau \ \exp\left[i\left(\omega - k_\parallel v_\parallel - n\Omega\right)\tau\right] = \frac{i}{\omega - k_\parallel v_\parallel - n\Omega}$$

and

$$V_{T\alpha}^2 = \frac{2T_\alpha}{m_\alpha}$$

we obtain

$$\left\langle g_{\mathbf{k},\omega}^\alpha \right\rangle_\varphi = \frac{e_\alpha}{T_\alpha} F_M^\alpha \sum_{n=-\infty}^{\infty} \left\{ \left(v_\perp \ E_r^{rf} \frac{n}{z} + v_\parallel E_\parallel^{rf} \right) J_n^2 + i v_\perp E_\theta^{rf} J_n{}' J_n \right\} \frac{i}{\omega - k_\parallel v_\parallel - n\Omega}$$

Restricts the calculus only to $n = -1, 0, 1$ results

$$\left\langle g_{\mathbf{k},\omega}^\alpha \right\rangle_\varphi \approx F_M^\alpha \frac{e_\alpha}{T_\alpha} \frac{v_\perp E_\theta^{rf} J_1 J_2 + i v_\parallel E_\parallel^{rf} \left(J_0^2 + 2J_1^2\right)}{\omega - k_\parallel v_\parallel - n\Omega}$$

3 Entropy production

A natural definition of the kinetic form of entropy per unit volume of species α of a plasma is in terms of the gyrophase averaged distribution function $\overline{f}^\alpha$,

$$S^\alpha = -\int d\mathbf{v} \ \overline{f}^\alpha \ \ln \overline{f}^\alpha \tag{6}$$

With F^α the distribution function of species α in absence of rf waves and g^α the distribution function of species α interacting with rf waves,

$$f^\alpha = F^\alpha + g^\alpha$$

The rate of change of the entropy is

$$\dot{S}^\alpha \equiv -\int d\mathbf{v} \left(1 + \ln \overline{f}^\alpha\right) \partial_t \overline{f}^\alpha \tag{7}$$

Using the kinetic equation (1) in (7) result

$$\dot{S}^\alpha \equiv \dot{S}_1^\alpha + \dot{S}_2^\alpha + \dot{S}_3^\alpha + \dot{S}_4^\alpha$$

with

$$\begin{aligned}
\dot{S}_1^\alpha &\equiv -\int d\mathbf{v} \left(1 + \ln \overline{f}^\alpha\right) C^\alpha \\
\dot{S}_2^\alpha &\equiv \int d\mathbf{v} \left(1 + \ln \overline{f}^\alpha\right) \nabla \cdot \left(\mathbf{v} f^{\ \alpha}\right) \\
\dot{S}_3^\alpha &\equiv \int d\mathbf{v} \left(1 + \ln \overline{f}^\alpha\right) L_0^\alpha f^{\ \alpha} \\
\dot{S}_4^\alpha &\equiv \int d\mathbf{v} \left(1 + \ln \overline{f}^\alpha\right) L_{rf}^\alpha \ f^{\ \alpha}
\end{aligned}$$

3.0.1 Evaluation of $\overset{.}{S}{}_1^{\alpha}$

$$\overset{.}{S}{}_1^{\alpha} \equiv -\int d\mathbf{v} \left(1 + \ln \overline{f}^{\alpha}\right) C^{\alpha}$$

Conservation of the particles number leads to

$$\int d\mathbf{v} C^{\alpha} = 0$$

so

$$\overset{.}{S}{}_1^{\alpha} \equiv -\int d\mathbf{v} \ln \overline{f}^{\alpha} C^{\alpha} = -\int d\mathbf{v} \ln \left(\overline{F^{\alpha}} + \overline{g^{\alpha}}\right) C^{\alpha}$$

This will describe collisional rate of heat and momentum exchange.

If we put

$$\overline{F^{\alpha}} = \overline{F_M^{\alpha}} \quad , \quad \overline{g^{\alpha}} = \chi^{\alpha} \overline{F_M^{\alpha}} \tag{8}$$

then

$$\begin{aligned} \overset{.}{S}{}_1^{\alpha} &\equiv -\int d\mathbf{v}\, C^{\alpha} \ln \left(\overline{F_M^{\alpha}} + \chi^{\alpha} \overline{F_M^{\alpha}}\right) \\ &= -\int d\mathbf{v}\, C^{\alpha} \ln \overline{F_M^{\alpha}} - \int d\mathbf{v}\, C^{\alpha} \ln \left(1 + \chi^{\alpha}\right) \end{aligned}$$

The collisional rate of heat exchange is

$$-\int d\mathbf{v}\, C^{\alpha} \frac{m_{\alpha}}{2} \left(\mathbf{v} - \mathbf{u}^{\alpha}\right)^2 = Q_c^{\alpha}$$

and the collisional rate of momentum exchange is

$$\mathbf{F}_c^{\alpha} = \int d\mathbf{v}\, m_{\alpha} \mathbf{v} C^{\alpha}$$

So,

$$\sigma_{11}^{\alpha} = -\int d\mathbf{v}\, C^{\alpha} \ln \overline{F_M^{\alpha}} = \frac{1}{T^{\alpha}} \left(Q_c^{\alpha} + \mathbf{u}^{\alpha} \cdot \mathbf{F}_c^{\alpha}\right)$$

with

$$\sum_{\alpha} T^{\alpha} \sigma_{11}^{\alpha} = \sum_{\alpha} \left(Q_c^{\alpha} + \mathbf{u}^{\alpha} \cdot \mathbf{F}_c^{\alpha}\right) = 0$$

because of energy conservation in the collisions.

The entropy production

$$\sigma_{12} = -\int d\mathbf{v}\, C^{\alpha} \ln \left(1 + \chi^{\alpha}\right)$$

is due to collisional process and the rf waves.

3.0.2 Evaluation of $\overset{.}{S}{}_2^{\alpha}$

Integrating by parts we obtain

$$\overset{.}{S}{}_2^{\alpha} \equiv \int d\mathbf{v} \left(1 + \ln \overline{f}^{\alpha}\right) \nabla \cdot \left(\mathbf{v} f^{\ \alpha}\right)$$

$$
\begin{aligned}
&= \nabla\cdot \int d\mathbf{v} \left(1+\ln \overline{f}^{\alpha}\right) \mathbf{v} f^{\ \alpha} - \int d\mathbf{v} f^{\ \alpha} \mathbf{v}\cdot\nabla \ln \overline{f}^{\alpha} \\
&= -\boldsymbol{\nabla}\cdot \left[\mathbf{J}_S^{\alpha} + S^{\alpha}\mathbf{u}^{\alpha}\right] + \sigma_2^{\alpha}
\end{aligned}
$$

where

$$
\mathbf{J}_S^{\alpha} = -\int d\mathbf{v}\left(\mathbf{v}-\mathbf{u}^{\alpha}\right) f^{\ \alpha}\left(1+\ln \overline{f}^{\alpha}\right)
$$

represents the conductive entropy flux and

$$
S^{\alpha}\mathbf{u}^{\alpha} = -\mathbf{u}^{\alpha}\int d\mathbf{v}\left(1+\ln \overline{f}^{\alpha}\right) f^{\ \alpha}
$$

is the convective entropy flux. Let us evaluate the entropy source σ_2^{α}:

$$
\sigma_2^{\alpha} = -\int d\mathbf{v} f^{\ \alpha}\mathbf{v}\cdot\nabla \ln \overline{f}^{\alpha} = -\int d\mathbf{v} f^{\alpha}\mathbf{v}\cdot\nabla \ln\left(F^{\alpha}+g^{\alpha}\right)
$$

With (8) σ_2^{α} can be rewritten as

$$
\sigma_2^{\alpha} = -\int d\mathbf{v} f^{\alpha}\mathbf{v}\cdot\nabla \ln\left[F_M^{\alpha}\left(1+\chi^{\alpha}\right)\right] = \sigma_{21}^{\alpha} + \sigma_{22}^{\alpha}
$$

where

$$
\sigma_{21}^{\alpha} = -\int d\mathbf{v}\ f^{\alpha}\ \mathbf{v}\cdot\nabla \ln F_M^{\alpha} \tag{9}
$$

$$
\sigma_{22}^{\alpha} = -\int d\mathbf{v}\ f^{\alpha}\ \mathbf{v}\cdot\nabla \ln\left(1+\chi^{\alpha}\right)
$$

With

$$
\nabla \ln F_M^{\alpha} = -\frac{1}{T_{\alpha}}\left[X_1^{\alpha} + X_2^{\alpha}\left(\frac{m_{\alpha}v^2}{2} - \frac{5}{2}T_{\alpha}\right)\right]\nabla V \tag{10}
$$

where ε is the particle energy and thermodynamic forces X_1^{α} , X_2^{α} are defined as

$$
X_1^{\alpha} = -\frac{1}{n_0^{\alpha}}\frac{\partial p_{\alpha}}{\partial V} - e_{\alpha}\frac{\partial \Phi_0}{\partial V} \quad , \quad X_2^{\alpha} \equiv -\frac{\partial \ln T^{\alpha}}{\partial V} \tag{11}
$$

So, from (9) with (10) and (11) result

$$
\begin{aligned}
\sigma_{21}^{\alpha} &= \int d\mathbf{v}\ f^{\alpha}\ \mathbf{v}\cdot\frac{1}{T_{\alpha}}\left[X_1^{\alpha} + X_2^{\alpha}\left(\frac{m_{\alpha}v^2}{2} - \frac{5}{2}T_{\alpha}\right)\right]\nabla V \\
&= \frac{1}{T_{\alpha}}X_1^{\alpha}\left(\int d\mathbf{v}\ f^{\alpha}\ \mathbf{v}\right)\cdot\nabla V \\
&\quad + \frac{1}{T_{\alpha}}X_2^{\alpha}\left[\int d\mathbf{v}\ \frac{m_{\alpha}v^2}{2}f^{\alpha}\ \mathbf{v}\right]\cdot\nabla V \\
&\quad + \frac{1}{T_{\alpha}}X_2^{\alpha}\int d\mathbf{v}\ f^{\alpha}\ \mathbf{v}\cdot\left(-\frac{5}{2}T_{\alpha}\right)\nabla V
\end{aligned}
$$

respective

$$
\sigma_{21}^{\alpha} = \frac{1}{T_{\alpha}}X_1^{\alpha}\ \boldsymbol{\Gamma}^{\alpha}\cdot\nabla V + \frac{1}{T_{\alpha}}X_2^{\alpha}\mathbf{Q}^{\alpha}\cdot\nabla V + \frac{1}{T_{\alpha}}X_2^{\alpha}\left(-\frac{5}{2}T_{\alpha}\right)\boldsymbol{\Gamma}^{\alpha}\cdot\nabla V
$$

$$\sigma_{21}^{\alpha} = \frac{1}{T_\alpha}\left[X_1^\alpha\,\Gamma^{\alpha,r} + X_2^\alpha Q^{\alpha,r} + X_2^\alpha\left(-\frac{5}{2}T_\alpha\right)\Gamma^{\alpha,r}\right] \tag{12}$$

where the particle flux in the radial direction is

$$\Gamma^{\alpha,r} = \mathbf{\Gamma}^\alpha\cdot\nabla V = \left(\int d\mathbf{v}\ f^\alpha\ \mathbf{v}\right)\cdot\nabla V$$

and the energy flux in radial direction is

$$Q^{\alpha,r} = \mathbf{Q}^\alpha\cdot\nabla V = \left[\int d\mathbf{v}\ \frac{m_\alpha v^2}{2} f^\alpha\ \mathbf{v}\right]\cdot\nabla V$$

With forces X_1^α , X_2^α and fluxes J_1^α , J_2^α defined as

$$\begin{aligned} J_1^\alpha &\equiv \Gamma^{\alpha,r} \qquad (13)\\ J_2^\alpha &= Q^{\alpha,r} - \frac{5}{2}T_\alpha\Gamma^{\alpha,r} \qquad (14)\end{aligned}$$

we can write the entropy production σ_{21}^α in thermodynamic form as

$$\sigma_{21}^\alpha = \frac{1}{T_\alpha}\sum_{i=1}^{2} X_i^\alpha J_i^\alpha \tag{15}$$

3.0.3 Evaluation of $\dot{S}_3^\alpha$

Integrating by parts in

$$\dot{S}_3^\alpha \equiv \int d\mathbf{v}\left(1 + \ln\overline{f}^\alpha\right) L_0^\alpha f^{\ \alpha}$$

we obtain

$$\dot{S}_3^\alpha = -\int d\mathbf{v}\ f^{\ \alpha} L_0^\alpha\ \ln\overline{f}^\alpha$$

Because L_0^α do not depend on time can be neglected in the zero order approximation.

3.0.4 Evaluation of $\dot{S}_4^\alpha$

Integrating by parts in

$$\dot{S}_4^\alpha \equiv \int d\mathbf{v}\left(1 + \ln\overline{f}^\alpha\right) L_{rf}^\alpha\ f^{\ \alpha}$$

we obtain

$$\begin{aligned}\dot{S}_4^\alpha &= -\int d\mathbf{v}\ f^{\ \alpha}\frac{e_\alpha}{m_\alpha}\left(\mathbf{E}_{rf} + \frac{1}{c}\mathbf{v}\times\mathbf{B}_{rf}\right)\cdot\frac{\partial}{\partial\mathbf{v}}\ln\overline{f}^\alpha\\ &= \sigma_{41}^\alpha + \sigma_{42}^\alpha + \sigma_{43}^\alpha\end{aligned}$$

where

$$\begin{aligned}\sigma_{41}^\alpha &= -\frac{e_\alpha}{m_\alpha}\int d\mathbf{v}\ f^\alpha\mathbf{E}_{rf}\cdot\frac{\partial}{\partial\mathbf{v}}\ln F_M^\alpha\\ \sigma_{42}^\alpha &= -\frac{e_\alpha}{m_\alpha}\frac{1}{c}\int d\mathbf{v}\ f^\alpha\mathbf{v}\times\mathbf{B}_{rf}\cdot\frac{\partial}{\partial\mathbf{v}}\ln F_M^\alpha\end{aligned}$$

In leader order approximation

$$\begin{aligned}
\sigma_{41}^{\alpha} &= -\frac{e_\alpha}{m_\alpha}\int d\mathbf{v}\ f^{\alpha}\left(E_{\perp}^{rf}\frac{\partial}{\partial v_\perp}\ln\ F_M^{\alpha}+E_{\parallel}^{rf}\frac{\partial}{\partial v_\parallel}\ln\ F_M^{\alpha}\right)\\
&= \frac{2e_\alpha}{m_\alpha V_{Ti}^2}\int d\mathbf{v}\ f^{\alpha}\left(v_\perp E_\perp^{rf}+v_\parallel E_\parallel^{rf}\right)\\
&= \frac{2e_\alpha}{m_\alpha V_{Ti}^2}\left(\int d\mathbf{v}\ f^{\alpha}v_\perp\right)E_\perp^{rf}+\frac{2e_\alpha}{m_\alpha V_{Ti}^2}\left(\int d\mathbf{v}\ f^{\alpha}v_\parallel\right)E_\parallel^{rf}\\
&= \frac{e_\alpha}{T_\alpha}\mathbf{\Gamma}^{\alpha}\cdot\mathbf{E}_{rf}
\end{aligned}$$

and

$$\sigma_{42}^{\alpha}=0$$

Some other corrections terms can be appear from

$$\sigma_{43}^{\alpha}=-\frac{e_\alpha}{m_\alpha}\int d\mathbf{v}\ f^{\alpha}\left(\mathbf{E}_{rf}+\frac{1}{c}\mathbf{v}\times\mathbf{B}_{rf}\right)\cdot\frac{\partial}{\partial\mathbf{v}}\ln\left(1+\chi^{\alpha}\right)$$

with χ^{α} defined by (8).

4 Conclusions

In this work we have evaluated in the first significant order the entropy production due to plasma particles interaction with radio-frequency waves in the magnetically confined plasma. Both the ‘flux–force’ and ‘dissipation’ expressions are present. The second order non-linear terms must be also included to describe the influence of the radio-frequency waves on the transport.

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