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First-class Approaches of the Second-Class Systems. The Example of Massive Forms

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Abstract

Massive 2- and 3-forms are analyzed from the point of view of the Hamiltonian quantization using the gauge-unfixing approach. For both models the gauge-unfixing method finally output a manifestly Lorentz covariant path-integral.

1 Introduction

The purpose of this paper is to present the problem of the Hamiltonian quantization of the massive 2- and 3-forms in the framework of the gauge-unfixing (GU) approach [1]–[2] based on path integral. The main idea is to associate with the original second-class theory an equivalent first-class system. The associated first-class system has to satisfy the following requirements: its number of physical degrees of freedom coincides with that of the original second-class theory, the algebras of classical observables are isomorphic and the first-class Hamiltonian restricted to the original constraint surface reduces to the original canonical Hamiltonian. The above isomorphism renders the equivalence of the two systems also at the level of the path integral quantization and hence allows the replacement of the Hamiltonian path integral for the original second-class theory with that of the equivalent first-class system.

This paper is organized in five sections. In Section 2 we start from a bosonic second-class constrained system and briefly expose the GU method [1]–[2] of constructing first-class system equivalent with the original theory. In Section 3 we apply GU methods to massive 2-forms and meanwhile obtain the path integrals corresponding to the first-class systems associated with this model. After integrating out the auxiliary fields and performing some field redefinitions, we discover nothing but the manifestly Lorentz covariant path integrals corresponding to the Lagrangian formulation of the first-class systems, which reduce to the Lagrangian path integral for Stückelberg-coupled 1- and 2-forms. In Section 4 we exemplify the GU method in the case of massive 3-forms. Section 5 ends the paper with the main conclusions.

2 Gauge unfixing method

The starting point is a bosonic dynamic system with the phase-space locally parameterized by n canonical pairs $z^a = \left(q^i,\ p_i\right)$, endowed with the canonical Hamiltonian H_c, and subject to the purely second-class constraints

$$\chi_{\alpha_0}\left(z^a\right) \approx 0, \qquad \alpha_0 = \overline{1, 2M_0}, \tag{1}$$

Assume that one can split the second-class constraint set (1) into two subsets

$$\chi_{\alpha_0}\left(z^a\right) \equiv \left(G_{\bar{\alpha}_0}\left(z^a\right), C^{\bar{\beta}_0}\left(z^a\right)\right) \approx 0, \qquad \bar{\alpha}_0, \bar{\beta}_0 = \overline{1, M_0}. \tag{2}$$

such that

$$\left[G_{\bar{\alpha}_0}, G_{\bar{\beta}_0}\right] = D^{\bar{\gamma}_0}_{\bar{\alpha}_0\bar{\beta}_0} G_{\bar{\gamma}_0}. \tag{3}$$

Relations (3) yield the subset

$$G_{\bar{\alpha}_0}\left(z^a\right)\approx 0 \tag{4}$$

to be first-class. The second-class behaviour of the overall constraint set ensures that

$$C^{\bar{\alpha}_0}\left(z^a\right)\approx 0 \tag{5}$$

may be regarded as some gauge-fixing conditions for this first-class set.

We introduce an operator $\hat{X}$ [3] that associates with every smooth function F on the original phase-space an application $\hat{X}F$, which is in strong involution with the functions $G_{\bar{\alpha}_0}$

$$\hat{X}F = F - C^{\bar{\alpha}_0}\left[G_{\bar{\alpha}_0}, F\right] + \tfrac{1}{2}C^{\bar{\alpha}_0}C^{\bar{\beta}_0}\left[G_{\bar{\alpha}_0}, \left[G_{\bar{\beta}_0}, F\right]\right] - \cdots, \tag{6}$$

$$\left[\hat{X}F, G_{\bar{\alpha}_0}\right] = 0. \tag{7}$$

With the help of this operator we construct a first-class Hamiltonian $\hat{X}H_c$ with respect to (4).

The original second-class theory and respectively the gauge-unfixed system are classically equivalent since they possess the same number of physical degrees of freedom $\mathcal{N}_O = \frac{1}{2}\left(2n - 2M_0\right) = \mathcal{N}_{GU}$ and the corresponding algebras of classical observables are isomorphic. Consequently, the two systems become also equivalent at the level of the path integral quantization, which allows one to replace the Hamiltonian path integral of the original second-class theory

$$\begin{aligned} Z_O &= \int \mathcal{D}\left(z^a, \lambda^{\alpha_0}\right)\det\left(\left[G_{\bar{\alpha}_0}, C^{\bar{\beta}_0}\right]\right)\times \\ &\quad \times \exp\left[\mathrm{i}\int dt\left(\dot{q}^i p_i - H_c - \lambda^{\alpha_0}\chi_{\alpha_0}\right)\right], \end{aligned} \tag{8}$$

with that of the gauge-unfixed first-class system

$$\begin{aligned} Z_{GU} &= \int \mathcal{D}\left(z^a, \lambda^{\bar{\alpha}_0}\right)\left(\prod_{\bar{\alpha}_0}\delta\left(C^{\bar{\alpha}_0}\right)\right)\left(\det\left(\left[G_{\bar{\alpha}_0}, C^{\bar{\beta}_0}\right]\right)\right)\times \\ &\quad \times \exp\left[\mathrm{i}\int dt\left(\dot{q}^i p_i - \hat{X}H_c - \lambda^{\bar{\alpha}_0}G_{\bar{\alpha}_0}\right)\right]. \end{aligned} \tag{9}$$

3 Massive 2-forms

We start from the Lagrangian action of massive 2-forms in $D \geq 3$ [4]–[5]

$$S_0^L\left[A_{\mu\nu}\right] = \int d^D x\left(-\tfrac{1}{12}F_{\mu\nu\rho}F^{\mu\nu\rho} - \tfrac{m^2}{4}A_{\mu\nu}A^{\mu\nu}\right). \tag{10}$$

By performing the canonical analysis of this model [6]-[7], there result the constraints

$$\chi^{(1)i} \equiv \pi^{0i} \approx 0, \tag{11}$$

$$\chi_i^{(2)} \equiv 2\partial^j\pi_{ji} - m^2 A_{0i} \approx 0, \tag{12}$$

along with the canonical Hamiltonian

$$H_c = \int d^{D-1}x\left(-\pi_{ij}\pi^{ij} + \tfrac{1}{12}F_{ijk}F^{ijk} + \tfrac{m^2}{4}A_{\mu\nu}A^{\mu\nu} - 2A_{0i}\partial_j\pi^{ji}\right). \tag{13}$$

The constraints (11) and (12) are second-class and irreducible.

According to the GU method we consider (12) as the first-class constraint set and the remaining constraints (11) as the corresponding canonical gauge conditions [8]-[9] and redefine the first-class constraints as

$$G^i \equiv -\tfrac{1}{m^2}\left(2\partial_j\pi^{ji} - m^2 A^{0i}\right) \approx 0. \tag{14}$$

The first-class Hamiltonian with respect to (14) follows from relation (6)

$$\hat{X}H_c = H_c - \int d^{D-1}y\chi_i^{(1)}(y)\left[G^i(y), H_c(y^0)\right]$$
$$+\tfrac{1}{2}\int d^{D-1}yd^{D-1}z\chi_i^{(1)}(y)\chi_j^{(1)}\left(y^0,\mathbf{z}\right)\left[G^i(y)\left[G^j\left(y^0,\mathbf{z}\right), H_c y^0\right]\right] - \cdots$$
$$= H_c - \int d^{D-1}y\left[\pi_{0i}\partial_j A^{ji} - \tfrac{1}{2m^2}\partial_i\pi_{j0}\partial^{[i}\pi^{j]0}\right]. \qquad (15)$$

An irreducible set of constraints can always be replaced by a reducible one by introducing constraints that are consequences of the ones already at hand [10]. In view of this, we supplement (14) with one more constraint, $G \equiv -m^2\partial_i G^i \approx 0$, such that the new constraint set

$$G^i \equiv -\tfrac{1}{m^2}\left(2\partial_j\pi^{ji} - m^2 A^{0i}\right) \approx 0, \qquad (16)$$
$$G \equiv -m^2\partial_i A^{0i} \approx 0 \qquad (17)$$

remains first-class and, moreover, becomes off-shell first-order reducible, with first-order reducibility functions

$$(Z_\kappa) = \left(\begin{array}{cc}\partial_i & \tfrac{1}{m^2}\end{array}\right). \qquad (18)$$

At this stage, it is useful to make the canonical transformation

$$A_{0i} \longrightarrow -\tfrac{1}{m^2}\Pi_i, \qquad \pi^{0i} \longrightarrow m^2 B^i, \qquad (19)$$

The constraints (16) and (17) become

$$G^i \equiv -\tfrac{1}{m^2}\left(2\partial_j\pi^{ji} + \Pi^i\right) \approx 0, \qquad (20)$$
$$G \equiv \partial_i\Pi^i \approx 0, \qquad (21)$$

while the first-class Hamiltonian (15) takes the form

$$H_{GU} = \int d^{D-1}y\left[-\pi_{ij}\pi^{ij} + \tfrac{1}{12}F_{ijk}F^{ijk} + \tfrac{m^2}{4}A_{ij}A^{ij} + \tfrac{m^2}{2}A_{ij}\partial^{[i}B^{j]}\right.$$
$$\left. + \tfrac{m^2}{4}\partial_{[i}B_{j]}\partial^{[i}B^{j]} - \tfrac{1}{2m^2}\Pi_i\Pi^i + \tfrac{1}{m^2}\Pi_i\left(2\partial_j\pi^{ji} + \Pi^i\right)\right]. \qquad (22)$$

Due to the equivalence between the first-order reducible first-class system and the original second-class theory, one can replace the Hamiltonian path integral of massive 2-forms with that associated with the reducible first-class system. The argument of the exponential from the Hamiltonian path integral of the reducible first-class system read as

$$S_{GU} = \int d^D x\left[\left(\partial_0 A_{ij}\right)\pi^{ij} + \left(\partial_0 B_i\right)\Pi^i + \pi_{ij}\pi^{ij} - \tfrac{1}{12}F_{ijk}F^{ijk}\right.$$
$$-\tfrac{m^2}{4}A_{ij}A^{ij} - \tfrac{m^2}{2}A_{ij}\partial^{[i}B^{j]} - \tfrac{m^2}{4}\partial_{[i}B_{j]}\partial^{[i}B^{j]} + \tfrac{1}{2m^2}\Pi_i\Pi^i$$
$$\left.-\tfrac{1}{m^2}\Pi_i\left(2\partial_j\pi^{ji} + \Pi^i\right) + \tfrac{1}{m^2}\lambda_i\left(2\partial_j\pi^{ji} + \Pi^i\right) - \lambda\left(\partial_i\Pi^i\right)\right]. \qquad (23)$$

If we perform the transformation

$$\Pi^i \longrightarrow \Pi^i, \qquad \lambda_i \longrightarrow \bar{\lambda}_i = \lambda_i - \Pi_i \qquad (24)$$

in the path integral, the argument of the exponential becomes

$$S'_{GU} = \int d^D x\left[\left(\partial_0 A_{ij}\right)\pi^{ij} + \left(\partial_0 B_i\right)\Pi^i + \pi_{ij}\pi^{ij} - \tfrac{1}{12}F_{ijk}F^{ijk}\right.$$
$$-\tfrac{m^2}{4}A_{ij}A^{ij} - \tfrac{m^2}{2}A_{ij}\partial^{[i}B^{j]} - \tfrac{m^2}{4}\partial_{[i}B_{j]}\partial^{[i}B^{j]} + \tfrac{1}{2m^2}\Pi_i\Pi^i$$
$$\left.+\tfrac{1}{m^2}\bar{\lambda}_i\left(2\partial_j\pi^{ji} + \Pi^i\right) - \lambda\left(\partial_i\Pi^i\right)\right]. \qquad (25)$$

We enlarge the original phase-space with the Lagrange multipliers $\{\bar{\lambda}_i, \lambda\}$ and with their canonical momenta $\{p^i, p\}$ and we add the constraints

$$p^i \approx 0, \qquad p \approx 0. \tag{26}$$

The argument of the exponential from the Hamiltonian path integral for the first-class theory with the phase-space locally parameterized by the fields/momenta $\{A_{ij},\, B_i,\, \bar{\lambda}_i,\, \lambda,\, \pi^{ij},\, \Pi^i,\, p^i,\, p\}$ and subject to the first-class constraints (20), (21), and (26) reads as

$$\begin{aligned} S''_{GU} \;=\; & \int d^D x \left[(\partial_0 A_{ij})\, \pi^{ij} + (\partial_0 B_i)\, \Pi^i + \left(\partial_0 \bar{\lambda}_i\right) p^i + (\partial_0 \lambda)\, p \right. \\ & + \pi_{ij}\pi^{ij} - \tfrac{1}{12} F_{ijk} F^{ijk} - \tfrac{m^2}{4} A_{ij} A^{ij} - \tfrac{m^2}{2} A_{ij} \partial^{[i} B^{j]} \\ & - \tfrac{m^2}{4} \partial_{[i} B_{j]} \partial^{[i} B^{j]} + \tfrac{1}{2m^2} \Pi_i \Pi^i \\ & \left. + \tfrac{1}{m^2} \bar{\lambda}_i \left(2\partial_j \pi^{ji} + \Pi^i \right) - \lambda \left(\partial_i \Pi^i \right) - \Lambda_i p^i - \Lambda p \right]. \end{aligned} \tag{27}$$

Performing in (27) the integration over $\{\pi^{ij}, \Pi^i, p^i, p, \Lambda_i, \Lambda\}$ and making the notations

$$\tfrac{1}{m^2} \bar{\lambda}_i \equiv -\bar{A}_{i0}, \qquad \lambda \equiv -B_0, \tag{28}$$

then (27) can be written as

$$\begin{aligned} S'''_{GU} \;=\; & \int d^D x \left[-\tfrac{1}{12} F_{ijk} F^{ijk} - \tfrac{1}{4} \bar{F}_{0ij} \bar{F}^{0ij} - \tfrac{m^2}{4} A_{ij} A^{ij} - \tfrac{m^2}{2} \bar{A}_{i0} \bar{A}^{i0} \right. \\ & \left. - \tfrac{m^2}{2} A_{ij} F^{ij} - m^2 \bar{A}_{i0} F^{i0} - \tfrac{m^2}{4} F_{ij} F^{ij} - \tfrac{m^2}{2} F_{0i} F^{0i} \right], \end{aligned} \tag{29}$$

where

$$\bar{F}_{0ij} = \partial_0 A_{ij} + \partial_{[i} \bar{A}_{j]0},\;\; F_{ij} = -\partial_{[i} B_{j]},\;\; F_{0i} = -\left(\partial_0 B_i - \partial_i B_0\right). \tag{30}$$

$$F_{ij} = -\partial_{[i} B_{j]}, \qquad F_{0i} = -\left(\partial_0 B_i - \partial_i B_0\right). \tag{31}$$

The functional (29) associated with the reducible first-class system takes now a manifestly Lorentz covariant form

$$\tilde{S}_{GU}\left[\bar{B}_\mu, \bar{A}_{\mu\nu}\right] = \int d^D x \left[-\tfrac{1}{12} \bar{F}_{\mu\nu\rho} \bar{F}^{\mu\nu\rho} - \tfrac{1}{4} \left(F_{\mu\nu} - m\bar{A}_{\mu\nu}\right)\left(F^{\mu\nu} - m\bar{A}^{\mu\nu}\right) \right],$$

with

$$\bar{A}_{\mu\nu} = -\bar{A}_{\nu\mu}, \qquad \bar{A}_{\mu\nu} \equiv \left(\bar{A}_{0j}, A_{jk}\right), \qquad \bar{F}_{\mu\nu\rho} = \partial_{[\mu} \bar{A}_{\nu\rho]}, \tag{32}$$

$$\bar{B}_\mu = -\tfrac{1}{m} B_\mu, \qquad F_{\mu\nu} = \partial_{[\mu} \bar{B}_{\nu]}, \tag{33}$$

and describes precisely the (Lagrangian) Stückelberg coupling [11] between the one-form $\bar{B}_\mu$ and the two-form $\bar{A}_{\mu\nu}$.

4 Massive 3-forms

We start from the Lagrangian action of massive 3-forms in $D \geq 4$ [4]–[5]

$$S_0^L\left[A_{\mu\nu\rho}\right] = \int d^D x \left(-\tfrac{1}{48} F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\lambda} - \tfrac{m^2}{12} A_{\mu\nu\rho} A^{\mu\nu\rho} \right). \tag{34}$$

By performing the canonical analysis of this model [6]-[7], there result the constraints

$$\chi^{(1)ij} \equiv \pi^{0ij} \approx 0, \tag{35}$$

$$\chi^{(2)}_{ij} \equiv 3\partial^k \pi_{kij} - \tfrac{m^2}{2} A_{0ij} \approx 0, \tag{36}$$

along with the canonical Hamiltonian

$$H_c = \int d^{D-1}x \left(-3\pi_{ijk}\pi^{ijk} + \tfrac{1}{48}F_{ijk}F^{ijk} + \tfrac{m^2}{12}A_{\mu\nu\rho}A^{\mu\nu\rho} + \partial_{[k}A_{ij]}\pi^{kij}\right). \tag{37}$$

According to the GU method we consider (36) as the first-class constraint set and the remaining constraints (35) as the corresponding canonical gauge conditions [8]-[9] and redefine the first-class constraints as

$$G^{ij} \equiv -\tfrac{1}{m^2}\left(3\partial_k\pi^{kij} - \tfrac{m^2}{2}A^{0ij}\right) \approx 0. \tag{38}$$

The first-class Hamiltonian with respect to (38) follows from relation (6)

$$\begin{aligned} \hat{X}H_c = H_c &- \int d^{D-1}y\chi^{(1)}_{ij}(y)\left[G^{ij}(y), H_c(y^0)\right] \\ &+\tfrac{1}{2}\int d^{D-1}yd^{D-1}z\chi^{(1)}_{ij}(y)\chi^{(1)}_{kl}(y^0,\mathbf{z})\left[G^{ij}(y),\left[G^{kl}(y^0,\mathbf{z}),H_c(y^0)\right]\right] - \cdots \\ = H_c &- \int d^{D-1}y\left[\tfrac{1}{2}\pi_{0ij}\partial_k A^{kij} - \tfrac{1}{4m^2}\partial_i\pi_{jk0}\partial^{[i}\pi^{jk]0}\right]. \end{aligned} \tag{39}$$

An irreducible set of constraints can always be replaced by a reducible one by introducing constraints that are consequences of the ones already at hand [10]. In view of this, we supplement constraints (38) with one more constraint, $G^i \equiv -\frac{m^2}{2}\partial_j G^{ji} \approx 0$, such that the new constraint set

$$G^{ij} \equiv -\tfrac{1}{m^2}\left(3\partial_k\pi^{kij} - \tfrac{m^2}{2}A^{0ij}\right) \approx 0, \tag{40}$$

$$G^{i} \equiv -\tfrac{m^2}{2}\partial_j A^{0ji} \approx 0 \tag{41}$$

remains first-class and, moreover, becomes off-shell second-order reducible. At this stage, it is useful to make the canonical transformation

$$A_{0ij} \longrightarrow -\tfrac{1}{m^2}\Pi_{ij}, \qquad \pi^{0ij} \longrightarrow m^2 B^{ij}, \tag{42}$$

The constraints (40) and (41) become

$$G^{ij} \equiv -\tfrac{1}{m^2}\left(3\partial_j\pi^{ji} + \tfrac{1}{2}\Pi^{ij}\right) \approx 0, \tag{43}$$

$$G^{i} \equiv \tfrac{1}{2}\partial_j\Pi^{ji} \approx 0, \tag{44}$$

while the first-class Hamiltonian (39) takes the form

$$\begin{aligned} H_{GU} = \int d^{D-1}y\Big[&-3\pi_{ijk}\pi^{ijk} + \tfrac{1}{48}F_{ijkl}F^{ijkl} + \tfrac{m^2}{12}A_{ijk}A^{ijk} \\ &+\tfrac{m^2}{6}A_{ijk}\partial^{[i}B^{jk]} + \tfrac{m^2}{12}\partial_{[i}B_{jk]}\partial^{[i}B^{jk]} \\ &-\tfrac{1}{4m^2}\Pi_{ij}\Pi^{ij} + \tfrac{1}{m^2}\Pi_{ij}\left(3\partial_k\pi^{kij} + \tfrac{1}{2}\Pi^{ij}\right)\Big]. \end{aligned} \tag{45}$$

Due to the equivalence between the first-order reducible first-class system and the original second-class theory, one can replace the Hamiltonian path integral of massive 3-forms with that associated with the reducible first-class system. The argument of the exponential from the Hamiltonian path integral of the reducible first-class system read as

$$\begin{aligned} S_{GU} = \int d^Dx\Big[&(\partial_0 A_{ijk})\pi^{ijk} + (\partial_0 B_{ij})\Pi^{ij} + 3\pi_{ijk}\pi^{ijk} \\ &-\tfrac{1}{48}F_{ijkl}F^{ijkl} - \tfrac{m^2}{12}A_{ijk}A^{ijk} - \tfrac{m^2}{6}A_{ijk}\partial^{[i}B^{jk]} \\ &-\tfrac{m^2}{12}\partial_{[i}B_{jk]}\partial^{[i}B^{jk]} + \tfrac{1}{4m^2}\Pi_{ij}\Pi^{ij} - \tfrac{1}{m^2}\Pi_{ij}\left(3\partial_k\pi^{kij} + \tfrac{1}{2}\Pi^{ij}\right) \\ &+\tfrac{1}{m^2}\lambda_{ij}\left(3\partial_k\pi^{kij} + \tfrac{1}{2}\Pi^{ij}\right) - \tfrac{1}{2}\lambda_i\left(\partial_j\Pi^{ji}\right)\Big]. \end{aligned} \tag{46}$$

If we perform the transformation

$$\Pi^{ij} \longrightarrow \Pi^{ij}, \qquad \lambda_{ij} \longrightarrow \bar{\lambda}_{ij} = \lambda_{ij} - \Pi_{ij} \tag{47}$$

in the path integral, the argument of the exponential becomes

$$\begin{aligned} S'_{GU} = & \int d^D x \left[(\partial_0 A_{ijk}) \pi^{ijk} + (\partial_0 B_{ij}) \Pi^{ij} + 3\pi_{ijk}\pi^{ijk} - \tfrac{1}{48} F_{ijkl} F^{ijkl} \right. \\ & - \tfrac{m^2}{12} A_{ijk} A^{ijk} - \tfrac{m^2}{6} A_{ijk} \partial^{[i} B^{jk]} - \tfrac{m^2}{12} \partial_{[i} B_{jk]} \partial^{[i} B^{jk]} \\ & \left. + \tfrac{1}{4m^2} \Pi_{ij} \Pi^{ij} + \tfrac{1}{m^2} \bar{\lambda}_{ij} \left(3\partial_k \pi^{kij} + \tfrac{1}{2} \Pi^{ij} \right) - \tfrac{1}{2} \lambda_i \left(\partial_j \Pi^{ji} \right) \right]. \end{aligned} \tag{48}$$

We enlarge the original phase-space with the Lagrange multipliers $\{\bar{\lambda}_{ij}, \lambda_i\}$ and with their canonical momenta $\{p^{ij}, p^i\}$ and we add the constraints

$$p^{ij} \approx 0, \qquad p^i \approx 0. \tag{49}$$

The argument of the exponential from the Hamiltonian path integral for the first-class theory with the phase-space locally parameterized by $\{A_{ijk},\ B_{ij},\ \bar{\lambda}_{ij},\ \lambda_i,\ \pi^{ijk},\ \Pi^{ij},\ p^{ij},\ p^i\}$ and subject to the first-class constraints (20), (44), and (49) reads as

$$\begin{aligned} S''_{GU} = & \int d^D x \left[(\partial_0 A_{ijk}) \pi^{ijk} + (\partial_0 B_{ij}) \Pi^{ij} + \left(\partial_0 \bar{\lambda}_{ij} \right) p^{ij} + (\partial_0 \lambda_i) p^i \right. \\ & + 3\pi_{ijk}\pi^{ijk} - \tfrac{1}{48} F_{ijkl} F^{ijkl} - \tfrac{m^2}{12} A_{ijk} A^{ijk} - \tfrac{m^2}{6} A_{ijk} \partial^{[i} B^{jk]} \\ & - \tfrac{m^2}{12} \partial_{[i} B_{jk]} \partial^{[i} B^{jk]} + \tfrac{1}{4m^2} \Pi_{ij} \Pi^{ij} + \tfrac{1}{m^2} \bar{\lambda}_{ij} \left(3\partial_k \pi^{kij} + \tfrac{1}{2} \Pi^{ij} \right) \\ & \left. - \tfrac{1}{2} \lambda_j \left(\partial_j \Pi^{ji} \right) - \Lambda_{ij} p^{ij} - \Lambda_i p^i \right]. \end{aligned} \tag{50}$$

Performing in (50) the integration over $\{\pi^{ijk}, \Pi^{ij}, p^{ij}, p^i, \Lambda_{ij}, \Lambda_i\}$ and making the notations

$$\tfrac{1}{m^2} \bar{\lambda}_{ij} \equiv \bar{A}_{ij0}, \qquad \tfrac{1}{4} \lambda_i \equiv B_{i0}, \tag{51}$$

then (50) can be written as

$$\begin{aligned} S'''_{GU} = & \int d^D x \left[-\tfrac{1}{48} F_{ijkl} F^{ijkl} - \tfrac{1}{12} \bar{F}_{0ijk} \bar{F}^{0ijk} - \tfrac{m^2}{12} A_{ijk} A^{ijk} - \tfrac{m^2}{4} \bar{A}_{ij0} \bar{A}^{ij0} \right. \\ & \left. - \tfrac{m^2}{6} A_{ijk} F^{ijk} - \tfrac{m^2}{2} \bar{A}_{ij0} F^{ij0} - \tfrac{m^2}{12} F_{ijk} F^{ijk} - \tfrac{m^2}{4} F_{0ij} F^{0ij} \right], \end{aligned} \tag{52}$$

where

$$\bar{F}_{0ijk} = \partial_0 A_{ijk} - \partial_{[i} \bar{A}_{jk]0}, \tag{53}$$

$$F_{ijk} = \partial_{[i} B_{jk]}, \qquad F_{0i} = 2 \left(\partial_0 B_{ij} + \partial_{[i} B_{j]0} \right). \tag{54}$$

The functional (52) associated with the reducible first-class system takes now a manifestly Lorentz covariant form

$$\begin{aligned} S'''_{GU} \left[\bar{B}_{\mu\nu}, \bar{A}_{\mu\nu\rho} \right] = & \int d^D x \left[-\tfrac{1}{48} \bar{F}_{\mu\nu\rho\lambda} \bar{F}^{\mu\nu\rho\lambda} \right. \\ & \left. - \tfrac{1}{12} \left(F_{\mu\nu\rho} - m\bar{A}_{\mu\nu\rho} \right) \left(F^{\mu\nu\rho} - m\bar{A}^{\mu\nu\rho} \right) \right], \end{aligned} \tag{55}$$

with

$$\bar{A}_{\mu\nu\rho} \equiv \left(\bar{A}_{0ij}, A_{ijk} \right), \qquad \bar{F}_{\mu\nu\rho\lambda} = \partial_{[\mu} \bar{A}_{\nu\rho\lambda]}, \tag{56}$$

$$\bar{B}_{\mu\nu} = -\tfrac{1}{m} B_{\mu\nu}, \qquad F_{\mu\nu\rho} = \partial_{[\mu} \bar{B}_{\nu\rho]}, \tag{57}$$

and describes precisely the (Lagrangian) Stückelberg coupling [11] between the 2-form $\bar{B}_{\mu\nu}$ and 3-form $\bar{A}_{\mu\nu\rho}$.

5 Conclusion

In this paper we realized the path-integral quantization of the massive 2- and 3-forms using GU method. In the framework of GU approach, starting from the original canonical Hamiltonian, we generated a first-class Hamiltonian with respect to the first-class constraint subset. We built the Hamiltonian path integral of the GU first-class system and then eliminated the auxiliary fields and performed some variable redefinitions such that the path integral finally takes a manifestly Lorentz covariant form. It is interesting to remark that this approaches require an appropriate extension of the phase-space in order to render a manifestly covariant path integral. In the case of massive 2-forms the GU method allowed the identification of the Lagrangian path integral for Stückelberg-coupled 1- and 2-forms and for massive 3-forms the GU approach allowed the identification of the Lagrangian path integral for Stückelberg-coupled 2- and 3-forms.

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