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Interactions between Weyl Graviton and Massless Spin-3/2 Particles. No-go Results

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Abstract

The cross-couplings between one Weyl graviton [described in the free limit by the linearized Weyl actions] and various type of massless spinor-vectors are studied with the help of the deformation theory based on local BRST cohomology. Under the hypotheses of locality, analyticity of the interactions in the coupling constant, Poincaré invariance, (background) Lorentz invariance, and the preservation of the number of derivatives on each field, we prove that there are no consistent cross-interactions one Weyl graviton and a massless spin-3/2 particle.

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1 Introduction

The study of Weyl gravitons is important in view of the remarkable properties of conformal supergravity [1], as well as by the renewed interest in Weyl gravity [2] in connection with the ADS/CFT correspondence. On the other hand, the construction of conformal SUGRA models [3, 4, 5, 6], requires the investigation of consistent couplings that can be introduced between one Weyl graviton and a gauge spinor-vector.

The aim of this paper is to analyze the non-trivial cross-couplings that can be introduced between one Weyl graviton [described in the free limit by the linearized Weyl action] and one gauge spinor-vector [described in the free limit either by the Rarita-Schwinger action or by a Lagrangian action with three spacetime derivatives]. Thus, under the hypotheses of locality, analyticity of the interactions in the coupling constant, Poincaré invariance, (background) Lorentz invariance, and the preservation of the number of derivatives on each field, we prove that there are no consistent cross-interactions between one Weyl graviton and one Rarita-Schwinger/ Q-gravitino. Our results are obtained in the context of the deformation technique [7] combined with the local BRST cohomology [8]. The announced results strongly prove that a minimal conformal SUGRA model requires, besides a Weyl field and a gauge spinor-vector, other gauge/matter fields.

This paper is organized in five sections. In Section 2 briefly addresses the deformation procedure based on BRST symmetry. Section 3 is dedicated to the construction of the BRST symmetries of the free theories analyzed: i) one Weyl field and one massless Rarita-Schwinger field and ii) one Weyl field and one Q-gravitino field. In Section 4 we prove that there are no consistent cross-interactions between one Weyl graviton and one Rarita-Schwinger/ Q-gravitino. Section 5 exposes the main conclusions of the paper.

2 Construction of consistent interactions

2.1 Setting the problem

We begin with a "free" gauge theory, described by a Lagrangian action $S_0\left[\Phi^{\alpha_0}\right]$, which is assumed to be invariant under some gauge transformations

$$\delta_\epsilon \Phi^{\alpha_0} = Z^{\alpha_0}_{\ \alpha_1}\left(\Phi\right)\epsilon^{\alpha_1},\ \frac{\delta S_0}{\delta \Phi^{\alpha_0}} Z^{\alpha_0}_{\ \alpha_1}\left(\Phi\right) = 0, \tag{1}$$

such that the gauge algebra reads as

$$\begin{aligned} &Z^{\beta_0}_{\ \alpha_1}\left(\Phi\right)\frac{\delta Z^{\alpha_0}_{\ \beta_1}\left(\Phi\right)}{\delta \Phi^{\beta_0}} - Z^{\beta_0}_{\ \beta_1}\left(\Phi\right)\frac{\delta Z^{\alpha_0}_{\ \alpha_1}\left(\Phi\right)}{\delta \Phi^{\beta_0}} = \\ &C^{\lambda_1}_{\ \alpha_1\beta_1}\left(\Phi\right) Z^{\alpha_0}_{\ \lambda_1}\left(\Phi\right) + M^{\alpha_0\beta_0}_{\ \alpha_1\beta_1}\left(\Phi\right)\frac{\delta S_0}{\delta \Phi^{\beta_0}}. \end{aligned} \tag{2}$$

We consider the problem of constructing consistent interactions among the fields Φ^{α_0} such that the couplings preserve the field spectrum and the number of the independent gauge symmetries. In view of this, we deform the original action S_0

$$S_0 \longrightarrow \bar{S}_0 = S_0 + \lambda \overset{(1)}{S_0} + \lambda^2 \overset{(2)}{S_0} + \cdots \tag{3}$$

and the original gauge symmetries,

$$Z^{\alpha_0}_{\ \alpha_1} \longrightarrow \bar{Z}^{\alpha_0}_{\ \alpha_1} = Z^{\alpha_0}_{\ \alpha_1} + \lambda \overset{(1)}{Z}{}^{\alpha_0}_{\ \alpha_1} + \lambda^2 \overset{(2)}{Z}{}^{\alpha_0}_{\ \alpha_1} + \cdots \tag{4}$$

in such a way that the new gauge transformations $\bar{\delta}_\varepsilon \Phi^{\alpha_0} = \bar{Z}^{\alpha_0}_{\alpha_1}\varepsilon^{\alpha_1}$ are indeed gauge symmetries of the full action (3)

$$\frac{\delta\left(S_0 + \lambda \overset{(1)}{S_0} + \lambda^2 \overset{(2)}{S_0} + \cdots\right)}{\delta \Phi^{\alpha_0}}\left(Z^{\alpha_0}_{\ \alpha_1} + \lambda \overset{(1)}{Z}{}^{\alpha_0}_{\ \alpha_1} + \lambda^2 \overset{(2)}{Z}{}^{\alpha_0}_{\ \alpha_1} + \cdots\right) = 0. \tag{5}$$

By projecting the equation (5) on the various powers in the deformation parameter [also known as coupling constant] we obtain an equivalent tower of equations that simultaneously involves the objects $\overset{(k)}{S_0}$ and $\overset{(k)}{Z}{}^{\alpha_0}_{\ \alpha_1}$.

As it will be seen below, a more convenient way to construct the consistent interactions relies on the cohomological approach, based on the BRST symmetry. The cohomological approach systematizes the recursive construction to co-cycles of the BRST differential. Finally, by reformulating the problem of consistent interactions at a cohomological level, one can bring in the powerful tools of homological algebra.

2.2 Cohomological reformulation

At the level of the BRST formalism, the entire gauge structure of a theory is completely captured by the BRST differential, s. The main features of s are its nilpotency, $s^2 = 0$, and canonical action [in a structure named antibracket]. Denoting by $(,)$ the antibracket, and by S the canonical generator of the Lagrangian BRST symmetry

$$sF = (F, S) \tag{6}$$

the nilpotency of s is equivalent to the classical master equation

$$(S, S) = 0. \tag{7}$$

In agreement with the structure (1)–(2) of the gauge algebra, the solution to the master equation (7) starts like

$$S = S_0 + \Phi^*_{\alpha_0} Z^{\alpha_0}_{\ \ \alpha_1} \eta^{\alpha_1} + \frac{1}{2}\left(\eta^*_{\lambda_1} C^{\lambda_1}_{\ \ \alpha_1\beta_1} - \frac{1}{2}\Phi^*_{\alpha_0}\Phi^*_{\beta_0} M^{\alpha_0\beta_0}_{\ \ \alpha_1\beta_1}\right)\eta^{\alpha_1}\eta^{\beta_1} + \cdots , \tag{8}$$

where $\Phi^*_{\alpha_0}$ represent the antifields associated with the original fields, η^{α_1} are the ghosts corresponding to the gauge parameters ϵ^{α_1}, and $\eta^*_{\lambda_1}$ denote the antifields of the ghosts.

Due to the fact that the solution to the master equation contains all the information on the gauge structure of a given theory, we can reformulate the problem of introducing consistent interactions as a deformation problem of the solution to the master equation corresponding to the "free" theory. If an interacting gauge theory can be consistently constructed, then the solution S to the master equation associated with the "free" theory can be deformed into a solution $\bar{S}$

$$\begin{aligned} S \rightarrow \bar{S} &= S + \lambda S_1 + \lambda^2 S_2 + \cdots \\ &= S + \lambda \int d^D x\, a + \lambda^2 \int d^D x\, b + \lambda^3 \int d^D x\, c \cdots , \end{aligned} \tag{9}$$

of the master equation for the deformed theory

$$\left(\bar{S}, \bar{S}\right) = 0, \tag{10}$$

such that both the ghost and antifield spectra of the initial theory are preserved. The equation (10) splits, according to the various orders in λ, into

$$(S, S) = 0, \tag{11}$$

$$2\,(S_1, S) = 0, \tag{12}$$

$$2\,(S_2, S) + (S_1, S_1) = 0, \tag{13}$$

$$\vdots$$

If we denote by Δ the nonintegrated density of the antibracket (S_1, S_1) then the local expressions of the equations (12)–(13) are

$$sa = \partial_\mu m^\mu, \tag{14}$$

$$2sb + \Delta = \partial_\mu n^\mu, \tag{15}$$

$$\vdots$$

were m^μ and n^μ are some local currents.

3 BRST symmetries of the free models

3.1 Linearized Weyl graviton and massless Rarita-Schwinger gravitino

The Lagrangian action of the "free" theory is written as the sum between the linearized Weyl gravity action [9] and the massless Rarita-Schwinger action [10]

$$S_0^{\mathrm{W,RS}}\left[h_{\mu\nu},\psi_\mu\right]=\frac{1}{2}\int \mathrm{d}^4x\left(\mathcal{W}_{\mu\nu\alpha\beta}\mathcal{W}^{\mu\nu\alpha\beta}-\mathrm{i}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho\right), \tag{16}$$

where $\mathcal{W}_{\mu\nu\alpha\beta}$ is the linearized Weyl tensor in four space-time dimensions, given in terms of the linearized Riemann tensor $\mathcal{R}_{\mu\nu\alpha\beta}$ and of its traces by

$$\mathcal{W}_{\mu\nu\alpha\beta}=\mathcal{R}_{\mu\nu\alpha\beta}-\frac{1}{2}\left(\sigma_{\mu[\alpha}\mathcal{R}_{\beta]\nu}-\sigma_{\nu[\alpha}\mathcal{R}_{\beta]\mu}\right)+\frac{1}{6}\mathcal{R}\sigma_{\mu[\alpha}\sigma_{\beta]\nu}. \tag{17}$$

Throughout the paper we work with the flat metric of 'mostly minus' signature $\sigma_{\mu\nu}=(+---)$. The notation $[\mu\ldots\nu]$ signifies full antisymmetry with respect to the indices between brackets without normalization factors [i.e. the independent terms appear only once and are not multiplied by overall numerical factors]. The linearized Riemann tensor is expressed by

$$\begin{aligned}\mathcal{R}_{\mu\nu\alpha\beta} &= \frac{1}{2}\left(\partial_\mu\partial_\beta h_{\nu\alpha}+\partial_\nu\partial_\alpha h_{\mu\beta}-\partial_\nu\partial_\beta h_{\mu\alpha}-\partial_\mu\partial_\alpha h_{\nu\beta}\right)\\ &\equiv \frac{1}{2}\partial_{[\mu}h_{\nu][\alpha,\beta]},\end{aligned} \tag{18}$$

while its simple and respectively double traces read as

$$\mathcal{R}_{\mu\nu}=\sigma^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta},\qquad \mathcal{R}=\sigma^{\mu\nu}\mathcal{R}_{\mu\nu}. \tag{19}$$

The linearized Weyl tensor can be also expressed in terms of the symmetric tensor $\mathcal{K}_{\mu\nu}$ like

$$\mathcal{W}_{\mu\nu\alpha\beta}=\mathcal{R}_{\mu\nu\alpha\beta}-\left(\sigma_{\mu[\alpha}\mathcal{K}_{\beta]\nu}-\sigma_{\nu[\alpha}\mathcal{K}_{\beta]\mu}\right), \tag{20}$$

where

$$\mathcal{K}_{\mu\nu}=\frac{1}{2}\left(\mathcal{R}_{\mu\nu}-\frac{1}{6}\sigma_{\mu\nu}\mathcal{R}\right). \tag{21}$$

The spinor-vector ψ_μ has [Majorana] real components and the γ-matrices are in the Majorana representation

$$\gamma_\mu^*=-\gamma_\mu,\qquad \gamma_\mu^T=-\gamma_0\gamma_\mu\gamma_0,\qquad \left(\mu=\overline{0,3}\right), \tag{22}$$

where $*$ and T in (22) signifies the operations of complex conjugation and respectively of transposition. The theory described by (16) possesses an irreducible and abelian generating set of gauge transformations

$$\delta_{\epsilon,\theta}h_{\mu\nu} = \partial_{(\mu}\epsilon_{\nu)}+2\sigma_{\mu\nu}\epsilon, \tag{23}$$

$$\delta_{\epsilon,\theta}\psi_\mu = \partial_\mu\theta, \tag{24}$$

where the gauge parameters ϵ_μ [responsible for so-called linearized version of the diffeomorphisms] and ϵ [corresponding for the so-called conformal invariance of Weyl theory] are bosonic and θ is a fermionic spinor with real components. The notation $(\mu\nu)$ signifies symmetry with respect to the indices between parentheses without the factor 1/2.

The BRST generators together with their degrees and behavior under the complex involution are listed below

	$h_{\mu\nu}$	ψ_μ	η_μ	ξ	C	$h^{*\mu\nu}$	$\psi^{*\mu}$	$\eta^{*\mu}$	ξ^*	C^*
ε	0	1	1	1	0	1	0	0	0	1
agh	0	0	0	0	0	1	1	2	2	2
pgh	0	0	1	1	1	0	0	0	0	0
$\star$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$+$

Since the gauge generators of the free theory are field independent, it follows that the BRST differential simply reduces to

$$s = \delta + \gamma, \tag{25}$$

where δ represents the Koszul-Tate differential and γ stands for the exterior derivative along the gauge orbits. In our case the solution to the master equation reads as

$$S^{\mathrm{W,RS}} = S_0^{\mathrm{W,RS}}\left[h_{\mu\nu}, \psi_\mu\right] + \int \mathrm{d}^4x \left[h^{*\mu\nu}\left(\partial_{(\mu}\eta_{\nu)} + 2\sigma_{\mu\nu}\xi\right) + \psi^{*\mu}\partial_\mu C\right]. \tag{26}$$

3.2 Linearized Weyl graviton and massless Q-gravitino

The Lagrangian action of the "free" theory is written as the sum between the linearized Weyl gravity action [9] and the massless Q-gravitino action [1]

$$S_0^{\mathrm{W,Q}}\left[h_{\mu\nu}, \psi_\mu\right] = \tfrac{1}{2}\int \mathrm{d}^4x \left(\mathcal{W}_{\mu\nu\alpha\beta}\mathcal{W}^{\mu\nu\alpha\beta} - 8\mathrm{i}\bar{\phi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\phi_\rho\right), \tag{27}$$

where $\mathcal{W}_{\mu\nu\alpha\beta}$ are expressed in (17) and ϕ_μ are the components of a Majorana spinor-vector

$$\phi_\mu = \frac{\mathrm{i}}{3}\left(\gamma^\rho\partial_{[\rho}\psi_{\mu]} + \frac{1}{2}\gamma_{\mu\nu\rho}\partial^\nu\psi^\rho\right). \tag{28}$$

The action (27) is invariant under the irreducible and abelian generating set of gauge transformations consisting in (23) and

$$\delta_{\epsilon,\theta}\psi_\mu = \partial_\mu\theta + \mathrm{i}\gamma_\mu\varepsilon, \tag{29}$$

where θ and ε are fermionic spinors with real components.

The BRST generators together with their degrees and behavior under the complex involution are listed below

	$h_{\mu\nu}$	ψ_μ	η_μ	ξ	C	χ	$h^{*\mu\nu}$	$\psi^{*\mu}$	$\eta^{*\mu}$	ξ^*	C^*	χ^*
ε	0	1	1	1	0	0	1	0	0	0	1	1
agh	0	0	0	0	0	0	1	1	2	2	2	2
pgh	0	0	1	1	1	1	0	0	0	0	0	0
$\star$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$+$

Since the gauge generators of the free theory are field independent, it follows that the BRST differential simply reduces to (25). In the present situation, the solution to the master equation reads as

$$S^{\mathrm{W,Q}} = S_0^{\mathrm{W,Q}}\left[h_{\mu\nu}, \psi_\mu\right] + \int \mathrm{d}^4x \left[h^{*\mu\nu}\left(\partial_{(\mu}\eta_{\nu)} + 2\sigma_{\mu\nu}\xi\right) + \psi^{*\mu}\left(\partial_\mu C + \mathrm{i}\gamma_\mu\chi\right)\right]. \tag{30}$$

4 Results

4.1 Case I: Weyl and Rarita-Schwinger fields

4.1.1 First-order deformation

The non-integrated density of the first-order deformation [the solution to (14)] can be naturally decomposed as

$$a = a^{\mathrm{RS}} + a^{\mathrm{W}} + a^{\mathrm{W-RS}}. \tag{31}$$

The first term in the right hand side of (31) depends only on the BRST generators due to the Rarita-Schwinger field, the second one contains only BRST generators due to the Weyl field and the last term in the right hand side on (31) effectively mixes both sectors. Each of the terms in the right hand side of (31) satisfies an individual equation of the type (14) [obtained by the projection of (14) on various sectors].

The piece a^{RS} is known [11] that can be chosen to be trivial

$$a^{\mathrm{RS}} = 0 \tag{32}$$

and a^{W} [9] reads as

$$\begin{aligned} a^{\mathrm{W}} &= \eta^{*\mu}\left(\frac{1}{2}\eta^{\nu}\partial_{[\mu}\eta_{\nu]} + \eta_{\mu}\xi\right) - \xi^{*}\eta_{\mu}\partial^{\mu}\xi \\ &\quad -\frac{1}{2}h^{*\mu\nu}\eta^{\rho}\left(\partial_{(\mu}h_{\nu)\rho} - 2\partial_{\rho}h_{\mu\nu}\right) \\ &\quad +2h^{*\mu\nu}h_{\mu\nu}\xi + \frac{1}{2}h^{*\mu\nu}h_{\rho(\mu}\partial_{\nu)}\eta^{\rho} + a_0^{\mathrm{W}} \end{aligned} \tag{33}$$

where a_0^{W} is the cubic vertex of the Weyl Lagrangian density.

It can be shown that the last term in the right hand side of (31) can be decomposed accordingly antighost number as

$$a^{\mathrm{W-RS}} = \sum_{i=0}^{2} a_i^{\mathrm{W-RS}}, \quad \mathrm{agh}\left(a_i^{\mathrm{W-RS}}\right) = \mathrm{pgh}\left(a_i^{\mathrm{W-RS}}\right) = i, \quad i = \overline{0,2}, \tag{34}$$

where the terms in (34) are subject to the equations

$$\gamma a_2^{\mathrm{W-RS}} = 0, \tag{35}$$

$$\delta a_i^{\mathrm{W-RS}} + \gamma a_{i-1}^{\mathrm{W-RS}} = \partial^{\mu} \overset{(i-1)}{m}{}_{\mu}, \quad i = 1, 2. \tag{36}$$

The results concerning the pieces of the non-integrated density of the first-order deformation in the interacting sector are summarized in the following theorems [12].

Theorem 1 *Without any derivative assumption, the decomposition (34) reduces to*

$$a^{\mathrm{W-RS}} = a_0^{\mathrm{W-RS}}, \tag{37}$$

where $a_0^{\mathrm{W-RS}}$ satisfies the equation

$$\gamma a_0^{\mathrm{W-RS}} = \partial^{\mu} \overset{(0)}{m}_{\mu}. \tag{38}$$

Theorem 2 *The general solution to the equation (38) that satisfies the assumptions made in the beginning [including the derivative order hypothesis] reduces to the trivial one*

$$a_0^{\mathrm{W-RS}} = 0. \tag{39}$$

4.1.2 Higher-order deformations

On behalf of the outcomes concerning first-order deformations, it results that the second order-deformation reduces to

$$b = b^{\mathrm{W}} \tag{40}$$

and consequently the higher-order deformations depend only on the BRST generators in the Weyl. These imply that the deformed solution of the master equation reduces to

$$\bar{S}^{\mathrm{W,RS}} = \bar{S}^{\mathrm{W}} + \int \mathrm{d}^4x \left(-\frac{\mathrm{i}}{2}\bar{\psi}_{\mu}\gamma^{\mu\nu\rho}\partial_{\nu}\psi_{\rho} + \psi^{*\mu}\partial_{\mu}C\right), \tag{41}$$

where $\bar{S}^{\mathrm{W}}$ represents the solution of the master equation corresponding to Weyl gravity, written in terms of

$$g_{\mu\nu} = \sigma_{\mu\nu} + \lambda h_{\mu\nu}. \tag{42}$$

4.2 Case II: Weyl and Q-gravitino fields

4.2.1 First-order deformation

As in the first case, the non-integrated density of the first-order deformation reduces to

$$a = a^{\mathrm{Q}} + a^{\mathrm{W}} + a^{\mathrm{W-Q}}. \tag{43}$$

The first term in the right hand side of (43) depends only on the BRST generators due to the Q-gravition field, the second one contains only BRST generators due to the Weyl field and the last term in the right hand side on (43) effectively mixes both sectors. As in the previous situation, each of the terms in the right hand side of (43) satisfies an individual equation of the type (14).

The first-term in the right-hand side of the decomposition (43), that comply with the assumptions made in the beginning, reads as

$$a^{\mathrm{Q}} = q_1 \overline{\mathcal{F}}^{\mu\nu} \mathcal{F}_{\mu\nu} + q_2 \overline{\mathcal{F}}_{\mu\nu} \gamma^{\mu\nu\rho\lambda} \mathcal{F}_{\rho\lambda} \tag{44}$$

where q_1 and q_2 are arbitrary real constants and the objects $\mathcal{F}_{\mu\nu}$ are

$$\mathcal{F}_{\mu\nu} = \partial_{[\mu}\psi_{\nu]} + \mathrm{i}\gamma_{[\mu}\phi_{\nu]}. \tag{45}$$

The second term in (43) is expressed by (33).

The results concerning the pieces of the non-integrated density of the first-order deformation in the interacting sector are summarized in the following theorem.

Theorem 3 *The general solution of the equation (14) in the interacting sector that satisfies the assumptions made in the beginning [including the derivative order hypothesis] is parametrized by a real constant and its concrete form is reads as*

$$\begin{aligned}
a^{\mathrm{W-Q}} \quad = \quad & k\left\{\frac{\mathrm{i}}{2}\eta^*_\mu \bar{C}\gamma^\mu C - \xi^*\bar{C}\chi + C^*\left[C\xi + \frac{1}{4}\gamma^{\mu\nu}C\partial_{[\mu}\eta_{\nu]} + 2\left(\partial^\mu C\right)\eta_\mu\right]\right. \\
& +\chi^*\left[\mathrm{i}\gamma^\mu C\partial_\mu\xi - \chi\xi + 2\left(\partial^\mu\chi\right)\eta_\mu + \frac{1}{4}\gamma^{\mu\nu}\chi\partial_{[\mu}\eta_{\nu]}\right] \\
& +\psi^{*\mu}\left[\psi^\nu\partial_{[\mu}\eta_{\nu]} + 2\left(\partial^\nu\psi_\mu\right)\eta_\nu - \psi_\mu\xi - \left(\partial^\nu C\right)h_{\mu\nu}\right. \\
& \left.-\frac{1}{4}\gamma^{\alpha\beta}\left(C\partial_{[\alpha}h_{\beta]\mu} - \psi_\mu\partial_{[\mu}\eta_{\nu]}\right)\right] - h^{*\mu\nu}\bar{C}\gamma_{(\mu}\psi_{\nu)} \\
& -2\mathrm{i}\mathcal{W}^{\mu\alpha\nu\beta}\left[\bar{\psi}_\mu\left(\gamma_\alpha\partial_{[\nu}\psi_{\beta]} + 2\gamma_\beta\partial_\nu\psi_\alpha\right) - \bar{\psi}_\mu\gamma_\alpha\mathcal{F}_{\nu\beta}\right] \\
& +\left[\left(\partial_{[\alpha}h_{\beta]\nu}\right)\bar{\psi}_\mu\gamma^{\alpha\beta} + 4h_{\mu\rho}\left(\partial^\rho\bar{\psi}_\nu\right) - 2h\left(\partial_{[\mu}\bar{\psi}_{\nu]}\right) - 2\left(\partial_{[\mu}h_{\nu]\rho}\right)\psi^\rho\right. \\
& \left.+2h\bar{\mathcal{F}}_{\mu\nu}\right]\hat{F}^{\mu\nu} - 2\left[2\mathrm{i}\mathcal{K}_{\nu\rho}\bar{\psi}_\mu\gamma^\rho - \frac{1}{2}\left(\partial_{[\alpha}h_{\beta]\mu}\right)\bar{\phi}_\nu\gamma^{\alpha\beta} + \left(\partial_{[\mu}h_{\nu]\rho}\right)\bar{\phi}^\rho\right. \\
& \left.\left.+2\left(\partial_\nu h - \partial^\rho h_{\nu\rho}\right)\bar{\phi}_\mu\right]\mathcal{F}^{\mu\nu} - 4\bar{\phi}_\nu\left(\partial^\rho\mathcal{F}^{\mu\nu}\right)h_{\mu\rho}\right\}.
\end{aligned} \tag{46}$$

4.2.2 Higher-order deformations

The nonintegrated density of the second-order deformation admits a decomposition similar to (43)

$$b = b^{\mathrm{Q}} + b^{\mathrm{W}} + b^{\mathrm{W-Q}}. \tag{47}$$

The terms in the right-hand side of (47) are subject to the equations

$$\begin{aligned}
2sb^{\mathrm{Q}} + \Delta^{\mathrm{Q}} &= \partial^\mu n^{\mathrm{Q}}_\mu, &(48)\\
2sb^{\mathrm{W}} + \Delta^{\mathrm{W}} &= \partial^\mu n^{\mathrm{W}}_\mu, &(49)\\
2sb^{\mathrm{W-Q}} + \Delta^{\mathrm{W-Q}} &= \partial^\mu n^{\mathrm{W-Q}}_\mu. &(50)
\end{aligned}$$

In (48)–(50) Δ^{Q}, Δ^{W} and $\Delta^{\mathrm{W-Q}}$ represent the projections of Δ on various sectors. By direct computation it infers that Δ^{Q} admits the development [according with antighost number]

$$\Delta^{\mathrm{Q}} = \Delta_0^{\mathrm{Q}} + \Delta_1^{\mathrm{Q}} + \Delta_2^{\mathrm{Q}} \tag{51}$$

that induces a similar decomposition for b^{Q}

$$b^{\mathrm{Q}} = b_0^{\mathrm{Q}} + b_1^{\mathrm{Q}} + b_2^{\mathrm{Q}}. \tag{52}$$

Inserting (51) and (52) in (48) and projecting the obtained equation on various antighost numbers, we derive the tower of equations

$$2\gamma b_2^{\mathrm{Q}} + \Delta_2^{\mathrm{Q}} = \partial^\mu \overset{(2)}{n}{}^{\mathrm{Q}}_{\mu}, \tag{53}$$

$$2\delta b_2^{\mathrm{Q}} + 2\gamma b_1^{\mathrm{Q}} + \Delta_1^{\mathrm{Q}} = \partial^\mu \overset{(1)}{n}{}^{\mathrm{Q}}_{\mu}, \tag{54}$$

$$2\delta b_1^{\mathrm{Q}} + 2\gamma b_0^{\mathrm{Q}} + \Delta_0^{\mathrm{Q}} = \partial^\mu \overset{(0)}{n}{}^{\mathrm{Q}}_{\mu}. \tag{55}$$

According to (53), Δ_2^{Q} has to be γ-exact modulo d. But, by direct computation we get

$$\begin{aligned}\Delta_2^{\mathrm{Q}} = \; & \gamma \left\{ \mathrm{i}k^2 \left[C^* \left(2\psi_\mu \bar{C}\gamma^\mu C - \gamma^{\mu\nu} C\bar{C}\gamma_\mu\psi_\nu \right) \right.\right. \\ & \left.\left. +2\chi^*\gamma^\mu C \left(\bar{\chi}\psi_\mu - \bar{C}\phi_\mu \right) + \chi^* \left(2\phi_\mu \bar{C}\gamma^\mu C - \gamma^{\mu\nu}\chi\bar{C}\gamma_\mu\psi_\nu \right) \right] \right\} \\ & + \frac{3k^2}{4} \left[C^* \left(2\gamma_\mu \chi\bar{C}\gamma^\mu C + \gamma^{\mu\nu}\chi\bar{C}\gamma_{\mu\nu} C \right) \right. \\ & \left. + \chi^* \left(\gamma_\mu C\bar{\chi}\gamma^\mu\chi + \gamma^{\mu\nu} C\bar{\chi}\gamma_{\mu\nu}\chi \right) . \right. \end{aligned} \tag{56}$$

Comparing (56) with (53) we infer that k must vanishes

$$k = 0. \tag{57}$$

Replacing the result (57) into (46) we conclude

$$a^{\mathrm{W-Q}} = 0. \tag{58}$$

The last result together with (44) reduce the decomposition (47) to

$$b = b^{\mathrm{W}} \tag{59}$$

and consequently the higher-order deformations depend only on the BRST generators in the Weyl. These imply that the deformed solution of the master equation reduces to

$$\begin{aligned}\bar{S}^{\mathrm{W,Q}} = \; & \bar{S}^{\mathrm{W}} + \int \mathrm{d}^4x \left[-4\mathrm{i}\bar{\phi}_\mu \gamma^{\mu\nu\rho}\partial_\nu\phi_\rho + \psi^{*\mu}\left(\partial_\mu C + \mathrm{i}\gamma_\mu\chi \right) \right. \\ & \left. + \lambda \left(q_1 \overline{\mathcal{F}}^{\mu\nu}\mathcal{F}_{\mu\nu} + q_2 \overline{\mathcal{F}}_{\mu\nu}\gamma^{\mu\nu\rho\lambda}\mathcal{F}_{\rho\lambda} \right) \right], \end{aligned} \tag{60}$$

where $\bar{S}^{\mathrm{W}}$ has the same significance as in the above.

5 Conclusion

To conclude with, in the above we have investigated the cross-couplings that can be introduced between the Weyl graviton and the massless spin-3/2 fields from the BRST formalism point of view. Thus, under the general conditions of locality, smoothness, (background) Lorentz invariance, Poincaré invariance and preservation of the number of derivatives with respect to each field [the last hypothesis was made only in antighost number zero], we have proved that there are no such cross-couplings. This represents an elegant proof of the fact that a minimal conformal SUGRA model requires, besides a Weyl field and a gauge spinor-vector, other gauge/matter fields.

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