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Non(Anti)commutative Field Theories: Model Building and Renormalizability Properties

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Abstract

We discuss one particular model of non(anti)commutative superspace. The deformation is nonhermitian and given in terms of the SUSY covariant derivatives D_α. We construct a deformed Wess-Zumino action and analyze its renormalizability properties. One-loop divergences in the two-point, three-point and four-point Green functions are calculated. In the general model we find that divergences in the four-point function cannot be absorbed and thus our model is not renormalizable. However, there is a special choice of the free parameters in the model that renders renormalizability. We discuss this choice and other possibilities to render the model renormalizable.

Keywords: supersymmetry, nonhermitian twist, deformed Wess-Zumino model, renormalizability, one-loop effective action

1 Introduction

It is well known that Quantum Field Theory encounters problems at very high energies and very short distances. This suggests that the structure of space-time has to be modified at these scales. A way to modify the structure of space-time is to deform the usual commutation relations between coordinates; this gives a noncommutative (NC) space. Different models of noncommutativity were discussed in the literature, see [1], [2] and [3] for references. A deformation of Standard Model on the canonically deformed space-time was constructed in [4] and some phenomenological consequences were analyzed in [5]. Renormalizability of different noncommutative field theory models was discussed in [6].

A step further in this direction is the modification of superspace and introduction of so-called non(anti)commutativity. A strong motivation for this comes from string theory. Namely, it was discovered that a noncommutativity of the superspace coordinates can arise when a superstring moves in a constant gravitino or graviphoton background [7], [8]. There has been a lot of work on this subject and different ways of deforming superspace have been discussed. We shall mention some of them in Section 3.

In this paper we analyze one particular model of the deformed superspace. In the next section we describe our approach. We use the example of twisted Poincaré symmetry [9] to explain both the motivation and the technical details. In Section 3 we apply the twist approach to superspace and construct a deformation of the Wess-Zumino model. In order to see how a deformation (nonanticommutativity) affects the renormalizability properties of the model, we calculate the divergent part of the one-loop effective action. Our model is not renormalizable in general; however, there exists a special choice of the free parameters that renders renormalizability. In the last section we discuss our results.

2 Noncommutative spaces and symmetries: twist approach

There are different ways to realize a noncommutative space and to formulate a physical model on it, see [1] and [2]. We will follow the approach of [2]. In this section we will describe the most important steps of that approach, the details can be found in [2].

The noncommutative space $\hat{\mathcal{A}}_{\hat{x}}$ can be introduced as a quotient

$$\hat{\mathcal{A}}_{\hat{x}} = \frac{\mathbb{C}[\hat{x}^0, \ldots, \hat{x}^3][[h]]}{I_{\hat{\mathcal{R}}}}. \tag{1}$$

Here a two-sided ideal $I_{\hat{\mathcal{R}}}$ is given by the linear span of elements

$$I_{\hat{\mathcal{R}}}: \qquad (\hat{x}\ldots\hat{x})\Big([\hat{x}^m, \hat{x}^n] - i\Theta^{mn}(\hat{x})\Big)(\hat{x}\ldots\hat{x}), \tag{2}$$

where $(\hat{x}\ldots\hat{x})$ stands for an arbitrary product of the coordinates $\hat{x}^m$ in the algebra $\mathbb{C}[\hat{x}^0, \ldots, \hat{x}^3][[h]]$. Note that we work in four dimensions[1] and Latin indices go from 0 to 3. The algebra $\mathbb{C}[\hat{x}^0, \ldots, \hat{x}^3][[h]]$ is freely generated by $\hat{x}^m$ coordinates and formal power series in the parameter h are included. We also have that $\Theta^{mn}(\hat{x}) \in \mathbb{C}[\hat{x}^0, \ldots, \hat{x}^3][[h]]$ and for $h = 0$ the usual algebra of commuting coordinates is obtained.

The defining relation of the deformed space

$$[\hat{x}^m, \hat{x}^n] = i\Theta^{mn}(\hat{x}) \tag{3}$$

is very general and one usually considers some special examples of it. Among them there are three very important ones

$$\begin{aligned} \text{Canonically deformed} \quad [\hat{x}^m, \hat{x}^m] &= i\theta^{mn}, && (4)\\ \text{Lie algebra deformed} \quad [\hat{x}^m, \hat{x}^n] &= iC_l^{mn}\hat{x}^l, && (5)\\ q\text{-deformed} \quad \hat{x}^m\hat{x}^m &= \frac{1}{q}R^{mn}_{\ \ rs}\hat{x}^r\hat{x}^s. && (6) \end{aligned}$$

In the case of the canonically deformed spaces $\theta^{mn} = -\theta^{nm}$ is an antisymmetric constant matrix of mass dimension -2. For the Lie algebra deformed spaces C_l^{mn} are Lie algebra structure constants of mass dimension -1. Finally, $R^{mn}_{\ \ rs}$ is the dimensionless R-matrix of the quantum space. These three examples are important because they fulfill the Poincaré-Birkoff-Witt (PBW) property. This property enables us to map an arbitrary element $\hat{f}(\hat{x})$ of $\hat{\mathcal{A}}_{\hat{x}}$ to $f(x)$ in the space of commuting coordinates $\mathcal{A}_x$. The (noncommutative) algebra multiplication is then mapped to the so-called $\star$-product:

$$\hat{f} \cdot \hat{g}(\hat{x}) \mapsto f \star g(x) \in \mathcal{A}_x. \tag{7}$$

This product is bilinear and associative but noncommutative. The algebra of noncommuting coordinates $\hat{\mathcal{A}}_{\hat{x}}$ is then isomorphic to the algebra of commuting coordinates with the $\star$-product (instead of the usual pointwise multiplication) as multiplication. As an example, we write here the $\star$-product for the canonically deformed space. It is given by the Moyal $\star$-product

$$\begin{aligned} f \star g\,(x) &= \lim_{x\to y} e^{\frac{i}{2}\theta^{rs}\frac{\partial}{\partial x^r}\frac{\partial}{\partial y^s}} f(x)g(y) && (8)\\ &= \sum_{n=1}^{\infty}\Big(\frac{i}{2}\Big)^n\frac{1}{n!}\theta^{r_1s_1}\ldots\theta^{r_ns_n}\Big(\partial_{r_1}\ldots\partial_{r_n}f(x)\Big) \\ &\qquad\Big(\partial_{s_1}\ldots\partial_{s_n}g(x)\Big). \end{aligned}$$

Now one can define a noncommutative space as the usual space of commuting coordinates with the pointwise multiplication replaced by a noncommutative $\star$-product. This approach is very popular and different models were constructed. A noncommutative extension of the Standard Model was constructed in [4] and some phenomenological consequences were analyzed in [5].

However, there is a drawback of this approach. Namely, it is not clear what happens with symmetries of the theory in this approach. For example, the commutation relations (4) obviously break the global Lorentz symmetry, since θ^{mn} is constant. Is there a deformed symmetry which replaces the

[1] Everything said here can easily be generalized to an arbitrary number of dimensions.

global Lorentz symmetry in this case? If it exists, what is it? An answer to these question could be given using the twist formalism.

The main idea of this formalism is to first deform the symmetry of the theory and then see the consequences this deformation has on the space-time itself. Let us take the example of four dimensional Minkowski space-time and the global Poincare symmetry. The Poincaré algebra Ξ is generated by the translation generators ∂_m and the Lorentz generators M_{mn}. The full Hopf algebra reads:

$$\begin{aligned}
[\partial_m, \partial_n] &= 0, \quad [M_{mn}, \partial_r] = i(\eta_{nr}\partial_m - \eta_{mr}\partial_n), \\
[M_{mn}, M_{rs}] &= i(\eta_{nr}M_{ms} + \eta_{ms}M_{nr} - \eta_{ns}M_{mr} - \eta_{mr}M_{ns}), \\
\Delta\partial_m &= \partial_m \otimes 1 + 1 \otimes \partial_m, \quad \Delta M_{mn} = M_{mn} \otimes 1 + 1 \otimes M_{mn}, \\
\varepsilon(\partial_m) &= 0, \quad \varepsilon(M_{mn}) = 0, \\
S(\partial_m) &= -\partial_m, \quad S(M_{mn}) = -M_{mn}.
\end{aligned} \tag{9}$$

The comultiplications encodes the Leibniz rule; for example:

$$\begin{aligned}
\partial_m(f \cdot g) &= \mu\Big(\Delta(\partial_m)(f \otimes g)\Big) \\
&= \mu\Big((\partial_m f) \otimes g + f \otimes (\partial_m g)\Big) \\
&= (\partial_m f) \cdot g + f \cdot (\partial_m g),
\end{aligned} \tag{10}$$

with μ being the pointwise multiplication.

There is a well defined way to deform the symmetry Hopf algebra. In his papers [10] Drinfel'd introduced the notion of twist. The twist $\mathcal{F}$ is a bidifferential operator which belongs to $U\Xi \otimes U\Xi$, where $U\Xi$ is the universal enveloping algebra of the symmetry Lie algebra Ξ. Let us choose the twist $\mathcal{F}$ for our example in the following way

$$\mathcal{F} = e^{-\frac{i}{2}\theta^{rs}\partial_r \otimes \partial_s}, \tag{11}$$

where θ^{rs} is a constant antisymmetric matrix. Now we apply the twist (11) to the Hopf algebra (9). In this way the twisted Poincaré Hopf algebra is obtained

$$\begin{aligned}
[\partial_m, \partial_n] &= 0, \quad [M_{mn}, \partial_r] = i(\eta_{nr}\partial_m - \eta_{mr}\partial_n), \\
[M_{mn}, M_{rs}] &= i(\eta_{nr}M_{ms} + \eta_{ms}M_{nr} - \eta_{ns}M_{mr} - \eta_{mr}M_{ns}), \\
\Delta^{\mathcal{F}}\partial_m &= \mathcal{F}\Delta(\partial_m)\mathcal{F}^{-1} \\
&= \partial_m \otimes 1 + 1 \otimes \partial_m, \\
\Delta^{\mathcal{F}}M_{mn} &= \mathcal{F}\Delta(M_{mn})\mathcal{F}^{-1} \\
&= M_{mn} \otimes 1 + 1 \otimes M_{mn} \\
&\quad + \frac{1}{2}\theta^{ab}\Big(\eta_{am}\partial_n - \eta_{an}\partial_m) \otimes \partial_b + \partial_a \otimes (\eta_{bm}\partial_n - \eta_{bn}\partial_m\Big), \\
\varepsilon(\partial_m) &= 0, \quad \varepsilon(M_{mn}) = 0, \\
S(\partial_m) &= -\partial_m, \quad S(M_{mn}) = -M_{mn}.
\end{aligned} \tag{12}$$

We see immediately that the algebra remains the same, while the comultiplication changes, becomes "twisted". The twisted comultiplication is related with the deformed Leibniz rule, as we shall see in the following.

The inverse of the twist (11),

$$\mathcal{F}^{-1} = e^{\frac{i}{2}\theta^{rs}\partial_r \otimes \partial_s}, \tag{13}$$

defines the $\star$-product. For arbitrary functions f and g the $\star$-product reads

$$\begin{aligned}
f \star g &= \mu_\star\{f \otimes g\} \\
&= \mu\{\mathcal{F}^{-1} f \otimes g\} \\
&= \mu\{e^{\frac{i}{2}\partial_r \otimes \partial_s} f \otimes g\} \\
&= f \cdot g + \frac{i}{2}\theta^{mn}(\partial_m f) \cdot (\partial_n g) + \mathcal{O}(\theta^2).
\end{aligned} \tag{14}$$

We see immediately that (14) is exactly the Moyal $\star$-product (8). Therefore "twisting" of the commutative space-time with the twist (11) gives the canonically deformed space-time.

Now we can define the action of the twisted Poincaré transformations on fields and $\star$-product of fields. Let us take scalar fields as an example. The deformed infinitesimal translations and Lorentz transformations are given by:

$$\begin{aligned}
\delta^\star_\epsilon \phi &= -\epsilon^m \partial_m \phi, \\
\delta^\star_\omega \phi &= -\frac{1}{2}\omega^{mn} M_{mn}\phi,
\end{aligned} \tag{15}$$

with ϵ and $\omega^{mn} = -\omega^{nm}$ constant parameters. On the $\star$-product of two scalar fields we have

$$\begin{aligned}
\delta^\star_\epsilon(\phi_1 \star \phi_2) &= (\delta^\star_\epsilon \phi_1)\star\phi_2 + \phi_1 \star (\delta^\star_\epsilon \phi_2) \\
&= -\epsilon^m \partial_m(\phi_1\star\phi_2),
\end{aligned} \tag{16}$$

$$\begin{aligned}
\delta^\star_\omega(\phi_1 \star \phi_2) &= (\delta^\star_\omega \phi_1)\star\phi_2 + \phi_1 \star (\delta^\star_\omega \phi_2) \\
&\quad + \frac{1}{2}\theta^{ab}\omega^{mn}\Big(\eta_{am}(\partial_n\phi_1) - \eta_{an}(\partial_m\phi_1)) \star (\partial_b\phi_2) \\
&\quad + (\partial_a\phi_1)\star(\eta_{bm}(\partial_n\phi_2 - \eta_{bn}(\partial_m\phi_2)\Big) \\
&= -\frac{1}{2}\omega^{mn} M_{mn}(\phi_1\star\phi_2).
\end{aligned} \tag{17}$$

The first lines in (16) and (17) follow from the deformed coproduct in (12). We see that the $\star$-product of two scalar fields is again a scalar field.

In the similar way we have

$$\begin{aligned}
\delta^\star_\omega(V^m \star \phi) &= -(\omega^l{}_n x^n)\partial_l(V^m\star\phi) + \omega^m_{\ l}(V^l\star\phi), \\
\delta^\star_\omega(V_m \star V^m) &= -(\omega^l{}_n x^n)\partial_l(V_m\star V^m), \\
&\dots
\end{aligned}$$

In this way we learned what is the symmetry of the canonically deformed space-time. The global Poincaré symmetry is replaced by the twisted Poincaré symmetry. The algebra remains the same, while the comultiplication changes. This leads to the deformed Leibniz rule (17).

This method is quite general[2] and it can be applied to different symmetries: diffeomorphisms, gauge symmetry, supersymmetry, see [2]. In the next section we describe the application to the supersymmetry.

3 Noncommutative SUSY field theory: D-deformation

We have already said in the beginning of this paper that the field theory on non(anti)commutative space-time is a very active field of research. There are different ways of introducing non(anti)commutativity and let us briefly mention some of them.

In [12] the authors combine SUSY with the κ-deformation of space-time, while in [13] SUSY is combined with the canonical deformation of space-time. In [7] a version of non(anti)commutative superspace is defined and analyzed. The anticommutation relations between the fermionic coordinates are modified in the following way

$$\{\theta^\alpha \stackrel{\star}{,} \theta^\beta\} = C^{\alpha\beta}, \quad \{\bar\theta_{\dot\alpha} \stackrel{\star}{,} \bar\theta_{\dot\beta}\} = \{\theta^\alpha \stackrel{\star}{,} \bar\theta_{\dot\alpha}\} = 0\ , \tag{18}$$

where $C^{\alpha\beta} = C^{\beta\alpha}$ is a complex, constant symmetric matrix. Such deformation is well defined only when undotted and dotted spinors are not related by the usual complex conjugation. In [7] the notion

[2] $\star$-products obtained by twist are just a subclass of the $\star$-products obtained by the Kontsevich general method [11]. Therefore, this method does not generate the most general $\star$-products.

of chirality is preserved, i.e. the deformed product of two chiral superfields is again a chiral superfield. On the other hand, one half of $\mathcal{N} = 1$ supersymmetry is broken and this is the so-called $\mathcal{N} = 1/2$ supersymmetry. Another type of deformation is introduced in [14] and [15]. There the product of two chiral superfields is not a chiral superfield but the model is invariant under the full supersymmetry. Renormalizability of different models (both scalar and gauge theories) has been discussed in [16], [17] and [15]. The twist approach was discussed in [18].

Now, let us go back to our model. We work in the superspace generated by x^m, θ^α and $\bar\theta_{\dot\alpha}$ coordinates which fulfill

$$[x^m, x^n] = [x^m, \theta^\alpha] = [x^m, \bar\theta_{\dot\alpha}] = 0, \quad \{\theta^\alpha, \theta^\beta\} = \{\bar\theta_{\dot\alpha}, \bar\theta_{\dot\beta}\} = \{\theta^\alpha, \bar\theta_{\dot\alpha}\} = 0, \tag{19}$$

with $m = 0, \dots 3$ and $\alpha, \beta = 1, 2$. These coordinates we call supercoordinates, to x^m we refer as bosonic and to θ^α and $\bar\theta_{\dot\alpha}$ we refer as fermionic coordinates. We work in Minkowski space-time with the metric $(-,+,+,+)$ and $x^2 = x^m x_m = -(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2$.

A general superfield $F(x, \theta, \bar\theta)$ can be expanded in powers of θ and $\bar\theta$,

$$\begin{aligned} F(x,\theta,\bar\theta) \ &= f(x) + \theta\phi(x) + \bar\theta\bar\chi(x) + \theta\theta m(x) + \bar\theta\bar\theta n(x) + \theta\sigma^m\bar\theta v_m(x) \\ &\quad + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\varphi(x) + \theta\theta\bar\theta\bar\theta d(x). \end{aligned} \tag{20}$$

Under the infinitesimal $\mathcal{N} = 1$ SUSY transformations[3] it transforms as

$$\delta_\xi F = (\xi Q + \bar\xi\bar Q)F, \tag{21}$$

where ξ^α and $\bar\xi_{\dot\alpha}$ are constant anticommuting parameters and Q^α and $\bar Q_{\dot\alpha}$ are SUSY generators,

$$Q_\alpha = \partial_\alpha - i\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m, \quad \bar Q_{\dot\alpha} = -\bar\partial_{\dot\alpha} + i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m. \tag{22}$$

Following the steps outlined in Section 2 we now introduce a deformation of the infinitesimal SUSY transformations by choosing the twist $\mathcal{F}$ in the following way

$$\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}D_\alpha\otimes D_\beta}, \tag{23}$$

with the complex constant matrix $C^{\alpha\beta} = C^{\beta\alpha} \in \mathbb{C}$ and $D_\alpha = \partial_\alpha + i\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m$. Note that this twist[4] is not hermitian, $\mathcal{F}^* \neq \mathcal{F}$; the usual complex conjugation is denoted by "$*$". It can be shown that (23) satisfies all requirements for a twist, [2]. The Hopf algebra of infinitesimal SUSY transformations does not change since

$$\{Q_\alpha, D_\beta\} = \{\bar Q_{\dot\alpha}, D_\beta\} = 0. \tag{24}$$

In particular, the comultiplication of the SUSY generators Q_α and $\bar Q_{\dot\alpha}$ remains undeformed. This means that the full commutative supersymmetry is preserved.

The inverse of the twist (23),

$$\mathcal{F}^{-1} = e^{-\frac{1}{2}C^{\alpha\beta}D_\alpha\otimes D_\beta}, \tag{25}$$

defines the $\star$-product. For arbitrary superfields F and G the $\star$-product reads

$$\begin{aligned} F\star G \ &= \ \mu_\star\{F\otimes G\} \\ &= \ \mu\{\mathcal{F}^{-1}\, F\otimes G\} \\ &= \ F\cdot G - \frac{1}{2}(-1)^{|F|}C^{\alpha\beta}(D_\alpha F)\cdot(D_\beta G) \\ &\quad -\frac{1}{8}C^{\alpha\beta}C^{\gamma\delta}(D_\alpha D_\gamma F)\cdot(D_\beta D_\delta G), \end{aligned} \tag{26}$$

[3]Everything said here can be applied to $\mathcal{N} = 2$ and higher supersymmetries. We work with $\mathcal{N} = 1$ for simplicity.

[4]Strictly speaking, the twist $\mathcal{F}$ (23) does not belong to the universal enveloping algebra of the Lie algebra of infinitesimal SUSY transformations. Therefore to be mathematically correct we should enlarge the algebra by introducing the relations for the operators D_α as well. In this way the deformed SUSY Hopf algebra remains the same as the undeformed one. However, since $[D_\alpha, M_{mn}] \neq 0$ the super Poincaré algebra becomes deformed and different from the super Poincaré algebra in the commutative case.

where $|F| = 1$ if F is odd (fermionic) and $|F| = 0$ if F is even (bosonic). The second line is in fact the definition of the multiplication $\mu_\star$. No higher powers of $C^{\alpha\beta}$ appear since derivatives D_α are Grassmanian. The $\star$-product (26) is associative, noncommutative and in the zeroth order in the deformation parameter $C_{\alpha\beta}$ it reduces to the usual pointwise multiplication. One should also note that it is not hermitian,

$$(F\star G)^* \neq G^* \star F^*. \tag{27}$$

In this way we define the deformed superspace as a superspace generated by the usual bosonic and fermionic coordinates (19) while the deformation is contained in the new product (26).

The deformed infinitesimal SUSY transformation is defined in the following way

$$\delta^\star_\xi F = (\xi Q + \bar\xi\bar Q)F. \tag{28}$$

Since the coproduct is not deformed, the usual Leibniz rule follows. The $\star$-product of two superfields is again a superfield; its transformation law is given by

$$\begin{aligned}\delta^\star_\xi (F\star G) &= (\xi Q + \bar\xi\bar Q)(F\star G)\\ &= (\delta^\star_\xi F)\star G + F\star(\delta^\star_\xi G).\end{aligned} \tag{29}$$

4 D-deformed Wess-Zumino model

Being interested in a deformation of the Wess-Zumino model, we need to analyze properties of the $\star$-products of chiral fields. A chiral field Φ fulfills $\bar D_{\dot\alpha}\Phi = 0$, where $\bar D_{\dot\alpha} = -\bar\partial_{\dot\alpha} - i\theta^\alpha\sigma^m_{\ \alpha\dot\alpha}\partial_m$ and $\bar D_{\dot\alpha}$ is related to D_α by the usual complex conjugation. In terms of the component fields, Φ is given by

$$\begin{aligned}\Phi(x,\theta,\bar\theta) &= A(x) + \sqrt{2}\theta^\alpha\psi_\alpha(x) + \theta\theta H(x) + i\theta\sigma^l\bar\theta(\partial_l A(x))\\ &\quad -\frac{i}{\sqrt{2}}\theta\theta(\partial_m\psi^\alpha(x))\sigma^m_{\ \alpha\dot\alpha}\bar\theta^{\dot\alpha} + \frac14\theta\theta\bar\theta\bar\theta(\Box A(x)).\end{aligned} \tag{30}$$

The $\star$-product of two chiral fields reads

$$\begin{aligned}\Phi\star\Phi &= \Phi\cdot\Phi - \frac18 C^{\alpha\beta}C^{\gamma\delta}D_\alpha D_\gamma\Phi D_\beta D_\delta\Phi\\ &= \Phi\cdot\Phi - \frac{1}{32}C^2(D^2\Phi)(D^2\Phi)\\ &= A^2 - \frac{C^2}{2}H^2 + 2\sqrt{2}A\theta^\alpha\psi_\alpha\\ &\quad -i\sqrt{2}C^2 H\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}(\partial_m\psi_\alpha) + \theta\theta\Big(2AH - \psi\psi\Big)\\ &\quad +C^2\bar\theta\bar\theta\Big(-H\Box A + \frac12(\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi)\Big)\Big)\\ &\quad +i\theta\sigma^m\bar\theta\Big(\partial_m(A^2) + C^2 H\partial_m H\Big)\\ &\quad +i\sqrt{2}\theta\theta\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}(\partial_m(\psi_\alpha A))\\ &\quad +\frac{\sqrt{2}}{2}\bar\theta\bar\theta C^2(-H\theta\Box\psi + \theta\sigma^m\bar\sigma^n\partial_n\psi\partial_m H)\\ &\quad +\frac14\theta\theta\bar\theta\bar\theta(\Box A^2 - \frac12 C^2\Box H^2),\end{aligned} \tag{31}$$

where $C^2 = C^{\alpha\beta}C^{\gamma\delta}\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}$. Due to the $\bar\theta$, $\bar\theta\bar\theta$ and the $\theta\bar\theta\bar\theta$ terms in (31),$\Phi\star\Phi$ is not a chiral field. Following the method developed in [19] we decompose all $\star$-products of the chiral fields into their irreducible components by using the projectors defined in [20].

Finally, the deformed Wess-Zumino action is constructed by requiring that the action is invariant under the deformed SUSY transformations (28) and that in the commutative limit it reduces to the undeformed Wess-Zumino action. In addition, we try to make a minimal deformation in the sense that we deform by $\star$-multiplication only the terms already present in the undeformed Wess-Zumino action. However, as we shall see latter, renormalizability will in fact imply the addition of some 'nonminimal' terms. Thus, we propose the following action

$$\begin{aligned} S &= \int \mathrm{d}^4x \, \Big\{ \Phi^+ \star \Phi\Big|_{\theta\theta\bar\theta\bar\theta} + \Big[\frac{m}{2}\Big(P_2(\Phi\star\Phi)\Big|_{\theta\theta} + 2a_1 P_1(\Phi\star\Phi)\Big|_{\bar\theta\bar\theta}\Big) \\ &+ \frac{\lambda}{3}\Big(P_2(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta} + 3a_2 P_1(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\bar\theta\bar\theta} \\ &+ 2a_3(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta\bar\theta\bar\theta} + 3a_4 P_1(\Phi\star\Phi)\star\Phi^+\Big|_{\bar\theta\bar\theta} \\ &+ 3a_5 \bar C^2 P_2(\Phi\star\Phi)\star\Phi^+\Big|_{\theta\theta\bar\theta\bar\theta}\Big) + \text{ c.c. } \Big]\Big\}. \end{aligned} \tag{32}$$

Here P_2 and P_1 are chiral and antichiral projectors respectively. Coefficients $a_1,\dots,a_5$ are real and constant. The terms with the coefficients a_4 and a_5 obviously represent a non-minimal deformation. They are both SUSY invariant and vanish in the commutative limit. Note that the vanishing of the a_5-term in the commutative limit was done by hand by multiplication with $\bar C^2$. We shall see in the following that on the level of three-point functions, the a_3-term generates divergences of the form $P_1(\Phi\star\Phi)\star\Phi^+\Big|_{\bar\theta\bar\theta}$ while the a_1-term and the a_4-term generate divergences of the form $P_2(\Phi\star\Phi)\star\Phi^+\Big|_{\theta\theta\bar\theta\bar\theta}$. In order to absorb these divergences one needs to introduce the a_4-term and the a_5-term in the action (32) from the very beginning.

Using the background field method and the supergraph technique, one can calculate the one-loop divergent part of the effective action. The details of the calculation are given in [15], here we just summarize the results. The divergent parts of the one-point, two-point, three-point and four-point functions are given by:

$$\begin{aligned} \Gamma_1^{(1)} &= 0, \\ \Gamma_1^{(2)}\Big|_{dp} &= \frac{\lambda^2(1-(2a_1+\frac{a_4}{2}+\frac{2a_5}{m})m^2(C^2+\bar C^2))}{4\pi^2\varepsilon}\int \mathrm{d}^8z\; \Phi^+(z)\Phi(z) \\ &+ \frac{\lambda^2(m^2a_3 - ma_4 - a_5)}{16\pi^2\varepsilon}\int \mathrm{d}^8z\; \big[C^2\Phi(z)D^2\Phi(z) + c.c.\big], \end{aligned} \tag{33}$$

$$\begin{aligned} \Gamma_1^{(3)}\Big|_{dp} &= -\frac{\lambda^3(ma_1+ma_4+2a_5)}{2\pi^2\varepsilon}\int \mathrm{d}^8z\; \big[\bar C^2\Phi(z)\Phi(z)\Phi^+(z) + c.c.\big] \\ &+ \frac{\lambda^3(2ma_3 - a_4)}{8\pi^2\varepsilon}\int \mathrm{d}^8z\; \big[C^2\Phi(z)\Phi^+(z)D^2\Phi(z) + c.c.\big], \end{aligned} \tag{34}$$

$$\begin{aligned} \Gamma_1^{(4)}\Big|_{dp} &= \frac{\lambda^4}{8\pi^2\varepsilon}\int \mathrm{d}^8z\; \Big[\bar C^2\Phi(z)\Phi(z)\Phi^+(z)(a_3\bar D^2\Phi^+(z) \\ &\qquad -2a_4\Phi(z)) + c.c.\Big]. \end{aligned} \tag{35}$$

One can check that the five-point function and higher order functions are convergent.

5 Discussion and outlook

Let us first repeat the results for the undeformed Wess-Zumino model. There the divergences appear only in the two-point function. They lead to renormalization of the superfield and there is no mass counterterm. Three-point and higher-point functions are convergent, which means that there are no divergent counterterms for the coupling constants; all redefinitions can be expressed in terms of the field strength renormalization Z. We see that introducing the deformation (23) changes this behavior: we obtain divergences both in the three-point and in the four-point functions.

From (33), (34) and (35) we see that the two-point and the three-point functions are renormalizable, while the four-point function is not. Thus, the model with arbitrary coefficients $a_1,\dots,a_5$ is not renormalizable.

However, there is a special choice of the coefficients $a_1, \dots a_5$ which renders the model renormalizable. If we fix $a_3 = a_4 = 0$, the divergent part of the four-point function vanishes. In that case the divergent parts of the two- and three-point functions are

$$\begin{aligned} \Gamma_1^{(2)}\Big|_{dp} &= \frac{\lambda^2(1-(2a_1+\frac{2a_5}{m})m^2(C^2+\bar{C}^2))}{4\pi^2\varepsilon}\int \mathrm{d}^8 z\ \Phi^+(z)\Phi(z) \\ &\quad -\frac{\lambda^2 a_5}{16\pi^2\varepsilon}\int \mathrm{d}^8 z\ \left[C^2\Phi(z)D^2\Phi(z) + c.c.\right], \qquad (36) \\ \Gamma_1^{(3)}\Big|_{dp} &= -\frac{\lambda^3(ma_1+2a_5)}{2\pi^2\varepsilon}\int \mathrm{d}^8 z\ \left[\bar{C}^2\Phi(z)\Phi(z)\Phi^+(z) + c.c.\right]. \qquad (37) \end{aligned}$$

All divergences in (36) and (37) have the same form as terms in the classical action (32). But this is only a necessary condition for a theory to be renormalizable; one has to check the consistency of the field and the coupling constants redefinitions. This was done in [15] and the final conclusion was that the model is indeed renormalizable if $a_3 = a_4 = 0$. Additionally, if $a_5 = -\frac{1}{2}ma_1$ the divergent part of the three-point function also vanishes and the results are then the same as the results for the undeformed Wess-Zumino model.

Another problem that could be addressed is the choice of deformation. Namely, renormalizability can be chosen as a criterion to test the deformation. We could chose a dierent deformation compared to that discussed in this paper. Using our principles (SUSY invariance, commutative limit, minimal deformation) we could construct an in- variant action and check whether the obtained model has a better behavior. This could give us an important insight into which deformation of the superspace is preferred.

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