Physics AUC, vol. 21-Sp.Issue, 186-194 (2011) PHYSICS AUC

Evolution of Continuous Variable Entanglement and Discord in Open Quantum Systems

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Abstract

In the framework of the theory of open systems based on completely positive quantum dynamical semigroups, we give a description of the continuous-variable quantum entanglement and quantum discord for a system consisting of two non-interacting modes embedded in a thermal environment. Entanglement and discord are used to quantify the quantum correlations of the system. For all values of the temperature of the thermal reservoir, an initial separable Gaussian state remains separable for all times. The time evolution of logarithmic negativity, which characterizes the degree of entanglement, indicates that in the case of an entangled initial Gaussian state, entanglement suppression (entanglement sudden death) takes place, for non-zero temperatures of the environment. Only for a zero temperature of the thermal bath the initial entangled state remains entangled for finite times. Analysis of the time evolution of the Gaussian quantum discord, which is a measure of all quantum correlations in the bipartite state, including entanglement, shows that quantum discord decays asymptotically in time under the effect of the thermal bath. This is contrast with the sudden death of entanglement. Before the suppression of the entanglement, the qualitative evolution of quantum discord is very similar to that of the entanglement.

PACS numbers: 03.65.Yz, 03.67.Bg, 03.67.Mn

1 Introduction

The study of quantum correlations is a key issue in quantum information theory [1] and quantum entanglement represents the indispensable physical resource for the description and performance of quantum information processing tasks [2]. However, entanglement does not describe all the non-classical properties of quantum correlations. Zurek [3, 4] defined the quantum discord as a measure of quantum correlations which includes entanglement of bipartite systems and it can also exist in separable states.

In recent years there is an increasing interest in using non-classical entangled states of continuous variable systems in applications of quantum information processing, communication and computation [5]. A full characterization of the non-classical properties of such states exists, at present, only for the class of Gaussian states. In this special case there exist necessary and sufficient criteria of entanglement [6, 7] and quantitative entanglement measures [8, 9].

Quantum coherence and entanglement of quantum systems are inevitably influenced during their interaction with the external environment. In order to describe realistically quantum information processes it is necessary to take decoherence and dissipation into consideration.

In this work we describe, in the framework of the theory of open systems based on completely positive quantum dynamical semigroups, the dynamics of the continuous variable quantum entanglement and quantum discord of a subsystem consisting of two uncoupled modes (harmonic oscillators) interacting with a common thermal environment. We are interested in discussing the correlation effect of the environment, therefore we assume that the two modes are independent, i.e. they do not interact directly. The initial state of the subsystem is taken of Gaussian form and the evolution under the quantum dynamical semigroup assures the preservation in time of the Gaussian form of the state.

The paper is organized as follows. In Sec. 2 we write the Markovian master equation in the Heisenberg representation for two uncoupled harmonic oscillators interacting with a general environment and give the general solution of the evolution equation for the covariance matrix, i.e. we derive the variances and covariances of coordinates and momenta corresponding to a generic two-mode Gaussian state. By using the Peres-Simon necessary and sufficient condition for separability of two-mode Gaussian states [6, 10], we investigate in Sec. 3 the dynamics of entanglement for the considered subsystem. For all values of the temperature of the thermal reservoir, an initial separable Gaussian state remains separable for all times. We analyze the time evolution of the logarithmic negativity, which characterizes the degree of entanglement of the quantum state, and show that in the case of an entangled initial Gaussian state, entanglement suppression (entanglement sudden death) takes place, for non-zero temperatures of the environment. Only for a zero temperature of the thermal bath the initial entangled state remains entangled for all finite times, but in the limit of infinite time it evolves asymptotically to an equilibrium state which is always separable. We analyze the time evolution of the Gaussian quantum discord, which is a measure of all quantum correlations in the bipartite state, including entanglement, and show that quantum discord decays asymptotically in time under the effect of the thermal bath. This is contrast with the sudden death of entanglement. Before the suppression of the entanglement, the qualitative evolution of quantum discord is very similar to that of the entanglement. A summary is given in Sec. 4.

2 Equations of motion

We study the dynamics of the subsystem composed of two non-interacting modes in weak interaction with a thermal environment. In the axiomatic formalism based on completely positive quantum dynamical semigroups, the irreversible time evolution of an open system is described in the Heisenberg representation by the following quantum Markovian Kossakowski-Lindblad master equation for an operator A († denotes Hermitian conjugation) [11, 12]:

$$\frac{dA}{dt}=\frac{i}{\hbar}[H,A]+\frac{1}{\hbar}\sum_j(V_j^\dagger[A,V_j]+[V_j^\dagger,A]V_j). \tag{1}$$

Here, H denotes the Hamiltonian of the open system and the operators $V_j, V_j^\dagger$, defined on the Hilbert space of H, represent the interaction of the open system with the environment.

We are interested in the set of Gaussian states, therefore we introduce such quantum dynamical semigroups that preserve this set during time evolution of the system and in this case our model represents a Gaussian noise channel. Consequently H is taken a polynomial of second degree in the coordinates x, y and momenta p_x, p_y of the two quantum oscillators and $V_j, V_j^\dagger$ are taken polynomials of first degree in these canonical observables. Then in the linear space spanned by the coordinates and momenta there exist only four linearly independent operators $V_{j=1,2,3,4}$ [13]:

$$V_j=a_{xj}p_x+a_{yj}p_y+b_{xj}x+b_{yj}y, \tag{2}$$

where $a_{xj}, a_{yj}, b_{xj}, b_{yj}$ are complex coefficients. The Hamiltonian of the two uncoupled non-resonant harmonic oscillators of identical mass m and frequencies ω_1 and ω_2 is

$$H=\frac{1}{2m}(p_x^2+p_y^2)+\frac{m}{2}(\omega_1^2x^2+\omega_2^2y^2). \tag{3}$$

The fact that the evolution is given by a dynamical semigroup implies the positivity of the matrix formed by the scalar products of the four vectors $\mathbf{a}_x, \mathbf{b}_x, \mathbf{a}_y, \mathbf{b}_y$, whose entries are the components $a_{xj}, b_{xj}, a_{yj}, b_{yj}$, respectively. We take this matrix of the following form, where all coefficients $D_{xx}, D_{xp_x}, ...$ and λ are real quantities (we put from now on $\hbar=1$):

$$\begin{pmatrix} D_{xx} & -D_{xp_x}-i\lambda/2 & D_{xy} & -D_{xp_y} \\ -D_{xp_x}+i\lambda/2 & D_{p_xp_x} & -D_{yp_x} & D_{p_xp_y} \\ D_{xy} & -D_{yp_x} & D_{yy} & -D_{yp_y}-i\lambda/2 \\ -D_{xp_y} & D_{p_xp_y} & -D_{yp_y}+i\lambda/2 & D_{p_yp_y} \end{pmatrix}. \tag{4}$$

We introduce the following 4×4 bimodal covariance matrix:

$$\sigma(t) = \begin{pmatrix} \sigma_{xx}(t) & \sigma_{xp_x}(t) & \sigma_{xy}(t) & \sigma_{xp_y}(t) \\ \sigma_{xp_x}(t) & \sigma_{p_xp_x}(t) & \sigma_{yp_x}(t) & \sigma_{p_xp_y}(t) \\ \sigma_{xy}(t) & \sigma_{yp_x}(t) & \sigma_{yy}(t) & \sigma_{yp_y}(t) \\ \sigma_{xp_y}(t) & \sigma_{p_xp_y}(t) & \sigma_{yp_y}(t) & \sigma_{p_yp_y}(t) \end{pmatrix}. \tag{5}$$

From Eq. (1) we obtain the following system of equations for the quantum correlations of the canonical observables (T denotes the transposed matrix) [13]:

$$\frac{d\sigma(t)}{dt} = Y\sigma(t) + \sigma(t)Y^{\mathrm{T}} + 2D, \tag{6}$$

where

$$Y = \begin{pmatrix} -\lambda & 1/m & 0 & 0 \\ -m\omega_1^2 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 1/m \\ 0 & 0 & -m\omega_2^2 & -\lambda \end{pmatrix}, \tag{7}$$

$$D = \begin{pmatrix} D_{xx} & D_{xp_x} & D_{xy} & D_{xp_y} \\ D_{xp_x} & D_{p_xp_x} & D_{yp_x} & D_{p_xp_y} \\ D_{xy} & D_{yp_x} & D_{yy} & D_{yp_y} \\ D_{xp_y} & D_{p_xp_y} & D_{yp_y} & D_{p_yp_y} \end{pmatrix}. \tag{8}$$

The time-dependent solution of Eq. (6) is given by [13]

$$\sigma(t) = M(t)[\sigma(0) - \sigma(\infty)]M^{\mathrm{T}}(t) + \sigma(\infty), \tag{9}$$

where the matrix $M(t) = \exp(Yt)$ has to fulfill the condition $\lim_{t\to\infty} M(t) = 0$. In order that this limit exists, Y must only have eigenvalues with negative real parts. The values at infinity are obtained from the equation

$$Y\sigma(\infty) + \sigma(\infty)Y^{\mathrm{T}} = -2D. \tag{10}$$

3 Dynamics of entanglement and discord

3.1 Time evolution of entanglement and logarithmic negativity

A well-known sufficient condition for inseparability is the so-called Peres-Horodecki criterion [10, 14], which is based on the observation that the non-completely positive nature of the partial transposition operation of the density matrix for a bipartite system (transposition with respect to degrees of freedom of one subsystem only) may turn an inseparable state into a non-physical state. The signature of this non-physicality, and thus of quantum entanglement, is the appearance of a negative eigenvalue in the eigenspectrum of the partially transposed density matrix of a bipartite system. The characterization of the separability of continuous variable states using second-order moments of quadrature operators was given in Refs. [6, 7]. For Gaussian states, whose statistical properties are fully characterized by just second-order moments, this criterion was proven to be necessary and sufficient: A Gaussian continuous variable state is separable if and only if the partial transpose of its density matrix is non-negative (positive partial transpose (PPT) criterion).

The two-mode Gaussian state is entirely specified by its covariance matrix (5), which is a real, symmetric and positive matrix with the following block structure:

$$\sigma(t) = \begin{pmatrix} A & C \\ C^{\mathrm{T}} & B \end{pmatrix}, \tag{11}$$

where A, B and C are 2×2 Hermitian matrices. A and B denote the symmetric covariance matrices for the individual reduced one-mode states, while the matrix C contains the cross-correlations between modes.

The 4×4 covariance matrix (11) (where all first moments can be set to zero by means of local unitary operations which do not affect the entanglement) contains four local symplectic invariants in form of the determinants of the block matrices A, B, C and covariance matrix σ. Based on these invariants, Simon [6] derived the following PPT criterion for bipartite Gaussian continuous variable states: the necessary and sufficient condition for separability is $S(t) \geq 0$, where

$$\begin{aligned} S(t) \equiv \det A \det B + (\frac{1}{4} - |\det C|)^2 \\ -\mathrm{Tr}[AJCJBJC^{\mathrm{T}}J] - \frac{1}{4}(\det A + \det B) \end{aligned} \tag{12}$$

and J is the 2×2 symplectic matrix

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{13}$$

We suppose that the asymptotic state of the considered open system is a Gibbs state corresponding to two independent quantum harmonic oscillators in thermal equilibrium at temperature T. Then the quantum diffusion coefficients have the following form [12]:

$$\begin{aligned} m\omega_1 D_{xx} = \frac{D_{p_x p_x}}{m\omega_1} = \frac{\lambda}{2}\coth\frac{\omega_1}{2kT}, \\ m\omega_2 D_{yy} = \frac{D_{p_y p_y}}{m\omega_2} = \frac{\lambda}{2}\coth\frac{\omega_2}{2kT}, \\ D_{xp_x} = D_{yp_y} = D_{xy} = D_{p_x p_y} = D_{xp_y} = D_{yp_x} = 0. \end{aligned} \tag{14}$$

The elements of the covariance matrix can be calculated from Eqs. (9), (10). Solving for the time evolution of the covariance matrix elements, we can obtain the entanglement dynamics by using the Simon criterion.

In order to quantify the degree of entanglement of the infinite-dimensional bipartite system states of the two oscillators it is suitable to use the logarithmic negativity. For a Gaussian density operator, the logarithmic negativity is completely defined by the symplectic spectrum of the partial transpose of the covariance matrix. It is given by $E_N = -\log_2 2\tilde{\nu}_-$, where $\tilde{\nu}_-$ is the smallest of the two symplectic eigenvalues of the partial transpose $\tilde{\sigma}$ of the 2-mode covariance matrix σ :

$$2\tilde{\nu}_{\mp}^2 = \tilde{\Delta} \mp \sqrt{\tilde{\Delta}^2 - 4\det\sigma} \tag{15}$$

and $\tilde{\Delta}$ is the symplectic invariant (seralian), given by $\tilde{\Delta} = \det A + \det B - 2\det C$.

In our model, the logarithmic negativity is calculated as

$$E_N(t) = -\frac{1}{2}\log_2[4g(\sigma(t))], \tag{16}$$

where

$$\begin{aligned} g(\sigma(t)) = \frac{1}{2}(\det A + \det B) - \det C \\ -\left(\left[\frac{1}{2}(\det A + \det B) - \det C\right]^2 - \det\sigma(t)\right)^{1/2}. \end{aligned} \tag{17}$$

It determines the strength of entanglement for $E_N(t) > 0$, and if $E_N(t) \leq 0$, then the state is separable.

In the following, we analyze the dependence of the Simon function $S(t)$ and of the logarithmic negativity $E_N(t)$ on time t and temperature T of the thermal bath, with the diffusion coefficients given by Eqs. (14). We consider two types of the initial Gaussian states: 1) separable and 2) entangled.

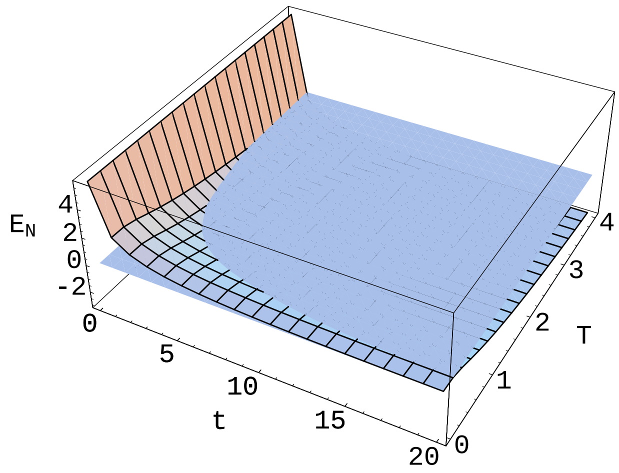

Figure 1: Logarithmic negativity E_N versus time t and temperature T for an entangled initial vacuum squeezed state with squeezing parameter $r = 4$ and $\lambda = 0.1, \omega_1 = \omega_2 = 1$. We take $m = \hbar = k = 1$.

1) We consider a separable initial Gaussian state, with the two modes initially prepared in their single-mode squeezed states (unimodal squeezed state) and with its initial covariance matrix taken of the form

$$\sigma_s(0) = \frac{1}{2}\begin{pmatrix} \cosh r & \sinh r & 0 & 0 \\ \sinh r & \cosh r & 0 & 0 \\ 0 & 0 & \cosh r & \sinh r \\ 0 & 0 & \sinh r & \cosh r \end{pmatrix}, \tag{18}$$

where r denotes the squeezing parameter. In this case $S(t)$ becomes strictly positive after the initial moment of time ($S(0) = 0$), so that the initial separable state remains separable for all values of the temperature T and for all times.

2) The evolution of an entangled initial state is illustrated in Figure 1, where we represent the dependence of the logarithmic negativity $E_N(t)$ on time t and temperature T for an entangled initial Gaussian state, taken of the form of a two-mode vacuum squeezed state, with the initial covariance matrix given by

$$\sigma_e(0) = \frac{1}{2}\begin{pmatrix} \cosh r & 0 & \sinh r & 0 \\ 0 & \cosh r & 0 & -\sinh r \\ \sinh r & 0 & \cosh r & 0 \\ 0 & -\sinh r & 0 & \cosh r \end{pmatrix}. \tag{19}$$

We observe that for a non-zero temperature T, at certain finite moment of time, which depends on T, $E_N(t)$ becomes zero and therefore the state becomes separable. This is the so-called phenomenon of entanglement sudden death. It is in contrast to the quantum decoherence, during which the loss of quantum coherence is usually gradual. For $T = 0$, $E_N(t)$ remains strictly positive for finite times and tends asymptotically to 0 for $t \to \infty$. Therefore, only for zero temperature of the thermal bath the initial entangled state remains entangled for all finite times and this state tends asymptotically to a separable one for infinitely large time. One can also show that the dissipation favorizes the phenomenon of entanglement sudden death – with increasing the dissipation parameter λ, the entanglement suppression happens earlier [15].

Using the diffusion coefficients given by Eqs. (14), we obtain from Eq. (10) the following elements of the asymptotic matrices $A(\infty)$ and $B(\infty)$:

$$\begin{aligned} m\omega_1\sigma_{xx}(\infty) &= \frac{\sigma_{p_xp_x}(\infty)}{m\omega_1} = \frac{1}{2}\coth\frac{\omega_1}{2kT}, \quad \sigma_{xp_x}(\infty) = 0, \\ m\omega_2\sigma_{yy}(\infty) &= \frac{\sigma_{p_yp_y}(\infty)}{m\omega_2} = \frac{1}{2}\coth\frac{\omega_2}{2kT}, \quad \sigma_{yp_y}(\infty) = 0 \end{aligned} \tag{20}$$

and of the entanglement matrix $C(\infty)$:

$$\sigma_{xy}(\infty)=\sigma_{xp_y}(\infty)=\sigma_{yp_x}(\infty)=\sigma_{p_xp_y}(\infty)=0. \tag{21}$$

In our case, the asymptotic logarithmic negativity is given by (for $\omega_1 \leq \omega_2$)

$$E_N(\infty)=-\log_2 \coth\frac{\omega_2}{2kT}. \tag{22}$$

It depends only on temperature, and does not depend on the initial Gaussian state [16, 17, 18]. $E_N(\infty)<0$ for $T\neq 0$ and $E_N(\infty)=0$ for $T=0$. Consequently, the equilibrium asymptotic state is always separable in the case of two non-interacting harmonic oscillators immersed in a common thermal reservoir.

3.2 Gaussian quantum discord

The separability of quantum states has often been described as a property synonymous with the classicality. However, recent studies have shown that separable states, usually considered as being classically correlated, might also contain quantum correlations. Quantum discord was introduced [3, 4] as a measure of all quantum correlations in a bipartite state, including – but not restricted to – entanglement. Quantum discord has been defined as the difference between two quantum analogues of classically equivalent expression of the mutual information, which is a measure of total correlations in a quantum state. For pure entangled states quantum discord coincides with the entropy of entanglement. Quantum discord can be different from zero also for some mixed separable state and therefore the correlations in such separable states with positive discord are an indicator of quantumness. States with zero discord represent essentially a classical probability distribution embedded in a quantum system.

For an arbitrary bipartite state ρ_{12}, the total correlations are expressed by quantum mutual information [19]

$$I(\rho_{12})=\sum_{i=1,2}S(\rho_i)-S(\rho_{12}), \tag{23}$$

where ρ_i represents the reduced density matrix of subsystem i and $S(\rho)=-\mathrm{Tr}(\rho\ln\rho)$ is the von Neumann entropy. Based on a complete set of local projectors $\{\Pi_2^k\}$ on the subsystem 2, the classical correlation in the bipartite quantum state ρ_{12} can be given by

$$C(\rho_{12})=S(\rho_1)-\inf_{\{\Pi_2^k\}}\{S(\rho_{1|2})\}, \tag{24}$$

where $S(\rho_{1|2})=\sum_k p^k S(\rho_1^k)$ is the conditional entropy of subsystem 1 and $\inf\{S(\rho_{1|2})\}$ represents the minimal value of the entropy with respect to a complete set of local measurements $\{\Pi_2^k\}$. Here, p^k is the measurement probability for the kth local projector and ρ_1^k denotes the reduced state of subsystem 1 after the local measurements. Then the quantum discord is defined by

$$D(\rho_{12})=I(\rho_{12})-C(\rho_{12}). \tag{25}$$

Originally the quantum discord was defined and evaluated mainly for finite dimensional systems. Very recently [20, 21] the notion of discord has been extended to the domain of continuous variable systems, in particular to the analysis of bipartite systems described by two-mode Gaussian states. Closed formulas have been derived for bipartite thermal squeezed states [20] and for all two-mode Gaussian states [21].

The Gaussian quantum discord of a general two-mode Gaussian state ρ_{12} can be defined as the quantum discord where the conditional entropy is restricted to generalized Gaussian positive operator valued measurements (POVM) on the mode 2 and in terms of symplectic invariants it is given by (the symmetry between the two modes 1 and 2 is broken) [21]

$$D=f(\sqrt{\beta})-f(\nu_-)-f(\nu_+)+f(\sqrt{\varepsilon}), \tag{26}$$

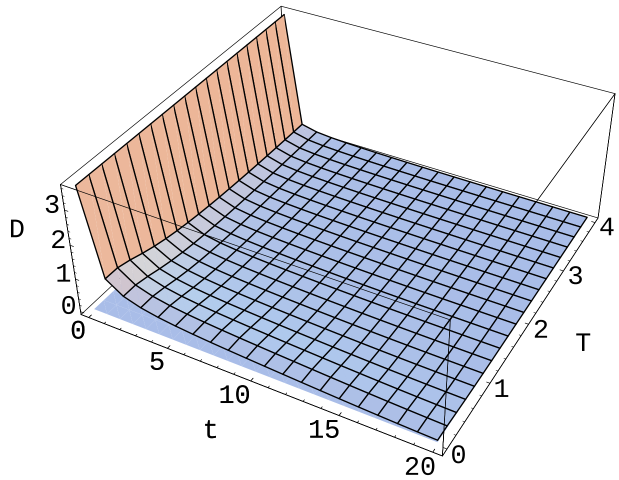

Figure 2: Gaussian quantum discord D versus time t and temperature T for an entangled initial vacuum squeezed state with squeezing parameter $r = 4$ and $\lambda = 0.1, \omega_1 = \omega_2 = 1$. We take $m = \hbar = k = 1$.

where

$$f(x) = \frac{x+1}{2} \log \frac{x+1}{2} - \frac{x-1}{2} \log \frac{x-1}{2}, \tag{27}$$

$$\varepsilon = \begin{cases} \dfrac{2\gamma^2 + (\beta-1)(\delta-\alpha) + 2|\gamma|\sqrt{\gamma^2 + (\beta-1)(\delta-\alpha)}}{(\beta-1)^2}, \\ \quad \text{if } (\delta - \alpha\beta)^2 \leq (\beta+1)\gamma^2(\alpha+\delta) \\ \\ \dfrac{\alpha\beta - \gamma^2 + \delta - \sqrt{\gamma^4 + (\delta-\alpha\beta)^2 - 2\gamma^2(\delta+\alpha\beta)}}{2\beta}, \\ \quad \text{otherwise,} \end{cases} \tag{28}$$

$$\alpha = 4 \det A, \quad \beta = 4 \det B, \quad \gamma = 4 \det C, \quad \delta = 16 \det \sigma, \tag{29}$$

and $\nu_\mp$ are the symplectic eigenvalues of the state, given by

$$2\nu_\mp^2 = \Delta \mp \sqrt{\Delta^2 - 4 \det \sigma}, \tag{30}$$

where $\Delta = \det A + \det B + 2 \det C$. Notice that Gaussian quantum discord only depends on $|\det C|$, i.e., entangled ($\det C < 0$) and separable states are treated on equal footing.

The evolution of the Gaussian quantum discord D is illustrated in Figure 2, where we represent the dependence of D on time t and temperature T for an entangled initial Gaussian state, taken of the form of a two-mode vacuum squeezed state (19), for such values of the parameters which satisfy for all times the first condition in formula (28). The Gaussian discord has nonzero values for all finite times and this fact certifies the existence of nonclassical correlations in two-mode Gaussian states – either separable or entangled. Gaussian discord asymptotically decreases in time, compared to the case of the logarithmic negativity, which has an evolution leading to a sudden suppression of the entanglement. For entangled initial states the Gaussian discord remains strictly positive in time and in the limit of infinite time it tends asymptotically to zero, corresponding to the thermal product (separable) state, with no correlation at all. One can easily show that for a separable initial Gaussian state with covariance matrix (18) the quantum discord is zero and it keeps this value during the whole time evolution of the state.

From Figure 2 we notice that, in concordance with the general properties of the Gaussian quantum discord [21], the states can be either separable or entangled for $D \leq 1$ and all the states above the threshold $D = 1$ are entangled. We also notice that the decay of quantum discord is stronger when the temperature T is increasing.

It should be remarked that the decay of quantum discord is very similar to that of the entanglement before the time of the sudden death of entanglement. In the vicinity of a zero logarithmic negativity ($E_N = 0$), the nonzero values of the discord can quantify the nonclassical correlations for separable mixed states and one considers that this fact could make possible some tasks in quantum computation [22].

4 Summary

In the framework of the theory of open systems based on completely positive quantum dynamical semigroups, we reviewed the Markovian dynamics of quantum correlations for a subsystem composed of two non-interacting modes embedded in a thermal bath. We have presented and discussed the influence of the environment on the dynamics of quantum entanglement and quantum discord for different initial states. We have described the time evolution of the logarithmic negativity, which characterizes the degree of entanglement of the quantum state, in terms of the covariance matrix for Gaussian input states, for the case when the asymptotic state of the considered open system is a Gibbs state corresponding to two independent quantum harmonic oscillators in thermal equilibrium. The dynamics of the quantum entanglement strongly depends on the initial states and the parameters characterizing the environment (dissipation coefficient and temperature). For all values of the temperature of the thermal reservoir, an initial separable Gaussian state remains separable for all times. In the case of an entangled initial Gaussian state, entanglement suppression (entanglement sudden death) takes place for non-zero temperatures of the environment. Only for a zero temperature of the thermal bath the initial entangled state remains entangled for finite times, but in the limit of infinite time it evolves asymptotically to an equilibrium state which is always separable. The time when the entanglement is suppressed, decreases with increasing the temperature and dissipation.

We described also the time evolution of the Gaussian quantum discord, which is a measure of all quantum correlations in the bipartite state, including entanglement. The values of quantum discord decrease asymptotically in time. This behaviour is qualitatively different from the sudden death of entanglement. The time evolution of quantum discord is very similar to that of the entanglement before the sudden suppression of the entanglement. After the sudden death of the entanglement, the nonzero values of quantum discord manifest the existence of quantum correlations for separable mixed states. Quantum discord is decreasing with increasing the temperature. One considers that the robustness of quantum discord could favorize the realization of scalable quantum computing in contrast to the fragility of the entanglement [22].

Acknowledgments

The author acknowledges the financial support received from the Romanian Ministry of Education and Research, through the Projects IDEI 497/2009 and PN 09 37 01 02/2010.

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