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Heterotic Type IIA Duality with Fluxes*

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Abstract

In this I present evidence that the most general $d = 4$, $N = 2$ compactifications of heterotic string with fluxes is dual to M or F-theory compactifications on manifolds with $SU(3)$ structure.

1 Introduction

String theory is one of the most important candidates to a quantum theory of gravity. It also naturally includes gauge interactions and therefore it is a candidate for a unified theory of gravity and particle physics. If this is so, it should be possible to determine the known physics (the Standard Model of Particle Physics) from string theory. Even though at high energies, string theory is very constrained (and predictive) we still do not understand how to extract information of phenomena which take place at much lower energy scales, like, standard model physics.

The general lore is that superstring theory predicts the number of the space-time dimensions to be ten. However, caution has to be taken here when interpreting the theory in this way. String theory is in principle a two-dimensional (super)conformal field theory and considering that the target space is indeed "our" space-time can be misleading. One of the most known examples is string theory on orbifolds which is perfectly well defined. However, orbifolds, from a naive geometric perspective do not make much sense.

I will nevertheless consider in the following that string theory is defined on a ten-dimensional space-time, but we shall see towards the end of this talk that non-geometric backgrounds naturally appear also in this setup. I shall work in the low energy approximation of string theory which is ten-dimensional supergravity.

In order to extract some low-energy behavior one needs to compactify, ie to consider that the space-time is a product of a four-dimensional space-time times some internal (six-dimensional) manifold. Upon compactification, the various string theories we know of are related by certain duality relations. The duality I will be interested in the following is the so called heterotic-type IIA duality. This duality states that heterotic string compactified on $K3 \times T^2$ is dual to type IIA strings compactified on $K3$ fibered Calabi–Yau manifolds.

Besides dualities, in this talk I will need another important ingredient, which is termed flux compactification. The usual compactifications suffer from the moduli problem in that after the compactification there exist plenty scalar fields without a potential (moduli). These moduli fields, if unfixed ruin all the predictive power of string theory. Therefore one has to find a mechanism of stabilising such fields. One such method is precisely flux compactification. In such compactifications, non-vanishing values for the field strengths of various fields on the internal manifold are considered

$$\int_{\gamma_p} F_p = integer \neq 0 \; . \tag{1.1}$$

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Here F_p is the p-form field strength off some $p-1$ gauge potential and γ_p denotes some appropriate p-cycle in the internal manifold. After the compactification, the fluxes generate potential and masses for various moduli fields. Understanding how fluxes change the original compactification is not often easy. A whole lot can be learned about how fluxes work and many new fluxes have been discovered by applying duality relations to certain flux backgrounds. This is precisely what I will show in this talk in the context of heterotic-type IIA duality in four dimensions.

2 Heterotic compactifications on $K3 \times T^2$

Heterotic compactifications are in general complicated due to the presence of non-Abelian gauge fields in ten dimensions, but for the discussion which follows I shall ignore this and work directly with the $U(1)^{16}$ Cartan subalgebra of the original gauge group. This choice can be motivated by the fact that at a generic point in the moduli space of the resulting $N=2$ theory in four dimensions the gauge group is broken to the maximal Abelian subgroup, $U(1)^{n_v+1}$, where n_v denotes the number of vector multiplets in four dimensions. Part of these gauge fields come directly form the Cartan subalgebra of the original gauge group[1] while four of the gauge fields come from the Kaluza-Klein vectors on the torus as well as from from the B-field with one leg on the torus. Roughly I will use the following assignment

$$A^0 = g_{\mu 4} \ , \quad A^1 = B_{\mu 4} \ , \quad A^2 = g_{\mu 5} \ , \quad A^3 = B_{\mu 5} \ ,$$

where by 4 and 5 I denote the directions along the T^2 torus. The resulting theory also contains a certain number of hyper-multiplets, but in the following I will only concentrate on the vector multiplets and the way they couple to the $N=2$ supergravity. The important thing to note is that vector multiplet sector is governed by the T^2 compactification and the $K3$ part is more like a spectator. For this reason I shall not discuss any detail of the $K3$ compactification and assume that it works as usual.

2.1 Turning on fluxes on T^2

In order to have a clearer picture of what will follow it is important to understand the heterotic-type IIA duality when gauge field fluxes are turned on T^2. This was studied in [1]. The fluxes on T^2 can be parameterised as

$$\int_{T^2} F^a = f^a \ , \tag{2.1}$$

where F^a denote the field strengths for the gauge fields in the Cartan subalgebra of the original gauge group and f^a denote the flux parameters, which, up to some normalization, are integer numbers. The effective action generated by this compactification was derived in [2, 3]. One feature that I will be mostly interested in the following is the fact that the gauge group becomes non-Abelian due to the fluxes and the structure constants are given precisely by the flux parameters f^a. In particular one finds the following field strengths

$$F^0 = dA^0 \ , \quad F^2 = dA^2 \ ,$$

$$F^1 = dA^1 + f^a A^a \wedge A^2 \ , \quad F^3 = dA^3 - f^a A^a \wedge A^0 \ , \qquad F^a = dA^a + f^a A^0 \wedge A^2 \ ,$$

from where the non-vanishing structure constants can be read off

$$f^1_{2a} = f^a \ ; \qquad F^3_{0a} = -f^a \ ; \qquad f^a_{02} = -f^a \ . \tag{2.2}$$

[1] Note that compactification of heterotic strings on curved backgrounds require that the gauge fields are non-trivial on the internal space. This in turn leads to a breakdown of the original gauge group. The precise resulting gauge group can only be determined once the form of the gauge bundle is known. For the case at hand I shall not be interested in these aspects as I will only consider the Cartan subalgebra. However the gauge bundle in general breaks also some of the Cartan generators and therefore the precise number of $U(1)$ gauge fields in four dimensions is left arbitrary.

2.2 Type IIA dual setup

Now if heterotic type IIA duality is indeed true, what would be the type IIA dual setup which would produce a theory similar to what I presented above on the heterotic side? In particular how can one obtain a non-Abelian gauge symmetry and charged vector multiplets in the context of Calabi–Yau compactification of type IIA theory? The answer to these questions turns out not to be within type IIA theory but in its strong coupling limit, M-theory. The key to this puzzle is the following. The four-dimensional duality I am interested in, has a correspondent in five dimensions which says that heterotic strings compactified on $K3 \times S^1_5$ is dual to M-theory compactifications on $K3$ fibered Calabi–Yau manifolds. Clearly, by compactifying on another circle – which I denote by S^1_4 – to four dimensions we recover the duality I started with. In this picture, the T^2 fluxes I turned on in the heterotic side appear only in the last step of the compactification from five to four dimensions as monodromies of the gauge field-scalars (scalar fields which come from the ten-dimensional gauge fields in the direction of S^1) around the S^1_4 circle. This should be clear from the fact that a constant field strength $F_{45} = k$, which defines a flux through $T^2 = S^1_5 \times S^1_4$, can be obtained from a gauge potential A_5 which is linear in the coordinate if the coordinate z_4 of the circle S^1_4, ie, $A_9 \sim k \cdot z_4$. This means that after going once completely around the circle, the gauge potential A_9 has a jump by $2\pi k$. Such shifts are symmetries of the theory and therefore compactification with such monodromies are consistent.

What I propose in the following is to do the same thing in M-theory. From the five-dimensional point of view, A_9 is a scalar field which sits in a vector multiplet. Therefore I will have to perform a compactification of the five-dimensional theory (which was obtained by compactifying M-theory on a Calabi–Yau manifold) on a circle to four dimensions with monodromies for the scalars in the vector multiplets. For Calabi–Yau manifolds which are $K3$ fibered, the isometry group of the vector moduli space in five dimensions is $SO(1,1) \times SO(1, n_v - 2)$. In principle only the subgroup corresponding to the shift symmetries discussed above should be taken into account to give the dual setup to the heterotic fluxes, but I propose to do something more general and allow monodromies which take values in the full isometry group.

Finally I want to give up this step-by-step procedure and package all the information together in a single step compactification of M-theory. For this I should mention that the scalars in the vector multiplet sector in five dimensions come from the Kähler moduli of the Calabi–Yau manifold. These scalar fields are supposed to vary along the circle and therefore the metric on the Calabi–Yau manifold depends on the point on the circle. This means that the full seven dimensional manifold is not simply a direct product between the Calabi–Yau manifold and the circle, but a fibration where the Calabi–Yau manifold is allowed to vary along the circle. Generically such manifold will be a manifold with SU(3) structure [1, 4].

There is an equivalent way of dealing with this problem which can be made more concrete. The Kähler moduli of the Calabi–Yau manifold are in one-to-one correspondence with the harmonic $(1,1)$ forms. Therefore, saying that the moduli vary over the circle is equivalent to saying that the harmonic forms of the Calabi–Yau manifold have a specific dependence on the circle coordinate. In particular I will choose

$$\omega_i(z + \epsilon) = \omega_i(z) + \epsilon M_i^j \omega_j(z) \, , \tag{2.3}$$

where M is the constant twist matrix and the monodromy matrix $\gamma_i^j = (e^M)_i^j$ is an element of the symmetry group of the vector moduli space as explained above. Even if the forms ω_i are harmonic on the Calabi–Yau slices, on the full seven dimensional manifold they obey the relation

$$d\omega_i = M_i^j \omega_j \wedge dz \, . \tag{2.4}$$

Such a relation imposes certain constraints on the manifold. In particular,

$$\int_{X_7} d(\omega_i \wedge \omega_j \wedge \omega_k) = 0 \, , \tag{2.5}$$

implies that the triple intersection numbers of the Calabi–Yau manifold

$$\mathcal{K}_{ijk} = \int_{CY} \omega_i \wedge \omega_j \wedge \omega_k \, , \tag{2.6}$$

obey the relation

$$\mathcal{K}_{ijk}M^l_k + \mathcal{K}_{jkl}M^l_i + \mathcal{K}_{kil}M^l_j = 0 \ . \tag{2.7}$$

This condition is precisely the condition that the Kähler moduli space of the Calabi–Yau manifold has an isometry. For the case I am interested in – namely a $K3$ fibered Calabi–Yau manifold – there is an isometry and therefore this condition must have some non-trivial solution. It is known that the intersection numbers for such manifolds are given by

$$\mathcal{K}_{123} = -1 \ , \qquad \mathcal{K}_{1ab} = 2\delta_{ab} \ , \quad a, b = 4, \ldots, h^{1,1} = n_v \ . \tag{2.8}$$

This can be insert in the constraint (2.7) to obtain a solution for the twist matrix M. The result can be parameterised as[2]

$$\begin{aligned} &M^2_2 = m \ , \quad M^2_a = m_a \ , \quad M^3_a = \tilde{m}_a \ , \quad M^b_a = -M^a_b = m^b_a \ , \\ &M^3_3 = -m \ , \quad M^a_2 = \tfrac{1}{2}\tilde{m}_a \ , \quad M^a_3 = \tfrac{1}{2}m_a \ , \end{aligned} \tag{2.9}$$

Compactification of M-theory on the seven-dimensional manifolds described above leads to $N = 2$ supergravity in four dimensions coupled to a non-Abelian vector multiplet sector where the non-vanishing structure constants are given in terms of the twist matrix M as [5]

$$f^j_{i0} = -M^j_i \ , \quad f^1_{23} = -m \ , \quad f^1_{2a} = \tilde{m}_a \ , \quad f^1_{3a} = m_a \ , \quad f^1_{ab} = 2{m_a}^b \ . \tag{2.10}$$

Allowing only $\tilde{m}_a$ to be non-vanishing in the twist matrix M it can be seen that precisely reproduces the situation which was found in heterotic compactifications [1].

It is now intriguing that on the type IIA (M-theory) side there are more fluxes than I originally started with in the heterotic case, namely m, m_a and ${m_a}^b$. So the question to ask now is whether these additional flux parameters have any interpretation on the heterotic side. At a first sight, m_a are T-dual to the parameters $\tilde{m}_a$ and therefore one should expect that these parameters somehow correspond to fluxes for the T-dual fields in the heterotic picture. But is this a meaningful compactification? And moreover, what is the dual of the other parameters like m and ${m_a}^b$? I will answer these questions in the next subsection.

2.3 Heterotic compactifications with double duality twists

The idea of the compactifications I want to discuss now comes from the double torus compactifications proposed in [6]. The main message is that in such compactifications the T-duality symmetry is made manifest. To be short, for the case at hand it means that I can split the T^2 compactification into two circle compactification. After the compactification on the first circle, the T-duality symmetry group is $SO(1, n_v - 2)$ and it can be used in order to twist the compactification on the second circle. The twist matrix as spelled out in [7] takes the form

$$ {N_I}^J = \begin{pmatrix} f & 0 & M^b \\ 0 & -f & W^b \\ -W_a & -M_a & {S_a}^b \ . \end{pmatrix}$$

In such compactifications it is argued that the resulting vector multiplet sector is non-Abelian with structure constants

$$f^P_{0N} = {N_N}^P \quad f^1_{NP} = N_{NP} \ , \ N, P = 2, 3, \ldots, n_v \ , \tag{2.11}$$

where the indices of the twist matrix (and structure constants as well) are raised and lowered with the $O(1, n_v - 2)$ invariant

$$L = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \mathbf{1}_{n_v-3} \end{pmatrix} \ . \tag{2.12}$$

[2]The case when $M^2_2 + M^3_3 \neq 0$ is more subtle and I will not discuss it here. Therefore note that I have already set $M^2_2 + M^3_3 = 0$.

The above structure constants suggest that we should perform the following identifications in order to match the heterotic and M-theory sides

$$f = m \ , \quad W^a = \frac{1}{\sqrt{2}} m_a \ , \quad M^a = -\frac{1}{\sqrt{2}} \tilde{m}_a \ , \quad S_{ab} = m_{ab} \ . \tag{2.13}$$

This assignment can be indeed verified to be correct also at the level of the gaugings and of the potential which are generated.

2.4 Generalization: R-fluxes and F-theory

The setup which I described above admits a certain generalization which I will shortly discuss in the following. One can think of using the full T-duality group, $SO(2, n_v - 1)$, in the twisted compactification although this symmetry is not present at the starting point of the compactification. Such deformations of the compactification manifold were termed in the literature as R-fluxes and they do not have a geometric description, not even locally. Formally it means to introduce a second twist matrix [7, 8]

$$\tilde{N}_I{}^J = \begin{pmatrix} q & 0 & P^b \\ 0 & -q & V^b \\ -V_a & -P_a & \tilde{S}_a{}^b \end{pmatrix} , \tag{2.14}$$

which commutes with the original one. The additional structure constants which are generated in this way have the form

$$f_{1N}^P = \tilde{N}_N{}^P \ , \quad f_{NP}^0 = \tilde{N}_{NP} \ . \tag{2.15}$$

The natural thing to ask now is whether this setup has any sort of type IIA dual. Recall that before, I needed to go M-theory because I needed the M-theory circle in order to twist the compactification around it. Now in order to implement a second twist matrix I would need another circle on the type IIA/M-theory side. This naturally takes us to F-theory compactified to six dimensions on Calabi–Yau manifolds and then further down to four dimensions on a T^2 with duality twists. Packaging things together as I did in the M-theory case, the setup to consider is F-theory compactifications on eight-dimensional manifolds with SU(3) structure. Now (2.4) is generalized to

$$d\omega_i = M_i^j \omega_j \wedge dz^1 + \tilde{M}_i^j \omega_j \wedge dz^2 \ , \tag{2.16}$$

where $z^{1,2}$ denote the two directions of the torus, while $\tilde{M}$ is the second twist matrix. The constraint (2.7) has to be satisfied for both matrices M and $\tilde{M}$ and moreover these matrices have to commute in order that the exterior derivative is nilpotent.

3 Conclusions

In this talk I have presented the aspects which appear in heterotic type IIA duality when fluxes which gauge isometries in the vector multiplet sector are involved. I showed that by firs considering gauge field fluxes on the torus on the heterotic side, one is lead to the most general picture, that heterotic compactifications with R-fluxes is dual to F-theory compactifications on eight-dimensional manifolds with SU(3) structure. One of the main messages that I wanted to convey is that non-geometric fluxes in the heterotic picture have a geometric correspondent within M- or even F-theory. The other important thing to realize is that by carefully considering duality aspects within supergravity compactifications one can get information about stringy effects of these setups.

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