# Note on novel interactions of dual linearized gravity coupled to BF-type topological field theories 

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#### Abstract

In this paper we announce some novel couplings of dual linearized gravity in $D=6$, described by a massless tensor field with the mixed symmetry $(3,1)$, to a topological BF theory with a maximal field spectrum. In this context all consistent selfinteractions of the BF model based on a maximal field spectrum in $D=6$ are also elucidated. The existence of couplings and of selfinteractions strongly depends on the existence of solutions to the so-called consistency equations, which involve all the functions that parameterize the Lagrangian interaction vertices. The method employed relies on the deformation of the solution to the classical master equation computed by means of the local cohomology of the BRST differential and then restricted to satisfy certain 'selection rules' commonly used in field theories, such as spacetime locality, Lorentz covariance, Poincaré invariance and so on.


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## 1 Introduction

Topological field theories [1] are important in view of the fact that certain interacting, nonAbelian versions are related to a Poisson structure algebra [2] present in various versions of Poisson sigma models, which are known to be useful at the study of two-dimensional gravity. It is well known that pure three-dimensional gravity is just a BF theory. Moreover, in higher dimensions general relativity and supergravity in Ashtekar formalism may also be formulated as topological BF theories with some extra constraints [3]-[6]. In view of these results, it is important to know the self-interactions in BF theories as well as the couplings between BF models and other theories.

On the other hand, tensor fields in "exotic" representations of the Lorentz group, characterized by a mixed Young symmetry type [7]-[12], held the attention lately on some important issues, like the dual formulation of field theories of spin two or higher [13][20], the impossibility of consistent cross-interactions in the dual formulation of linearized gravity(DFLG) [21], or the derivation of some exotic gravitational interactions [22, 23].

[^0]An important matter related to mixed symmetry type tensor fields is the study of their consistent interactions, among themselves as well as with other gauge theories.

The purpose of this work is to expose some novel results on the consistent interactions in $D=6$ between a massless tensor gauge field with the mixed symmetry of a twocolumn Young diagram of the type $(3,1)$ and an Abelian BF model with a maximal field spectrum (a scalar field, two sorts of one-forms, two types of two-forms and a three-form). Given the duality of a free massless tensor gauge field with the mixed symmetry $(3,1)$ to the Pauli-Fierz theory in $D=6$, we can rephrase our main task as searching for the consistent couplings in $D=6$ between a specific DFLG and a topological BF model. Our analysis relies on the deformation of the solution to the classical master equation by means of cohomological techniques with the help of the local BRST cohomology. The self-interactions in the $(3,1)$ sector have been investigated in [24]. The consistent selfcouplings in $D=6$ of an Abelian BF model with a maximal field spectrum have been announced in [25]. A similar problem in $D=5$ has been solved in relation with the cross-interactions between the DFLG in terms of a massless tensor field with the mixed symmetry $(2,1)$ and a topological BF model with a maximal field spectrum [26].

The construction of consistent interactions in gauge field theories employed here relies on the deformation of the solution to the classical master equation [27, 28] by means of cohomological techniques with the help of the local BRST cohomology [29]-[31].

All results have been obtained under some standard hypotheses from field theory regarding interacting (gauge field) theories: analyticity in the coupling constant, spacetime locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the preservation of the number of derivatives on each field. Finally, we obtain nontrivial interactions manifested on the one hand through cross-couplings vertices of order one in the coupling constant and on the other through self-interactions in the BF sector at orders one and respectively two in the deformation parameter. It is worth noting that: 1. the presence of cross-couplings is essentially due to the pair of forms of maximum form degree present in the BF field sector (two- and three-forms) and 2 . the appearance of some supplementary self-interactions in the BF sector at order two in the coupling constant that are strictly related to the presence of the $(3,1)$ theory (they all vanish in its absence). The gauge transformations of all fields are deformed and, in addition, some of them include gauge parameters from the complementary sector. The entire gauge structure of the coupled model is deformed with respect to that of the free one: open gauge algebra, on-shell reducibility structure, higher-order structure functions, etc.

## 2 Deformation procedure in brief

We begin with a "free" gauge theory, described by a Lagrangian action $S^{\mathrm{L}}\left[\Phi^{\alpha_{0}}\right]$, invariant under some gauge transformations

$$
\begin{equation*}
\delta_{\epsilon} \Phi^{\alpha_{0}}=Z_{\alpha_{1}}^{\alpha_{0}} \epsilon^{\alpha_{1}}, \quad \frac{\delta S^{\mathrm{L}}}{\delta \Phi^{\alpha_{0}}} Z_{\alpha_{1}}^{\alpha_{0}}=0 \tag{1}
\end{equation*}
$$

and consider the problem of constructing consistent interactions among the fields $\Phi^{\alpha_{0}}$ such that the couplings preserve both the field spectrum and the original number of gauge symmetries. This matter is addressed by means of reformulating the problem of constructing consistent interactions as a deformation problem of the solution to the classical master equation corresponding to the "free" theory [27, 28]. Such a reformulation is possible due to the fact that the solution to the classical master equation contains all
the information on the gauge structure of the theory. If a consistent interacting gauge theory can be constructed, then the solution $S$ to the classical master equation associated with the "free" theory, $(S, S)=0$, can be deformed into a solution $\bar{S}$,

$$
\begin{align*}
S \rightarrow \bar{S} & =S+\lambda S_{1}+\lambda^{2} S_{2}+\cdots \\
& =S+\lambda \int d^{D} x a+\lambda^{2} \int d^{D} x b+\lambda^{3} \int d^{D} x c+\cdots \tag{2}
\end{align*}
$$

of the classical master equation for the deformed theory

$$
\begin{equation*}
(\bar{S}, \bar{S})=0 \tag{3}
\end{equation*}
$$

such that both the ghost and antifield spectra of the initial theory are preserved. The symbol (, ) denotes the antibracket. Equation (3) splits, according to the various orders in the coupling constant (or deformation parameter) $\lambda$, into the equivalent tower of equations

$$
\begin{align*}
(S, S) & =0,  \tag{4}\\
2\left(S_{1}, S\right) & =0  \tag{5}\\
2\left(S_{2}, S\right)+\left(S_{1}, S_{1}\right) & =0, \tag{6}
\end{align*}
$$

Equation (4) is fulfilled by hypothesis. The next one requires that the first-order deformation of the solution to the classical master equation, $S_{1}$, is a cocycle of the "free" BRST differential $s \cdot=(\cdot, S)$. However, only cohomologically nontrivial solutions to (5) should be taken into account, as the BRST-exact ones can be eliminated by (in general nonlinear) field redefinitions. This means that $S_{1}$ pertains to the ghost number zero cohomological space of $s, H^{0}(s)$, which is generically nonempty due to its isomorphism to the space of physical observables of the "free" theory. It has been shown in [27, 28] (on behalf of the triviality of the antibracket map in the cohomology of the BRST differential) that there are no obstructions in finding solutions to the remaining equations, namely, (6) and so on. However, the resulting interactions may be nonlocal, and there might even appear obstructions if one insists on their locality. The analysis of these obstructions can be done with the help of cohomological techniques. As it will be seen below, all the interactions in the case of the model under study turn out to be local.

## 3 Free theory. Antifield-BRST symmetry

The starting point is a free theory in $D=6$, whose Lagrangian action is written as the sum between the Lagrangian action of an Abelian BF model with a maximal field spectrum (a scalar field, two sorts of one-forms, two types of two-forms and a three-form) and the Lagrangian action of a free, massless tensor field with the mixed symmetry $(3,1) t_{\lambda \mu \nu \mid \alpha}$ (meaning it is antisymmetric in its first three indices and fulfills the identity $t_{[\lambda \mu \nu \mid \alpha]} \equiv 0$ )

$$
S^{\mathrm{L}}\left[\Phi^{\alpha 0}\right]=S^{\mathrm{L}}\left[\left(\begin{array}{ll}
{[m]} & {[m+1)^{\mu_{1} \ldots \mu_{m+1}}}  \tag{7}\\
A_{\mu_{1} \ldots \mu_{m}} & B
\end{array}\right)_{m=\overline{0,2}}\right]+S^{\mathrm{L}}\left[t_{\lambda \mu \nu \mid \alpha}\right]
$$

where the Lagrangian actions of the BF model and respectively of a free, massless tensor field with the mixed symmetry $(3,1)$ read respectively as

$$
\begin{gather*}
S^{\mathrm{L}}\left[\left({\left.\left.\stackrel{[m]}{A_{\mu_{1} \ldots \mu_{m}}}, \stackrel{[m+1]^{\mu_{1} \ldots \mu_{m+1}}}{B}\right)_{m=\overline{0,2}}\right]=\sum_{m=0}^{2} \frac{1}{m+1}\left(\int d^{6} x^{[m+1]^{\mu_{1} \ldots \mu_{m+1}}}{ }_{\partial_{\left[\mu_{1}\right.}{ }^{[m]}}^{\left.A_{\left.\mu_{2} \ldots \mu_{m+1}\right]}\right]}\right)}^{S^{\mathrm{L}}\left[t_{\lambda \mu \nu \mid \alpha}\right]=-\frac{1}{48} \int d^{6} x\left(F_{\lambda \mu \nu \rho \mid \alpha} F^{\lambda \mu \nu \rho \mid \alpha}-4 F_{\lambda \mu \nu} F^{\lambda \mu \nu}\right),}\right.\right.
\end{gather*}
$$

where

$$
\begin{gather*}
\Phi^{\alpha_{0}}=\left(\left(\stackrel{[m]}{A}_{\mu_{1} \ldots \mu_{m}}, \frac{[m+1]^{\mu_{1} \ldots \mu_{m+1}}}{B}\right)_{m=\overline{0,2}}, t_{\lambda \mu \nu \mid \alpha}\right),  \tag{10}\\
F_{\lambda \mu \nu \rho \mid \alpha}=\partial_{[\lambda} t_{\mu \nu \rho] \mid \alpha}, \quad F_{\lambda \mu \nu}=F_{\lambda \mu \nu \rho \mid \alpha} \sigma^{\rho \alpha} . \tag{11}
\end{gather*}
$$

Everywhere in this paper the notations $[\mu \nu \ldots \rho]$ and $(\mu \nu \ldots \rho)$ signify complete antisymmetry and respectively complete symmetry with respect to the (Lorentz) indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. It is convenient to work with the Minkowski metric tensor of 'mostly plus' signature $\sigma_{\mu \nu}=\sigma^{\mu \nu}=\operatorname{diag}(-++++)$ and with the six-dimensional Levi-Civita symbol $\varepsilon^{\mu \nu \rho \lambda \sigma \theta}$ defined according to the convention $\varepsilon^{012345}=-\varepsilon_{012345}=-1$. The supplementary overscript between brackets appearing in relation with the BF sector signifies the form degree.

Action (7) is found invariant under the gauge transformations

$$
\begin{align*}
& \left.\delta_{\Omega^{\alpha_{1}}}{ }^{[0]}=0, \quad \delta_{\Omega^{\alpha_{1}}}{ }^{[m]} A_{\mu_{1} \ldots \mu_{m}}=\partial_{\left[\mu_{1}\right.}{ }^{[m-1]} \epsilon(m, 0) \mu_{2} \ldots \mu_{m}\right], \quad m=1,2,  \tag{12}\\
& { }^{[m+1]^{\mu_{1} \ldots \mu_{m+1}}} B{ }^{2}=-(m+2) \partial_{\rho}{ }^{[m+2]^{\rho \mu_{1} \ldots \mu_{m+1}}}{ }_{(m+1,0)}, \quad m=0,1,2,  \tag{13}\\
& \delta_{\Omega^{\alpha} 1} t_{\lambda \mu \nu \mid \alpha}=\partial_{[\lambda} \chi_{\mu \nu] \mid \alpha}+\partial_{[\lambda} \theta_{\mu \nu] \alpha}+3 \partial_{\alpha} \theta_{\lambda \mu \nu}, \tag{14}
\end{align*}
$$

where all the gauge parameters are bosonic, with $\binom{[m+2]^{\mu_{1} \ldots \mu_{m+2}}}{\xi_{(m+1,0)}}_{m=\overline{0,2}}$ and $\theta_{\lambda \mu \nu}$ completely antisymmetric and $\chi_{\mu \nu \mid \alpha}$ displaying the mixed symmetry $(2,1)$. By $\Omega^{\alpha_{1}}$ we denoted collectively all the gauge parameters as

$$
\begin{equation*}
\Omega^{\alpha_{1}} \equiv\left(\binom{[m-1]}{(m, 0) \mu_{1} \ldots \mu_{m-1}}_{m=1,2},\binom{[m+2]^{\mu_{1} \ldots \mu_{m+2}}}{(m+1,0)}_{m=\overline{0,2}}, \theta_{\lambda \mu \nu}, \chi_{\mu \nu \mid \alpha}\right) \tag{15}
\end{equation*}
$$

The gauge transformations given by (12)-(14) are off-shell reducible of order 4 (the reducibility relations hold everywhere in the space of field history, and not only on the stationary surface of field equations) in $D=6$. This means that: 1 . there exist some transformations of the gauge parameters (15), $\Omega^{\alpha_{1}}=\Omega^{\alpha_{1}}\left(\Omega^{\alpha_{2}}\right)$, such that the gauge transformations of all fields vanish strongly (first-order reducibility relations) $\delta_{\Omega^{\alpha_{1}}\left(\Omega^{\alpha_{2}}\right)} \Phi^{\alpha_{0}}=0$; 2. there exist some transformations of the reducibility parameters of order $k, \Omega^{\alpha_{k}}=$ $\Omega^{\alpha_{k}}\left(\Omega^{\alpha_{k+1}}\right), k=2,3,4$, such that the gauge parameters/reducibility parameters of or-$\operatorname{der}(k-1)$ vanish strongly ( $k$-order reducibility relations), $\Omega^{\alpha_{k-1}}\left(\Omega^{\alpha_{k}}\left(\Omega^{\alpha_{k+1}}\right)\right)=0, k=$
$2,3,4 ; 3$. there is no nontrivial transformation of the fourth-order reducibility parameters $\Omega^{\alpha_{5}}$ that annihilates all the third-order reducibility parameters, $\Omega^{\alpha_{4}}\left(\Omega^{\alpha_{5}}\right)=0 \Leftrightarrow \Omega^{\alpha_{5}}=0$. This is indeed the case for the model under study, where

$$
\begin{align*}
& \Omega^{\alpha_{2}}=\left(\stackrel{[0]}{\epsilon}_{(2,1)},\left(\begin{array}{c}
{[m+3]^{\mu_{1} . . . \mu_{m+3}}} \\
\xi \\
(m+1,1)
\end{array}\right)_{m=\overline{0,2}}, \theta_{\mu \nu}, \chi_{\mu \mid \nu}\right),  \tag{16}\\
& \Omega^{\alpha_{3}}=\left(\left(\begin{array}{c}
{[m+4]^{\mu_{1} \ldots \mu_{m+4}}} \\
\xi
\end{array}{ }_{(m+1,2)}\right)_{m=\overline{0,2}}, \theta_{\mu}\right),  \tag{17}\\
& \Omega^{\alpha_{4}}=\binom{[m+5]^{\mu_{1} \ldots \mu_{m+5}}}{{ }_{(m+1,3)}}_{m=\overline{0,1}}, \quad \Omega^{\alpha_{5}}=\stackrel{[6]^{\mu_{1} \ldots \mu_{6}}}{(1,4)}, \tag{18}
\end{align*}
$$

and all parameters denoted by $\xi$ together with $\theta_{\mu \nu}$ are antisymmetric, while $\chi_{\mu \mid \nu}$ is symmetric. The above discussed transformations of the reducibility parameters take the concrete form

$$
\begin{align*}
& \stackrel{[0]}{\epsilon}_{(1,0)}\left(\Omega^{\alpha_{2}}\right)=0, \quad{\stackrel{[1]}{\epsilon}(2,0) \mu_{1}}\left(\Omega^{\alpha_{2}}\right)=\partial_{\mu_{1}} \stackrel{[0]}{\epsilon}_{(2,1)},  \tag{19}\\
& \underset{\xi}{[m+2]^{\mu_{1} \ldots \mu_{m+2}}}\left(\Omega^{\alpha_{2}}\right)=-(m+3) \partial_{\rho} \stackrel{[m+3]^{\rho \mu_{1} \ldots \mu_{m+2}}}{\xi}{ }_{(m+1,1)}, \quad m=\overline{0,2},  \tag{20}\\
& \theta_{\lambda \mu \nu}\left(\Omega^{\alpha_{2}}\right)=-\frac{1}{2} \partial_{[\lambda} \theta_{\mu \nu]}, \quad \chi_{\mu \nu \mid \alpha}\left(\Omega^{\alpha_{2}}\right)=\partial_{[\mu} \chi_{\nu] \mid \alpha}+2 \partial_{\alpha} \theta_{\mu \nu}-\partial_{[\mu} \theta_{\nu] \alpha},  \tag{21}\\
& \stackrel{[m+3]^{\mu_{1} \ldots \mu_{m+3}}}{(m+1,1)}\left(\Omega^{\alpha_{3}}\right)=-(m+4) \partial_{\rho} \stackrel{[m+4]^{\rho \mu_{1} \ldots \mu_{m+3}} \xi}{(m+1,2)}, \quad m=\overline{0,2},  \tag{22}\\
& \stackrel{[0]}{\epsilon}(2,1)\left(\Omega^{\alpha_{3}}\right)=0, \quad \theta_{\mu \nu}\left(\Omega^{\alpha_{3}}\right)=\partial_{[\mu} \theta_{\nu]}, \quad \chi_{\mu \mid \nu}\left(\Omega^{\alpha_{3}}\right)=-3 \partial_{(\mu} \theta_{\nu)},  \tag{23}\\
& \stackrel{[m+4]^{\mu_{1} \ldots \mu_{m+4}}}{(m+1,2)}\left(\Omega^{\alpha_{4}}\right)=-(m+5) \partial_{\rho} \stackrel{[m+5]^{\rho \mu_{1} \ldots \mu_{m+4}} \xi}{(m+1,3)}, \quad m=\overline{0,1},  \tag{24}\\
& {[6]^{\mu_{1} \ldots \mu_{6}}} \\
& \xi_{(3,2)} \quad\left(\Omega^{\alpha_{4}}\right)=0, \quad \theta_{\mu}\left(\Omega^{\alpha_{4}}\right)=0,  \tag{25}\\
& \stackrel{[5]^{\mu_{1} \ldots \mu_{5}}}{\xi_{(1,3)}}\left(\Omega^{\alpha_{5}}\right)=-6 \partial_{\rho}{ }_{\rho}^{[6]}{ }_{(1,4)}{ }^{\rho \mu_{1} \ldots \mu_{5}}, \quad \stackrel{[6]^{[6]} \ldots \mu_{6}}{\xi}, \quad{ }_{(2,3)}^{\mu_{1}}\left(\Omega^{\alpha_{5}}\right)=0 . \tag{26}
\end{align*}
$$

The additional pair of lower indices appearing in connection with the BF sector signifies the form gauge field to which a certain parameter is associated and respectively the reducibility order.

We observe that the free theory under study is a usual linear gauge theory (its field equations are linear in the fields), whose generating set of gauge transformations is fourthorder reducible, such that we can define in a consistent manner its Cauchy order, which is found to be equal to six.

In order to construct the BRST symmetry of this free theory, we introduce the field/ghost and antifield spectra (10) and

$$
\begin{align*}
& \eta^{\alpha_{2}}=\left({\left.\stackrel{[0]}{\eta}]_{(2,1)},\left({\stackrel{[m+3]^{\prime}}{\mu_{1} \ldots \mu_{m+3}}}_{(m+1,1)}\right)_{m=\overline{0,2}}, \mathcal{A}_{\mu \nu}, \mathcal{G}_{\mu \mid \nu}\right), ~}_{\text {, }}\right.  \tag{28}\\
& \eta^{\alpha_{3}}=\left(\left({\stackrel{[m+4]}{\mu_{1} \ldots \mu_{m+4}}}_{(m+1,2)}^{\mu_{m=0,2}}, \mathcal{A}_{\mu}\right),\right.  \tag{29}\\
& \eta^{\alpha_{4}}=\binom{[m+5]^{\mu_{1} \ldots \mu_{m+5}}}{(m+1,3)}_{m=\overline{0,1}}, \quad \eta^{\alpha_{5}}=\stackrel{[6]}{C}(1,4)_{\mu_{1} \ldots \mu_{6}},  \tag{30}\\
& \Phi_{\alpha_{0}}^{*}=\left(\left({\left.\left.\stackrel{[m]^{*} \mu_{1} \ldots \mu_{m}}{A}, \stackrel{[m+1]^{*}}{\mu_{1} \ldots \mu_{m+1}}\right)_{m=\overline{0,2}}, t^{* \lambda \mu \nu \mid \alpha}\right), ~}_{\square}\right.\right.  \tag{31}\\
& \eta_{\alpha_{1}}^{*}=\left(\binom{[m-1]^{* \mu_{1} \ldots \mu_{m-1}}}{(m, 0)}_{m=1,2},\left({\stackrel{[m+2]^{*}}{C}(m+1,0) \mu_{1} \ldots \mu_{m+2}}^{)_{m=\overline{0,2}}, \mathcal{A}^{* \lambda \mu \nu}, \mathcal{G}^{* \mu \nu \mid \alpha}\right), ~, ~, ~}\right.\right.  \tag{32}\\
& \eta_{\alpha_{2}}^{*}=\left(\stackrel{[0 \mid}{\eta}_{(2,1)}^{*},\left({\stackrel{[m+3]^{*}}{ }}_{(m+1,1) \mu_{1} \ldots \mu_{m+3}}^{*}\right)_{m=\overline{0,2}}, \mathcal{A}^{* \mu \nu}, \mathcal{G}^{* \mu \mid \nu}\right),  \tag{33}\\
& \eta_{\alpha_{3}}^{*}=\left(\left({\left.\left.\stackrel{[m+4]^{*}}{(m+1,2) \mu_{1} \ldots \mu_{m+4}}\right)_{m=\overline{0,2}}, \mathcal{A}^{* \mu}\right), ~}\right.\right. \tag{34}
\end{align*}
$$

The fermionic ghosts (27) correspond to the bosonic gauge parameters (15), the bosonic ghosts for ghosts (28) are respectively associated with the first-order reducibility parameters (16), the fermionic ghosts for ghosts for ghosts $\eta^{\alpha_{3}}$ from (29) correspond to the secondorder reducibility parameters (17) and finally the bosonic ghosts for ghosts for ghosts for ghosts $\eta^{\alpha_{4}}$ and fermionic ghosts for ghosts for ghosts for ghosts for ghosts $\eta^{\alpha_{5}}$ from (30) are associated with the third-order and respectively fourth-order reducibility parameters (18). The star variables represent the antifields of the corresponding fields/ghosts. Their Grassmann parities are obtained via the usual rule $\varepsilon\left(\chi_{\Delta}^{*}\right)=\left(\varepsilon\left(\chi^{\Delta}\right)+1\right) \bmod 2$, where we employed the notations

$$
\begin{equation*}
\chi^{\Delta}=\left(\Phi^{\alpha_{0}}, \eta^{\alpha_{1}}, \eta^{\alpha_{2}}, \eta^{\alpha_{3}}, \eta^{\alpha_{4}}, \eta^{\alpha_{5}}\right), \quad \chi_{\Delta}^{*}=\left(\Phi_{\alpha_{0}}^{*}, \eta_{\alpha_{1}}^{*}, \eta_{\alpha_{2}}^{*}, \eta_{\alpha_{3}}^{*}, \eta_{\alpha_{4}}^{*}, \eta_{\alpha_{5}}^{*}\right) . \tag{36}
\end{equation*}
$$

It is understood that all ghosts and antifields are endowed with exactly the same symmetry or antisymmetry or mixed symmetry properties like the corresponding fields or reducibility parameters.

Since both the gauge generators and the reducibility functions are field-independent, it follows that the BRST differential, $s$, reduces to

$$
\begin{equation*}
s=\delta+\gamma, \tag{37}
\end{equation*}
$$

where $\delta$ is the Koszul-Tate differential, and $\gamma$ means the exterior longitudinal derivative. The Koszul-Tate differential is graded in terms of the antighost number (agh, agh $(\delta)=$ -1 , agh $(\gamma)=0)$ and enforces a resolution of the algebra of smooth functions defined on the stationary surface of field equations for action (7), $C^{\infty}(\Sigma), \Sigma: \delta S^{\mathrm{L}} / \delta \Phi^{\alpha_{0}}=0$.

The exterior longitudinal derivative is in this case a true differential, graded in terms of the pure ghost number ( $\mathrm{pgh}, \operatorname{pgh}(\gamma)=1, \operatorname{pgh}(\delta)=0$ ) and correlated with the original gauge symmetry via its cohomology in pure ghost number zero computed in $C^{\infty}(\Sigma)$, which is isomorphic to the algebra of physical observables for this free theory. The overall degree that grades the BRST complex is named ghost number (gh) and is defined like the difference between the pure ghost number and the antighost number, such that $\operatorname{gh}(\delta)=$ $\operatorname{gh}(\gamma)=\operatorname{gh}(s)=1$. The nilpotency of $s, s^{2}=0$, is equivalent to the individual nilpotency of the two composing differentials plus their anticommutation

$$
s^{2}=0 \Leftrightarrow\left(\delta^{2}=0, \quad \delta \gamma+\gamma \delta=0, \quad \gamma^{2}=0\right) .
$$

These two degrees of generators (10) and (27)-(35) from the BRST complex are valued like

$$
\begin{gather*}
\operatorname{pgh}\left(\Phi^{\alpha_{0}}\right)=0, \quad \operatorname{pgh}\left(\eta^{\alpha_{m}}\right)=m, \quad \operatorname{pgh}\left(\Phi_{\alpha_{0}}^{*}\right)=\operatorname{pgh}\left(\eta_{\alpha_{m}}^{*}\right)=0,  \tag{38}\\
\operatorname{agh}\left(\Phi^{\alpha_{0}}\right)=\operatorname{agh}\left(\eta^{\alpha_{m}}\right)=0, \quad \operatorname{agh}\left(\Phi_{\alpha_{0}}^{*}\right)=1,\left(\eta_{\alpha_{m}}^{*}\right)=m+1, \tag{39}
\end{gather*}
$$

for $m=\overline{1,5}$. The actions of the differentials $\delta$ and $\gamma$ on the above generators read as

$$
\begin{align*}
& \left(\delta \Phi^{\alpha_{0}}=0, \quad \delta \eta^{\alpha_{m}}=0, \quad m=\overline{1,5}\right) \Longleftrightarrow \delta \chi^{\Delta}=0,  \tag{40}\\
& \delta t^{* \lambda \mu \nu \mid \alpha}=-\frac{1}{3!}\left(\partial_{\rho} F^{\rho \lambda \mu \nu \mid \alpha}+\partial^{\alpha} F^{\lambda \mu \nu}-\sigma^{\alpha[\lambda} \partial_{\rho} F^{\mu \nu] \rho}\right),  \tag{41}\\
& \delta \mathcal{A}^{* \lambda \mu \nu}=-4 \partial_{\alpha} t^{* \lambda \mu \nu \mid \alpha}, \quad \delta \mathcal{G}^{* \mu \nu \mid \alpha}=-\partial_{\rho}\left(3 t^{* \rho \mu \nu \mid \alpha}-t^{* \mu \nu \alpha \mid \rho}\right) \text {, }  \tag{42}\\
& \delta \mathcal{A}^{* \mu \nu}=3 \partial_{\rho}\left(\mathcal{G}^{* \mu \nu \mid \rho}-\frac{1}{2} \mathcal{A}^{* \rho \mu \nu}\right),  \tag{43}\\
& \delta \mathcal{G}^{* \mu \mid \nu}=\partial_{\rho} \mathcal{G}^{* \rho(\mu \mid \nu)}, \quad \delta \mathcal{A}^{* \mu}=6 \partial_{\rho}\left(\mathcal{G}^{* \rho \mu}-\frac{1}{3} \mathcal{A}^{* \rho \mu}\right),  \tag{44}\\
& \delta A^{[m]^{* \mu_{1} \ldots \mu_{m}}}=\partial_{\rho}{ }^{[m+1]^{\rho \mu_{1} \ldots \mu_{m}}}, \quad m=\overline{0,2},  \tag{45}\\
& \delta{ }^{[m+1]^{*}}{ }_{\mu_{1} \ldots \mu_{m+1}}=-\frac{1}{m+1} \partial_{\left[\mu_{1}\right.} A_{\left.\mu_{2} \ldots \mu_{m+1}\right]}, \quad m=\overline{0,2},  \tag{46}\\
& \delta^{[m-1]^{*} \mu_{1} \ldots \mu_{m-1}}=-m \partial_{\rho} A^{[m]^{* \rho \mu_{1} \ldots \mu_{m-1}}}, \quad m=\overline{1,2}, \tag{47}
\end{align*}
$$

$$
\begin{align*}
& \delta \eta_{(2,1)}^{[0]^{*}}=\partial_{\rho} \eta_{(2,0)}^{[1]^{*}}, \quad \delta \stackrel{[m+k+3]^{*}}{C}{ }_{(m+1, k+1) \mu_{1} \ldots \mu_{m+k+3}}=(-)^{k+1} \partial_{\left[\mu_{1}\right.}^{[m+k+2]^{*}}{ }_{\left.(m+1, k) \mu_{2} \ldots \mu_{m+k+3}\right]}, \tag{48}
\end{align*}
$$

and respectively

$$
\begin{align*}
& \left(\gamma \Phi_{\alpha_{0}}^{*}=0, \quad \gamma \eta_{\alpha_{m}}^{*}=0, \quad m=\overline{1,4}\right) \Longleftrightarrow \gamma \chi_{\Delta}^{*}=0,  \tag{50}\\
& \gamma t_{\lambda \mu \nu \mid \alpha}=\partial_{[\lambda} \mathcal{G}_{\mu \nu] \mid \alpha}+\partial_{[\lambda} \mathcal{A}_{\mu \nu] \alpha}+3 \partial_{\alpha} \mathcal{A}_{\lambda \mu \nu},  \tag{51}\\
& \gamma \mathcal{A}_{\lambda \mu \nu}=-\frac{1}{2} \partial_{[\lambda} \mathcal{A}_{\mu \nu]}, \quad \gamma \mathcal{G}_{\mu \nu \mid \alpha}=\partial_{[\mu} \mathcal{G}_{\nu] \mid \alpha}+2 \partial_{\alpha} \mathcal{A}_{\mu \nu}-\partial_{[\mu} \mathcal{A}_{\nu] \alpha},  \tag{52}\\
& \gamma \mathcal{A}_{\mu \nu}=\partial_{[\mu} \mathcal{A}_{\nu]}, \quad \gamma \mathcal{G}_{\mu \mid \nu}=-3 \partial_{(\mu} \mathcal{A}_{\nu)}, \quad \gamma \mathcal{A}_{\mu}=0,  \tag{53}\\
& \gamma \stackrel{[0]}{A}=0, \quad \gamma^{[m]} A_{\mu_{1} \ldots \mu_{m}}=\partial_{\left[\mu_{1}\right.}{ }^{[m-1]}{ }_{\left.(m, 0) \mu_{2} \ldots \mu_{m}\right]}, \quad m=1,2,  \tag{54}\\
& \gamma \stackrel{[m+1]^{\mu_{1} \ldots \mu_{m+1}}}{B}=-(m+2) \partial_{\rho}{\stackrel{[m+2]^{\prime}}{C}{ }_{(m+1,0)}^{\rho \mu_{1} \ldots \mu_{m+1}}} \quad, \quad m=0,1,2,  \tag{55}\\
& \gamma^{[1]}{ }_{(2,0) \mu_{1}}=\partial_{\mu_{1}} \stackrel{[0]}{\eta}_{(2,1)}, \quad \gamma \stackrel{[m+k+2]^{\mu_{1} \ldots \mu_{m+k+2}}}{C}{ }_{(m+1, k)}=-(m+k+3) \partial_{\rho}{ }^{[m+k+3]^{\rho}{ }^{\rho \mu_{1} \ldots \mu_{m+k+2}}} \underset{(m+1, k+1)}{ } \text {, } \tag{56}
\end{align*}
$$

$$
\begin{equation*}
\stackrel{[0]}{\eta}_{(m, m-1)}=0, \quad m=1,2, \quad \gamma{ }_{[6]}^{C_{(m+1,4-m)}^{\mu_{1} \ldots \mu_{6}}}=0, \quad m=0,1,2 \tag{57}
\end{equation*}
$$

with $k=\overline{0,3-m}, m=\overline{0,2}$ in the latter relation from (49) and respectively from (56).
The BRST symmetry admits a canonical action $s \cdot=(\cdot, S)$, where its canonical generator $(\mathrm{gh}(S)=0, \varepsilon(S)=0)$ satisfies the classical master equation $(S, S)=0$. The symbol (,) denotes the antibracket, defined by decreeing the fields/ghosts conjugated with the corresponding antifields. In the case of the free theory under discussion the solution to the classical master equation takes the form

$$
\begin{align*}
& S=S^{\mathrm{L}}+\int d^{6} x\left[t^{* \lambda \mu \nu \mid \alpha}\left(\partial_{[\lambda} \mathcal{G}_{\mu \nu] \mid \alpha}+\partial_{[\lambda} \mathcal{A}_{\mu \nu] \alpha}+3 \partial_{\alpha} \mathcal{A}_{\lambda \mu \nu}\right)-\frac{1}{2} \mathcal{A}^{* \lambda \mu \nu} \partial_{[\lambda} \mathcal{A}_{\mu \nu]}\right. \\
& +\mathcal{G}^{* \mu \nu \mid \alpha}\left(\partial_{[\mu} \mathcal{G}_{\nu] \mid \alpha}+2 \partial_{\alpha} \mathcal{A}_{\mu \nu}-\partial_{[\mu} \mathcal{A}_{\nu] \alpha}\right)+\mathcal{A}^{* \mu \nu} \partial_{[\mu} \mathcal{A}_{\nu]}-3 \mathcal{G}^{* \mu \mid \nu} \partial_{(\mu} \mathcal{A}_{\nu)} \\
& +\sum_{m=1}^{2}{ }^{[m]^{*} \mu_{1} \ldots \mu_{m}} \partial_{\left[\mu_{1}\right.}{ }^{[m-1]}{ }_{\left.(m, 0) \mu_{2} \ldots \mu_{m}\right]}+\stackrel{[1)^{*} \mu_{(2,0)}}{ } \partial_{\mu_{1}}{ }_{\left[\begin{array}{l}
\eta \\
\eta
\end{array}\right.}^{(2,1)} \\
& -\sum_{m=0}^{2}(m+2){\stackrel{[m+1]^{*}}{B}}_{\mu_{1} \ldots \mu_{m+1}}^{*} \partial_{\rho}{\stackrel{[m+2]^{\rho}}{\rho \mu_{1} \ldots \mu_{m+1}}}_{(m+1,0)} \\
& \left.-\sum_{m=0}^{2} \sum_{k=0}^{3-m}(m+k+3) \stackrel{[m+k+2]^{*}}{C} \underset{(m+1, k) \mu_{1} \ldots \mu_{m+k+2}}{ } \partial_{\rho} \stackrel{[m+k+3]^{\rho \mu_{1} \ldots \mu_{m+k+2}}}{(m+1, k+1)}\right] . \tag{58}
\end{align*}
$$

The solution to the classical master equation encodes all the information on the gauge structure of a given theory. We remark that in our case solution (58) decomposes into terms with antighost numbers ranging from 0 to 5 . Let us briefly recall the significance of the various terms present in the solution to the master equation. Thus, the part with the antighost number equal to zero is nothing but the Lagrangian action of the gauge model under study. The components of antighost number equal to one are always proportional with the gauge generators. If the gauge algebra were non-Abelian, then there would appear terms simultaneously linear in the antighost number two antifields and quadratic in the pure ghost number one ghosts. The absence of such terms in our case shows that the gauge transformations are Abelian. The terms from (58) with higher antighost numbers give us information on the reducibility functions. If the reducibility relations held onshell, then there would appear components linear in the ghosts for ghosts (ghosts of pure ghost number strictly greater than one) and quadratic in the various antifields. Such pieces are not present in (58) since the reducibility relations hold off-shell. Other possible components in the solution to the master equation offer information on the higher-order structure functions related to the tensor gauge structure of the theory. There are no such terms in (58) as a consequence of the fact that all higher-order structure functions for the theory under study vanish.

## 4 Basic ingredients of the local BRST cohomology

In the sequel we determine all consistent Lagrangian interactions that can be added to the free theory described by (7) and (12)-(14). This is done by means of solving the deformation equations (5)-(6), etc., with the help of specific cohomological techniques. The interacting theory and its gauge structure are then deduced from the analysis of the deformed solution to the master equation that is consistent to all orders in the deformation parameter.

For obvious reasons, we consider only analytical, local, Lorentz covariant, and Poincaré invariant deformations (i.e., we do not allow explicit dependence on the spacetime coordinates). The analyticity of deformations refers to the fact that the deformed solution to the master equation, (2), is analytical in the coupling constant $\lambda$ and reduces to the original solution, (58), in the free limit $\lambda=0$. In addition, we require that the overall interacting Lagrangian satisfies two further restrictions related to the derivative order of its vertices: i) the maximum derivative order of each interaction vertex is equal to two; ii) the differential order of each interacting field equation is equal to that of the corresponding free equation (meaning that at most one spacetime derivative can act on each field from the BF sector and at most two spacetime derivatives on the tensor field $t_{\lambda \mu \nu \mid \alpha}$ ).

If we use the notation from (2), $S_{1}=\int d^{6} x a$, with $a$ local, then equation (5) (which controls the first-order deformation) takes the local form

$$
\begin{equation*}
s a=\partial_{\mu} m^{\mu}, \quad \operatorname{gh}(a)=0, \quad \varepsilon(a)=0, \tag{59}
\end{equation*}
$$

for some local $m^{\mu}$ and it shows that the nonintegrated density of the first-order deformation pertains to the local cohomology of $s$ at ghost number zero computed in the space of local forms, $a \in H^{0}(s \mid d)$, where $d$ denotes the exterior spacetime differential. In order to analyze the above equation, we develop $a$ according to the antighost number

$$
\begin{equation*}
a=\sum_{i=0}^{I} a_{i}, \quad \operatorname{agh}\left(a_{i}\right)=i, \quad \operatorname{gh}\left(a_{i}\right)=0, \quad \varepsilon\left(a_{i}\right)=0, \tag{60}
\end{equation*}
$$

and assume, without loss of generality, that the above decomposition stops at some finite value of $I$. By taking into account the splitting (37) of the BRST differential, equation (59) becomes equivalent to a tower of local equations, corresponding to the different decreasing values of the antighost number

$$
\begin{align*}
\gamma a_{I} & =\partial_{\mu}{ }_{(I)^{\mu}}^{\mu}  \tag{61}\\
\delta a_{I}+\gamma a_{I-1} & =\partial_{\mu}{ }^{(I-1)^{\mu}}  \tag{62}\\
\delta a_{i}+\gamma a_{i-1} & =\partial_{\mu}{ }^{(\stackrel{(-1}{m})^{\mu}}, \quad I-1 \geq i \geq 1, \tag{63}
\end{align*}
$$

for some local $\left(\stackrel{(i)}{m}_{m}^{\mu}\right)_{i=\overline{0, I}}$. Equation (61) can always be replaced in strictly positive values of the antighost number by

$$
\begin{equation*}
\gamma a_{I}=0, \quad I>0 . \tag{64}
\end{equation*}
$$

The nontriviality of the first-order deformation $a$ is translated at its highest antighost number component into the requirement that $a_{I} \in H^{I}(\gamma)$, where $H^{I}(\gamma)$ denotes the cohomology of the exterior longitudinal derivative $\gamma$ in pure ghost number equal to $I$ computed in the space of local forms. So, in order to solve equation (59) (equivalent with (64) and (62)-(63)), we need to compute the cohomology of $\gamma$ computed in the space of local forms, $H(\gamma)$, and, as it will be made clear below, also the local homology of $\delta$, $H(\delta \mid d)$.

From the actions of $\gamma$ on the BRST generators, given by definitions, (50)-(57), it can be shown that $H(\gamma)$ in the space of local forms is generated by the antifields $\chi_{\Delta}^{*}$ (see (36)) and their spacetime derivatives, by the quantities

$$
\begin{equation*}
F_{\bar{A}}=\left(\stackrel{[0]}{A},\left(\partial_{\left[\mu_{1}\right.}^{{ }^{[m]}} A_{\left.\mu_{2} \ldots \mu_{m+1}\right]}\right)_{m=1,2},\left(\partial_{\rho}^{[m+1]^{\rho \mu_{1} \ldots \mu_{m}}}\right)_{m=\overline{0,2}}, K_{\lambda \mu \nu \rho \mid \alpha \beta}\right) \tag{65}
\end{equation*}
$$

together with their spacetime derivatives, and by the undifferentiated objects (meaning that all their spacetime derivatives are trivial in $H(\gamma))$

$$
\begin{equation*}
\eta^{\bar{\Upsilon}}=\left(\left({\left.\stackrel{[0]}{\eta}{ }_{(m, m-1)}\right)_{m=1,2},\left(\stackrel{[6]}{C}_{(m+1,4-m)}^{\mu_{1} \ldots \mu_{6}}=0\right)}_{m=0,1,2}, \mathcal{F}_{\lambda \mu \nu \rho}, \mathcal{A}_{\mu}\right) .\right. \tag{66}
\end{equation*}
$$

In (65) and (66) we respectively used the notations

$$
\begin{gather*}
K_{\lambda \mu \nu \rho \mid \alpha \beta} \equiv F_{\lambda \mu \nu \rho \mid[\beta, \alpha]}=\partial_{\alpha} F_{\lambda \mu \nu \rho \mid \beta}-\partial_{\beta} F_{\lambda \mu \nu \rho \mid \alpha},  \tag{67}\\
\mathcal{F}_{\lambda \mu \nu \rho}=\partial_{[\lambda} \mathcal{A}_{\mu \nu \rho]}, \tag{68}
\end{gather*}
$$

with $f_{, \alpha} \equiv \partial_{\alpha} f$. The tensor of components $K_{\lambda \mu \nu \rho \mid \alpha \beta}$ defines the curvature tensor of the $(3,1)$ sector. It displays the mixed symmetry $(4,2)$, so it is antisymmetric in its first four indices and satisfies the Bianchi I identities $K_{[\lambda \mu \nu \rho \mid \alpha] \beta} \equiv 0$. In addition, it checks the Bianchi II identities $\partial_{[\lambda} K_{\mu \nu \rho \sigma] \mid \alpha \beta} \equiv 0$ and $K_{\lambda \mu \nu \rho \mid[\beta \gamma, \alpha]} \equiv 0$. The curvature tensor (67) and its spacetime derivatives are the most general gauge-invariant quantities constructed out of the tensor field $t_{\lambda \mu \nu \mid \alpha}$ and its derivatives. Its components are linear in some of the second-order derivatives of $t_{\lambda \mu \nu \mid \alpha}$, as it can be seen from (11) and (67)

$$
\begin{equation*}
K_{\lambda \mu \nu \rho \mid \alpha \beta}=\partial_{\alpha} \partial_{[\lambda} t_{\mu \nu \rho] \mid \beta}-\partial_{\beta} \partial_{[\lambda} t_{\mu \nu \rho] \mid \alpha} . \tag{69}
\end{equation*}
$$

In conclusion, the most general, nontrivial solution to (64) takes the form

$$
\begin{equation*}
a_{I}=\alpha_{I}\left(\left[F_{\bar{A}}\right],\left[\chi_{\Delta}^{*}\right]\right) e^{I}\left(\eta^{\bar{\gamma}}\right), \tag{70}
\end{equation*}
$$

where agh $\left(\alpha_{I}\right)=I$ and $\operatorname{pgh}\left(e^{I}\right)=I$. The notation $f([q])$ means that $f$ depends on $q$ and its spacetime derivatives up to a finite order and $e^{M}\left(\eta^{\bar{\Upsilon}}\right)$ denote the elements with pure ghost number $M$ of a basis in the space of polynomials in the objects (66). The objects $\alpha_{I}$ (obviously nontrivial in $H^{0}(\gamma)$ ) will be called invariant 'polynomials'. They are true polynomials with respect to all variables (65) and their spacetime derivatives, excepting the undifferentiated scalar field $\stackrel{[0]}{A}$, with respect to which $\alpha_{I}$ may be series. This is why we will keep the quotation marks around the word polynomial(s).

Inserting (70) in (62) we obtain that a necessary (but not sufficient) condition for the existence of (nontrivial) solutions $a_{I-1}$ is that the invariant 'polynomials' $\alpha_{I}$ are (nontrivial) objects from the local cohomology of Koszul-Tate differential in antighost number $I>0$ and in pure ghost number zero. Using the fact that the free model under study is a linear gauge theory of Cauchy order equal to 6 and the general result from the literature [29]-[31] according to which this local cohomology is trivial in antighost numbers strictly greater than its Cauchy order, we can state that

$$
\begin{equation*}
H_{J}(\delta \mid d)=0, \quad J>6 \tag{71}
\end{equation*}
$$

Moreover, it can be shown that if the invariant 'polynomial' $\alpha_{J}$, with agh $\left(\alpha_{J}\right)=J \geq 6$, is trivial in $H_{J}(\delta \mid d)$, then it can be taken to be trivial also in $H_{J}^{\text {inv }}(\delta \mid d)$. Here, $H_{J}^{\text {inv }}(\delta \mid d)$ denotes the local cohomology of Koszul-Tate differential in antighost number $J$ (and obviously in pure ghost number zero) computed in the space of invariant 'polynomials'. This result together with (71) ensures that

$$
\begin{equation*}
H_{J}^{\mathrm{inv}}(\delta \mid d)=0, \quad J>6 \tag{72}
\end{equation*}
$$

It is possible to show that no nontrivial representative of $H_{J}(\delta \mid d)$ or $H_{J}^{\text {inv }}(\delta \mid d)$ for $J \geq 2$ is allowed to involve the spacetime derivatives of the fields. Such a representative may [0] depend at most on the undifferentiated scalar field $A$. With the help of relations (40)-(49), it can be shown that both $H^{\text {inv }}(\delta \mid d)$ and $H(\delta \mid d)$ are spanned by the nontrivial elements:

$$
\begin{align*}
& J=6: \alpha_{6}=f_{1}^{\mu_{1} \ldots \mu_{6}} W_{\mu_{1} \ldots \mu_{6}},  \tag{73}\\
& J=5: \alpha_{5}=f_{1}^{\mu_{1} \ldots \mu_{5}} W_{\mu_{1} \ldots \mu_{5}}+g_{2}^{\mu_{1} \ldots \mu_{6}} C_{(2,3) \mu_{1} \ldots \mu_{6}}^{[6},  \tag{74}\\
& J=4: \alpha_{4}=f_{1}^{\mu_{1} \ldots \mu_{4}} W_{\mu_{1} \ldots \mu_{4}}+g_{2}^{\mu_{1} \ldots \mu_{5}} C_{(2,2) \mu_{1} \ldots \mu_{5}} \\
& +g_{3}^{\mu_{1} \ldots \mu_{6}}{ }_{C}^{[6]}{ }_{(3,2) \mu_{1} \ldots \mu_{6}}^{*}+h_{\mu} \mathcal{A}^{* \mu},  \tag{75}\\
& J=3: \alpha_{3}=f_{1}^{\mu_{1} \mu_{2} \mu_{3}} W_{\mu_{1} \mu_{2} \mu_{3}}+g_{2}^{\mu_{1} \ldots \mu_{4}} C_{(2,1) \mu_{1} \ldots \mu_{4}}^{\left[4{ }^{*}\right.} \\
& +g_{3}^{\mu_{1} \ldots \mu_{5}}{ }_{( }^{[5]}{ }_{(3,1) \mu_{1} \ldots \mu_{5}}^{*}+h_{\mu \nu} \mathcal{A}^{* \mu \nu}+\bar{h}_{\mu \mid \nu} \mathcal{G}^{* \mu \mid \nu}+k_{2} \eta_{(2,1)}^{[0)^{*}}  \tag{76}\\
& J=2: \alpha_{2}=f_{1}^{\mu_{1} \mu_{2}} W_{\mu_{1} \mu_{2}}+g_{2}^{\mu_{1} \mu_{2} \mu_{3}} C_{(2,0) \mu_{1} \mu_{2} \mu_{3}}^{[3]^{*}}+g_{3}^{\mu_{1} \ldots \mu_{4}} C_{(3,0) \mu_{1} \ldots \mu_{4}}^{[4]} \\
& +h_{\lambda \mu \nu} \mathcal{A}^{* \lambda \mu \nu}+\bar{h}_{\mu \nu \mid \alpha} \mathcal{G}^{* \mu \nu \mid \alpha}+k_{2 \mu_{1}}{ }^{[11} \eta_{(2,0)}^{* \mu_{1}}+k_{1}{ }^{[01}{ }^{[0}{ }_{(1,0)}^{*} . \tag{77}
\end{align*}
$$

In the above all coefficients denoted by $f, g, h, \bar{h}$, or $k$ are some constant, nonderivative tensors. The invariant polynomials $\left(W_{\mu_{1} \ldots \mu_{J}}\right)_{J=\overline{2,6}}$ appearing in (73)-(77) read as

$$
\begin{align*}
& \times{\stackrel{\left[J_{2}\right]^{*}}{C}}_{\left(1, J_{2}-2\right) \mu_{J_{1}+1} \cdots \mu_{J_{1}+J_{2}}} \cdots{\stackrel{\left[J_{k}\right]^{*}}{C}}_{\left.\left.\left(1, J_{k}-2\right) \mu_{J_{1}+J_{2}+\cdots+J_{k-1}+1} \ldots \mu_{\left.J_{1}+J_{2}+\cdots+J_{k}\right]}\right]\right)} \\
& +\frac{d^{J} W}{d(\stackrel{[0]}{A})}{ }^{J} \stackrel{[1]}{*}_{\mu_{1}}^{*} \cdots{\stackrel{[1]}{B}{ }_{\mu_{J}}^{*}}, \tag{78}
\end{align*}
$$

where $W=W\binom{[0]}{A}$ is an arbitrary, smooth function depending only on the undifferentiated scalar field $A$ and we used a special notation in the double sum from (78) in order to write it in the above compact form, namely

$$
\begin{equation*}
\stackrel{[1]}{B}_{\mu_{1}}^{*} \equiv \stackrel{[1]^{*}}{C}(1,-1) \mu_{1} . \tag{79}
\end{equation*}
$$

We recall that according to our notation $[\mu \nu \ldots \rho]$, each term from the double sum needs complete antisymmetrization over all $J$ Lorentz indices. If $J=2$, then only the first and
the last terms from (78) survive since the limits in the former sum are meaningless ( $k$ should run from 2 to 1 )

$$
\begin{equation*}
W_{\mu_{1} \mu_{2}}=\frac{d W}{d \stackrel{[0]}{A}} \stackrel{[2]}{ }_{(1,0) \mu_{1} \mu_{2}}+\frac{d^{2} W}{d(\stackrel{[0]}{A})^{2}} \stackrel{[1]}{B}_{\mu_{1}}^{*}{\stackrel{[1]}{B_{\mu}}}_{\mu_{2}} \tag{80}
\end{equation*}
$$

Let us exemplify the above expression of $W_{\mu_{1} \ldots \mu_{J}}$ for $J=6$. In this situation the double sum requires the (strictly) positive, integer solutions to the equations $J_{1}+J_{2}+\cdots+J_{k}=6$ for $k=\overline{2,5}$, subject to the additional ordering conditions $1 \leq J_{1} \leq J_{2} \leq \ldots \leq J_{k} \leq 5$. Consequently, we find the solutions

$$
\begin{gather*}
J=6, k=2:\left(J_{1}=1, J_{2}=5\right),\left(J_{1}=2, J_{2}=4\right),\left(J_{1}=3, J_{2}=3\right),  \tag{81}\\
J=6, k=3:\left(J_{1}=1, J_{2}=1, J_{3}=4\right),\left(J_{1}=1, J_{2}=2, J_{3}=3\right),\left(J_{1}=2, J_{2}=2, J_{3}=2\right),  \tag{82}\\
J=6, k=4:\left(J_{1}=1, J_{2}=1, J_{3}=1, J_{4}=3\right),\left(J_{1}=1, J_{2}=1, J_{3}=2, J_{4}=2\right),  \tag{83}\\
J=6, k=5:\left(J_{1}=1, J_{2}=1, J_{3}=1, J_{4}=1, J_{5}=2\right) . \tag{84}
\end{gather*}
$$

If we replace these solutions in (78) for $J=6$ and recall notation (79), then we have that

$$
\begin{aligned}
& W_{\mu_{1} \ldots \mu_{6}}=\frac{d W}{d \stackrel{[0]}{A}} \stackrel{[6]}{ }_{(1,4) \mu_{1} \ldots \mu_{6}}^{d(\stackrel{[0]}{A})^{2}}+\frac{d^{2} W}{\overbrace{[1]}^{*}}\left[\stackrel{[5]}{C}_{\left.(1,3) \mu_{2} \ldots \mu_{6}\right]}^{*}+\stackrel{[2]}{C}_{(1,0)\left[\mu_{1} \mu_{2}\right.}^{\left.\stackrel{[4]^{*}}{C}(1,2) \mu_{3} \ldots \mu_{6}\right]}\right. \\
& \left.+\stackrel{[3]}{C}_{(1,1)\left[\mu_{1} \mu_{2} \mu_{3}\right.}{\stackrel{[3]}{ }{ }^{*}}_{\left.(1,1) \mu_{4} \mu_{5} \mu_{6}\right]}\right)+\frac{d^{3} W}{d(\stackrel{[0]}{A})^{3}}\left({\stackrel{[1]}{B}]_{\left[\mu_{1}\right.}^{*} \stackrel{[1]}{*}_{\mu_{2}}^{*} \stackrel{[4]}{ }_{\left.(1,2) \mu_{3} \ldots \mu_{6}\right]}}^{(1)}\right. \\
& \left.+\stackrel{[1]^{*}}{\left[\mu_{1}\right.} \stackrel{[2]}{C}_{(1,0) \mu_{2} \mu_{3}}^{\stackrel{[3]}{ }{ }_{\left.(1,1) \mu_{4} \mu_{5} \mu_{6}\right]}}+{\stackrel{[2]^{*}}{C}}_{(1,0)\left[\mu_{1} \mu_{2}\right.}^{\stackrel{[2]}{C}}(1,0) \mu_{3} \mu_{4} \stackrel{[2]}{ }_{\left.(1,0) \mu_{5} \mu_{6}\right]}^{*}\right)
\end{aligned}
$$

In contrast to the spaces $\left(H_{J}(\delta \mid d)\right)_{J \geq 2}$ and $\left(H_{J}^{\mathrm{inv}}(\delta \mid d)\right)_{J \geq 2}$, which are finite-dimensional, the cohomology $H_{1}(\delta \mid d)$ (known to be related to global symmetries and ordinary conservation laws) is infinite-dimensional since the theory is free. Fortunately, it will not be needed in the sequel.

The previous results on $H(\delta \mid d)$ and $H^{\text {inv }}(\delta \mid d)$ in strictly positive antighost numbers are important because they control the obstructions to removing the antifields from the first-order deformation. More precisely, we can successively eliminate all the pieces of antighost number strictly greater that 6 from the nonintegrated density of the first-order deformation by adding solely trivial terms, so we can take, without loss of nontrivial objects, the condition $I \leq 6$ into (60). In addition, the last representative is of the form
(70), where the invariant 'polynomial' is necessarily a nontrivial object from $H_{6}^{\text {inv }}(\delta \mid d)$. We can further decompose $a$ in a natural manner as a sum between two kinds of deformations

$$
\begin{equation*}
a=a^{\mathrm{BF}}+a^{\mathrm{int}}, \tag{86}
\end{equation*}
$$

where $a^{\mathrm{BF}}$ contains only fields/ghosts/antifields from the BF sector and $a^{\text {int }}$ describes the cross-interactions between the two theories. Decomposition (86) does not include a component responsible for the self-interactions of the tensor field with the mixed symmetry $(3,1)$ since any such component has been proved in [24] to be trivial. The piece $a^{\mathrm{BF}}$ has been partially announced in [25]. It is parameterized by 5 smooth, but otherwise arbitrary functions of the undifferentiated scalar field, $M(\stackrel{[0]}{A})$ and $\left(W_{a}(\stackrel{[0]}{A})\right)_{a=\overline{1,4}}$. Due to the fact that $a^{\mathrm{BF}}$ and $a^{\text {int }}$ involve different types of fields and that $a^{\mathrm{BF}}$ separately satisfies an equation of the type (59), it follows that $a^{\text {int }}$ is subject to the equation

$$
\begin{equation*}
s a^{\mathrm{int}}=\partial^{\mu} m_{\mu}^{\mathrm{int}} \tag{87}
\end{equation*}
$$

for some local current $m_{\mu}^{\text {int }}$. It is possible to show that the general solution to the equation $s a^{\text {int }}=\partial^{\mu} m_{\mu}^{\text {int }}$ that complies with all the imposed hypotheses can be chosen to stop nontrivially at antighost number $I=4, a^{\text {int }}=a_{0}^{\mathrm{int}}+a_{1}^{\mathrm{int}}+a_{2}^{\mathrm{int}}+a_{3}^{\mathrm{int}}+a_{4}^{\mathrm{int}}$, where its components are subject to equations (64) and (62)-(63) for $I=4$. Without entering other technical details, to be reported elsewhere, we only mention that $a^{\text {int }}$ is parameterized by 2 arbitrary, smooth functions of the undifferentiated scalar field, $\left(U_{\bar{a}}(\stackrel{[0]}{A})\right)_{\bar{a}=\overline{1,2}}$.

## 5 Main result. Lagrangian formulation of the interacting theory

Finally, after performing all the necessary computations and use extensively the cohomological results from the previous section we find the fully deformed solution to the master equation that characterizes all consistent interactions that can be added in $D=6$ to a topological BF model and the DFLG based on the massless tensor field with the mixed symmetry ( 3,1 ). Consequently, we apply the considerations from the end of Section 3 and are thus able to identify the entire gauge structure of the interacting theory. We do not write here the concrete form of the deformation of the solution to the master equation, but rather insist on the main ingredients following from it. The Lagrangian action of the coupled model contains interactions vertices of orders 1 and 2 in the coupling constant $\lambda$ and reads as

$$
\begin{align*}
& \bar{S}^{\mathrm{L}}\left[\Phi^{\alpha_{0}}\right]=\int d^{6} x\left[\lambda M+\stackrel{[1]}{B}{ }^{\mu_{1}}\left(\partial_{\mu_{1}} \stackrel{[0]}{A}-\lambda W_{1} \stackrel{[1]}{A}_{\mu_{1}}\right)+\frac{1}{2} \stackrel{[2]}{B}{ }^{\mu_{1} \mu_{2}}\left(\partial_{\left[\mu_{1}\right.}^{{ }^{[1]}}{ }_{\left.\mu_{2}\right]}-2 \lambda W_{2}{ }^{[2]}{ }_{\mu_{1} \mu_{2}}\right)\right. \\
& +\frac{1}{3} \stackrel{[33}{B}{ }^{\mu_{1} \mu_{2} \mu_{3}}\left(\partial_{\left[\mu_{1}\right.} \stackrel{[2]}{A}_{\left.\mu_{2} \mu_{3}\right]}-3 \lambda W_{3}{ }_{\left[\mu_{1}\right]}^{[2]}{ }_{\left.\mu_{2} \mu_{3}\right]}^{[2]}\right)+\frac{\lambda}{8} \varepsilon^{\mu_{1} \ldots \mu_{6}} W_{4} \stackrel{[2]}{A}_{\mu_{1} \mu_{2}}{\stackrel{[2]}{A}{ }_{\mu_{3} \mu_{4}}{ }^{[2]}{ }_{\mu_{5} \mu_{6}}}^{[12} \\
& -\frac{1}{48}\left(F_{\mu \nu \rho \sigma \mid \alpha} F^{\mu \nu \rho \sigma \mid \alpha}-4 F_{\mu \nu \rho} F^{\mu \nu \rho}\right)+\frac{1}{9} \lambda F^{\mu \nu \rho}\left(\frac{1}{5!} U_{1} \stackrel{[3]}{B}_{\mu \nu \rho}+U_{2}{ }_{2}^{[1]}{ }_{[\mu}^{[2]}{ }_{\nu \rho]}\right) \tag{88}
\end{align*}
$$

where $\stackrel{[3}{B}_{\mu_{1} \mu_{2} \mu_{3}}$ is the Hodge dual of $\stackrel{[3]}{[3]^{\nu_{1} \nu_{2} \nu_{3}}}$, defined in general as

$$
\begin{equation*}
\stackrel{[p]}{\omega}_{\nu_{1} \ldots \nu_{p}} \rightarrow \stackrel{[6-p]}{\omega}^{\mu_{1} \ldots \mu_{6-p}}=\frac{1}{p!} \varepsilon^{\mu_{1} \ldots \mu_{6-p} \nu_{1} \ldots \nu_{p}} \stackrel{[p]}{\omega}_{\nu_{1} \ldots \nu_{p}}, \quad p=\overline{0,6} . \tag{89}
\end{equation*}
$$

Action (88) is invariant under some deformed gauge transformations that contain terms of orders 1 and 2 in the deformation parameter, namely,

$$
\begin{align*}
& \bar{\delta}_{\Omega^{\alpha_{1}}}{ }^{[0]}=\lambda W_{1}{ }^{[0]} \epsilon_{(1,0)},  \tag{90}\\
& \bar{\delta}_{\Omega^{\alpha_{1}}}{ }^{[1]}{ }_{\mu_{1}}=\partial_{\mu_{1}}{ }_{(1,0)}^{[0]}+2 \lambda W_{2}{ }_{(1,0) \mu_{1}}^{[1]}, \tag{91}
\end{align*}
$$

$$
\begin{align*}
& -\frac{\lambda}{6!} U_{1} \varepsilon_{\mu_{1} \ldots \mu_{6}} \partial^{\mu_{3}} \theta^{\mu_{4} \ldots \mu_{6}}, \tag{92}
\end{align*}
$$

$$
\begin{align*}
& -\frac{\lambda}{3}\left[F^{\mu_{1} \mu_{2} \mu_{3}}+\lambda\left(U_{2} \stackrel{[1]}{\left[1 \mu_{1}\right.} \stackrel{[2]}{A}{ }^{\left.\mu_{2} \mu_{3}\right]}+\frac{1}{5!} U_{1} \stackrel{[3]^{\mu_{1} \mu_{2} \mu_{3}}}{ }\right)\right]\left[\frac{1}{6!} \frac{d U_{1}}{d \theta} \varepsilon^{[0]} \varepsilon_{\mu_{2} \mu_{3} \nu_{1} \ldots \nu_{4}}{ }^{[4]{ }^{\nu_{1} \ldots \nu_{4}}}{ }_{(3,0)}\right. \\
& +\frac{d U_{2}}{d \|}\left(2 \stackrel{[1]}{A}_{\mu_{\mu_{2}}}^{\left.\left.{\stackrel{[1]}{(2,0) \mu_{3}}}^{[1]}-\stackrel{[2]}{A_{\mu_{2} \mu_{3}}} \stackrel{[0]}{\epsilon}_{(1,0)}\right)\right]}\right. \\
& -\frac{\lambda}{3}\left(\frac{d U_{2}}{[1]} \stackrel{[1]}{A}_{d A}^{\left[\mu_{2}\right.} \stackrel{[2]}{A}_{\left.\mu_{3} \mu_{4}\right]}+\frac{1}{5!} \frac{d U_{1}}{d 0]} \stackrel{[3]}{B}_{\mu_{2} \mu_{3} \mu_{4}}^{d A}\right) \partial^{\left[\mu_{1}\right.} \theta^{\left.\mu_{2} \mu_{3} \mu_{4}\right]}, \tag{93}
\end{align*}
$$

$$
\begin{align*}
& +6 \lambda W_{3}\left(2{\left.\stackrel{[2]}{A}{ }_{\mu_{3} \mu_{4}} \stackrel{\left[44^{\mu_{1} \mu_{2} \mu_{3} \mu_{4}}{ }_{(3,0)}\right.}{{ }^{[3]}{ }^{\mu_{1} \mu_{2} \mu_{3}}{ }_{[1]}^{\epsilon}}{ }_{(2,0) \mu_{3}}\right)}\right) \\
& -\frac{2 \lambda}{3} U_{2}\left[F^{\mu_{1} \mu_{2} \mu_{3}}+\lambda\left(U_{2} \stackrel{[1]}{\left[\mu_{1} \stackrel{12}{ }_{A}^{\left.\mu_{2} \mu_{3}\right]}\right.}+\frac{1}{5!} U_{1} \stackrel{\left[3 \tilde{B}^{\mu_{1} \mu_{2} \mu_{3}}\right.}{ }\right)\right]{\stackrel{[1]}{\epsilon}{ }_{(2,0) \mu_{3}}} \\
& -\lambda U_{2}{ }^{[2]}{ }_{\mu_{3} \mu_{4}} \partial^{\left[\mu_{1}\right.} \theta^{\left.\mu_{2} \mu_{3} \mu_{4}\right]}, \tag{94}
\end{align*}
$$

$$
\begin{align*}
& +\frac{3 \lambda}{2} W_{4} \varepsilon^{\mu_{1} \ldots \mu_{6}}{\left.\stackrel{[2]}{A}{ }_{\mu_{4} \mu_{5}}{ }^{[1]}{ }_{(2,0) \mu_{6}}-\lambda U_{2}{ }^{[1]}{ }_{\mu_{4}} \partial^{\left[\mu_{1}\right.} \theta^{\left.\mu_{2} \mu_{3} \mu_{4}\right]}\right]} \\
& +\frac{\lambda}{3} U_{2}\left[F^{\mu_{1} \mu_{2} \mu_{3}}+\lambda\left(U_{2} A^{[1]} \stackrel{\mu}{1}_{[2]}^{A}{ }^{\left.\mu_{2} \mu_{3}\right]}+\frac{1}{5!} U_{1} \widetilde{B}^{[3]}{ }^{\mu_{1} \mu_{2} \mu_{3}}\right)\right] \stackrel{[0]}{\epsilon}(1,0)^{[ }  \tag{95}\\
& \bar{\delta}_{\Omega^{\alpha} 1_{1}} t_{\mu \nu \rho \mid \alpha}=3 \partial_{\alpha} \theta_{\mu \nu \rho}+\partial_{[\mu} \theta_{\nu \rho] \alpha}+\partial_{[\mu} \chi_{\nu \rho] \mid \alpha} \tag{96}
\end{align*}
$$

It is very important to mention that all the parameterizing (smooth) functions depending only on the undifferentiated scalar field, $M,\left(W_{a}\right)_{a=1,4}$ and $U_{1,2}$ are no longer arbitrary. They are restricted to satisfy a system of 9 equations, among which 4 are purely algebraic and 5 partial differential equations of order one in the derivatives with respect to the scalar field. Also, 6 of them involve just the functions that parameterize the self-interactions among the BF fields, i.e. $M$ and $\left(W_{a}\right)_{a=1,4}$, and the others depend also on the functions controlling the cross-couplings between the BF and $(3,1)$ field sectors, $U_{1,2}$. This system reads as

$$
\begin{gather*}
W_{1} W_{2}=0, \quad W_{2} W_{3}=0, \quad W_{3} W_{4}=0, \quad W_{1} \frac{d M}{[0]}=0, \quad W_{1} \frac{d W_{2}}{d A}=0, \quad W_{1} \frac{d W_{4}}{d A}=0  \tag{97}\\
W_{2} U_{1}=0, \quad W_{4} U_{1}+160 W_{2} U_{2}=0, \quad-3 W_{3} U_{1}+W_{1} \frac{d U_{1}}{d A}=0  \tag{98}\\
d A
\end{gather*}
$$

These independent equations result as a consequence of the consistency condition of the first-order deformation of the solution to the master equation at order two in the coupling constant. On the one hand they provide the expression of the second-order deformation and on the other hand ensure that the fully deformed solution to the master equation stops at order two in the coupling constant. It can be shown that there exist solutions to this system such that at least one of the cross-interaction vertices (meaning at least one of the functions $U_{1}$ or $U_{2}$ ) survive. We do not insist here on the detailed solutions to this system, which will be reported elsewhere.

Let us briefly comment on the gauge structure of the cross-coupled theory. We observe that the cross-interaction terms,

$$
\begin{equation*}
\frac{1}{9} \lambda F^{\mu \nu \rho}\left(\frac{1}{5!} U_{1} \stackrel{[3]}{B}_{\mu \nu \rho}+U_{2}{\stackrel{[1]}{A}{ }_{[\mu}^{[2]} \stackrel{[ }{\nu \rho]}}^{2}\right), \tag{99}
\end{equation*}
$$

are only of order one in the deformation parameter and couple the tensor field $t_{\lambda \mu \mid \alpha}$ to all the forms denoted by $A$ from the BF sector (to the scalar field $A$ through $U_{1}$ and $U_{2}$ ), but only on the form of the type $B$ of maximal degree, equal to three. Also, it is interesting to see that there appear some terms in the Lagrangian density of order two in the coupling constant

$$
\begin{equation*}
+\frac{\lambda^{2}}{18}\left(\frac{1}{5!} U_{1} \stackrel{[3]}{B}_{\mu \nu \rho}+U_{2}{\left.\stackrel{[1]}{A} \stackrel{[2]}{[2]}_{\nu \rho]}\right)\left(\frac{1}{5!} U_{1} \stackrel{[3]}{B}^{\mu \nu \rho}+U_{2}{ }^{[11}{ }^{[\mu}{ }^{[2]^{\nu \rho]}}\right.}^{\nu}\right), \tag{100}
\end{equation*}
$$

which describe self-interactions in the BF sector, but are strictly due to the presence of the tensor $t_{\lambda \mu \nu \mid \alpha}$ (in its absence $U_{1}=U_{2}=0$, so they would vanish). In other words, if one studied only the BF self-interactions in $D=6$, then one would obtain only interaction vertices of order one in the coupling constant, namely those parameterized by the functions $M$ and $\left(W_{a}\right)_{a=1,4}$. Related to the gauge transformations in the BF sector, we notice that those of all forms denoted by $B$ are deformed with terms of orders 1 and respectively 2 in the coupling constant and, moreover, to include gauge parameters from the $(3,1)$ sector. Among the BF forms denoted by $A$, only the gauge transformations of the form of maximum degree, $\stackrel{[2]}{A_{\mu_{1} \mu_{2}}}$, include gauge parameters from the $(3,1)$ sector, but stops at order 1 in the deformation parameter. A remarkable feature is that the gauge transformations of the tensor field $t_{\lambda \mu \nu \mid \alpha}$ are modified by terms involving some of the gauge parameters from the BF sector. Other features of the gauge structure of the interacting model are withdrawn from the fully deformed solution to the classical master equation, like for instance: 1. the gauge algebra becomes open (the commutators among the deformed gauge transformations only close on shell, i.e., on the stationary surface of interacting field equations), by contrast to the free theory, where the gauge algebra is Abelian and 2 . the reducibility relations associated with the interacting model only hold on-shell, by contrast to those corresponding to the free theory, which hold off-shell. We do not provide here the concrete expressions of either the commutators among the gauge transformations or the reducibility functions/relations. They will be detailed elsewhere.

## 6 Conclusion

The most important conclusion of this paper is that under the hypotheses of analyticity in the coupling constant, spacetime locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the preservation of the number of derivatives on each field, the dual formulation of linearized gravity in $D=6$ based on a massless tensor field with the mixed symmetry $(3,1)$ allows for nontrivial cross-couplings to another gauge theory of interest from the point of view of gravity and supergravity theories - a topological BF model with a maximal field spectrum. The deformed Lagrangian contains interaction vertices of order 1 and 2 in the deformation parameter that couple the mixed symmetry tensor field to the BF forms. There appear some self-interactions in the BF sector at order 2 in the coupling constant that are strictly due to the presence of the mixed symmetry tensor field. One of the striking features of the deformed model is that the gauge transformations of all fields are deformed. All the ingredients of the gauge structure are strongly modified during the deformation procedure: the gauge algebra becomes open and the reducibility relations hold on-shell.

It can be concluded that the BF form pair of maximum degree, $\left(\stackrel{[2]}{A}_{\mu_{1} \mu_{2}}^{[3]}{ }_{B}^{\mu_{1} \mu_{2} \mu_{3}}\right)$ is crucial at the level of cross-couplings with the DFLG in $D=6$ based on a tensor field with the mixed symmetry ( 3,1 ) (in its absence we would get no cross-interacting vertices). We believe this is a general feature of cross-couplings between topological BF models and DFLG in $D=k+3$ based on a tensor field with the mixed symmetry $(k, 1)$ in the sense that the presence of the BF form pair of maximum degrees, $\left.\left(\begin{array}{l}{[m]} \\ A_{1} \ldots \mu_{m}\end{array}, \stackrel{[m+1]}{B}\right]^{\mu_{1} \ldots \mu_{m+1}}\right)$, with $m=(k+2) / 2$ if $k$ is even and respectively $m=(k+1) / 2$ if $k$ is odd, is essential.

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