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Note on yes-go and no-go interactions of massless tensor fields with the mixed symmetry $(k, 1)$. The case $k = 5$

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Abstract

The present paper is dedicated to a collection of yes-go and no-go results on consistent couplings of a massless tensor field with the mixed symmetry $(k, 1)$ in the case $k = 5$ under some well-defined hypotheses from QFT. This type of mixed symmetry tensor fields is important due to its duality to linearized Hilbert–Einstein gravity in $D = k + 3$. More precisely, it is shown that there are no consistent self-interactions of a single massless tensor fields with the mixed symmetry $(5, 1)$ and also no cross-interactions of this tensor field to the Pauli–Fierz model or to a generic matter theory, but there appear consistent cross-interactions with an arbitrary p-form gauge field for $p = 1$ in $D = 7$.

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1 Introduction

Tensor fields in "exotic" representations of the Lorentz group, characterized by a mixed Young symmetry type [1]–[6], are important in view of their involvement in superstring, supergravity, and supersymmetric high spin theories. This class of tensor gauge fields exhibits several desirable featured, such as the dual formulation of field theories of spin two or higher or the impossibility of consistent cross-interactions in the dual formulation of linearized gravity (LG) [7]. An important issue related to mixed symmetry-type tensor fields is the computation of their local BRST cohomology [8]–[10], since this can be helpful at solving many Lagrangian and Hamiltonian aspects, like the construction of their consistent interactions [11, 12] with higher-spin gauge theories [13]–[16].

The present paper proposes the investigation of the couplings between one of the versions dual to LG, namely that based on a massless tensor field that transforms in an irreducible representation of $GL(D, \mathbb{R})$, corresponding to a two-column Young diagram with 6 cells and 5 rows, and other field theories: self-couplings, cross-interactions with a Pauli–Fierz field, cross-couplings to purely matter theories, and interactions with an

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Abelian p-form for $p = 1$. The emerging findings emphasize no-go results in the first three situations and yes-go results for the last case. They generalize our previous results from [14] and [15] on the interactions involving a single massless tensor field with the mixed symmetry $(3, 1)$. Also, we mention the general results from [17], some of which have been studied in more details here in the case $k = 5$.

2 Deformation procedure in brief

We begin with a "free" gauge theory, described by a Lagrangian action $S^{\mathrm{L}}\left[\Phi^{\alpha_0}\right]$, invariant under some gauge transformations

$$\delta_\epsilon \Phi^{\alpha_0} = Z^{\alpha_0}_{\ \ \alpha_1} \epsilon^{\alpha_1}, \qquad \frac{\delta S^{\mathrm{L}}}{\delta \Phi^{\alpha_0}} Z^{\alpha_0}_{\ \ \alpha_1} = 0, \tag{1}$$

and consider the problem of constructing consistent interactions among the fields Φ^{α_0} such that the couplings preserve both the field spectrum and the original number of gauge symmetries. This matter is addressed by means of reformulating the problem of constructing consistent interactions as a deformation problem of the solution to the master equation corresponding to the "free" theory [11, 12]. Such a reformulation is possible due to the fact that the solution to the master equation contains all the information on the gauge structure of the theory. If a consistent interacting gauge theory can be constructed, then the solution S to the master equation associated with the "free" theory, $(S, S) = 0$, can be deformed into a solution $\bar{S}$,

$$\begin{aligned} S \rightarrow \bar{S} &= S + \lambda S_1 + \lambda^2 S_2 + \cdots \\ &= S + \lambda \int d^D x\, a + \lambda^2 \int d^D x\, b + \lambda^3 \int d^D x\, c + \cdots \end{aligned} \tag{2}$$

of the master equation for the deformed theory

$$\left(\bar{S}, \bar{S}\right) = 0, \tag{3}$$

such that both the ghost and antifield spectra of the initial theory are preserved. The symbol $(,)$ denotes the antibracket. Eq. (3) splits, according to the various orders in the coupling constant (or deformation parameter) λ, into the equivalent tower of equations

$$(S, S) = 0, \tag{4}$$

$$2\left(S_1, S\right) = 0, \tag{5}$$

$$2\left(S_2, S\right) + \left(S_1, S_1\right) = 0, \tag{6}$$

$$\vdots$$

Eq. (4) is fulfilled by hypothesis. The next one requires that the first-order deformation of the solution to the master equation, S_1, is a cocycle of the "free" BRST differential $s\cdot = (\cdot, S)$. However, only cohomologically nontrivial solutions to (5) should be taken into account, as the BRST-exact ones can be eliminated by (in general nonlinear) field redefinitions. This means that S_1 pertains to the ghost number zero cohomological space of s, $H^0(s)$, which is generically nonempty due to its isomorphism to the space of physical observables of the "free" theory. It has been shown in [11, 12] (on behalf of the triviality of the antibracket map in the cohomology of the BRST differential) that there are no obstructions in finding solutions to the remaining equations, namely, (6) and so on. However, the resulting interactions may be nonlocal, and there might even appear obstructions if one insists on their locality. The analysis of these obstructions can be done with the help of cohomological techniques. As it will be seen below, all the interactions in the case of the model under study turn out to be local.

3 Free massless tensor field with the mixed symmetry $(5,1)$

The most important argument in the study of massless tensor fields with the mixed symmetry $(k,1)$ is that they are dual to linearized gravity in $D = k+3$ spacetime dimensions. Next, we consider the Lagrangian action of a free, massless tensor field with the mixed symmetry $(5,1)$

$$S^{\mathrm{L}}\left[t_{\mu_1\ldots\mu_5|\alpha}\right] = -\frac{1}{2\cdot 6!}\int d^D x\left(F_{\mu_1\ldots\mu_6|\alpha}F^{\mu_1\cdots\mu_6|\alpha} - 6F_{\mu_1\ldots\mu_5}F^{\mu_1\cdots\mu_5}\right),\quad D\geq 7. \tag{7}$$

The tensor field $t_{\mu_1\ldots\mu_5|\alpha}$ is separately antisymmetric in its first 5 indices and satisfies the (algebraic) Bianchi I identity $t_{[\mu_1\ldots\mu_5|\alpha]}\equiv 0$, where the symbol $[\mu\ldots\nu]$ signifies the operation of complete antisymmetrization with respect to the indices between brackets, defined such as to include only the distinct terms for a tensor with given antisymmetry properties, without normalization factors. Assume that this tensor field is defined on a pseudo-Riemannian manifold of dimension D, like, for instance, a Minkowski-flat spacetime, endowed with a metric tensor of 'mostly plus' signature $\sigma_{\mu\nu}=\sigma^{\mu\nu}=(-+\cdots+)$. In (7) the tensor $F_{\mu_1\cdots\mu_6|\alpha}$ is defined by

$$F_{\mu_1\cdots\mu_6|\alpha} = \partial_{[\mu_1}t_{\mu_2\cdots\mu_6]|\alpha} \tag{8}$$

and its trace via $F_{\mu_1\cdots\mu_5} = F_{\mu_1\cdots\mu_6|\alpha}\sigma^{\mu_6\alpha}$, such that $F_{\mu_1\cdots\mu_6|\alpha}$ displays the mixed symmetry $(6,1)$ and its trace produces a completely antisymmetric tensor field of degree 5. The trace of $t_{\mu_1\ldots\mu_5|\alpha}$ itself is introduced in a similar manner, $t_{\mu_1\cdots\mu_4} = t_{\mu_1\cdots\mu_5|\alpha}\sigma^{\mu_5\alpha}$, providing a completely antisymmetric tensor field of degree 4.

Action (7) is invariant under a generating set of gauge transformations that can be taken under the form

$$\delta_{\theta,\epsilon}t_{\mu_1\cdots\mu_5|\alpha} = \partial_{[\mu_1}\overset{(1)}{\theta}_{\mu_2\ldots\mu_5]|\alpha} + \partial_{[\mu_1}\overset{(1)}{\epsilon}_{\mu_2\ldots\mu_5]\alpha} + 5\partial_\alpha\overset{(1)}{\epsilon}_{\mu_1\ldots\mu_5}, \tag{9}$$

where both types of gauge parameters are arbitrary, bosonic tensor fields on the spacetime, with $\overset{(1)}{\theta}_{\mu_1\ldots\mu_4|\alpha}$ displaying the mixed symmetry $(4,1)$ and $\overset{(1)}{\epsilon}_{\mu_1\ldots\mu_5}$ completely antisymmetric. The above set of gauge transformations is Abelian and off-shell reducible of order 4 [13], the corresponding sets of reducibility parameters being given by

$$\left(\overset{(2)}{\theta}_{\mu_1\mu_2\mu_3|\alpha},\overset{(2)}{\epsilon}_{\mu_1\mu_2\mu_3\mu_4}\right),\left(\overset{(3)}{\theta}_{\mu_1\mu_2|\alpha},\overset{(3)}{\epsilon}_{\mu_1\mu_2\mu_3}\right),\left(\overset{(4)}{\theta}_{\mu_1|\alpha},\overset{(4)}{\epsilon}_{\mu_1\mu_2}\right),\left(\overset{(5)}{\epsilon}_{\mu_1}\right), \tag{10}$$

with each $\overset{(m)}{\theta}_{\mu_1\mu_2\mu_{5-m}|\alpha}$ possessing the mixed symmetry $(5-m,1)$, for $m=2,3,4$ and all parameters denoted by ϵ completely antisymmetric where appropriate. The exact reducibility relations can be found in [13] for $k=5$.

Regarding the antifield-BRST symmetry of this free model, one starts with the identification of the BRST algebra on which the BRST differential s acts. The generators of the BRST algebra are of two kinds: fields/ghosts and antifields. The ghost spectrum for the model under study comprises the tensor fields

$$\left(\overset{(m+1)}{C}_{\mu_1\ldots\mu_{4-m}|\alpha},\overset{(m+1)}{\eta}_{\mu_1\ldots\mu_{5-m}}\right)_{m=\overline{0,3}},\overset{(5)}{\eta}_{\mu} \tag{11}$$

with the Grassmann parities given in [13] ,while the antifields are denoted by

$$t^{*\mu_1\cdots\mu_5|\alpha},\left(\overset{(m+1)}{C}{}^{*\mu_1\ldots\mu_{4-m}|\alpha},\overset{(m+1)}{\eta}{}^{*\mu_1\ldots\mu_{5-m}}\right)_{m=\overline{0,3}},\overset{(5)}{\eta}{}^{*\mu} \tag{12}$$

their statistics being respectively opposite to that of the corresponding field/ghost.

As both the gauge generators and reducibility functions for this model are field-independent, it follows that the associated BRST differential ($s^2=0$) splits into

$$s=\delta+\gamma, \tag{13}$$

where δ represents the Koszul–Tate differential ($\delta^2=0$), graded by the antighost number agh ($\mathrm{agh}\,(\delta)=-1$), while γ stands for the exterior derivative along the gauge orbits and turns out to be a true differential ($\gamma^2=0$) that anticommutes with δ ($\delta\gamma+\gamma\delta=0$), whose degree is named pure ghost number pgh ($\mathrm{pgh}\,(\gamma)=1$). These two degrees do not interfere ($\mathrm{agh}\,(\gamma)=0$, $\mathrm{pgh}\,(\delta)=0$). The overall degree that grades the BRST differential is known as the ghost number (gh) and is defined like the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}\,(s)=\mathrm{gh}\,(\delta)=\mathrm{gh}\,(\gamma)=1$. The corresponding degrees of the generators from the BRST complex are listed in [13] together with the actions of δ and γ on them.

The antifield-BRST differential is known to admit a canonical action in a structure named antibracket and defined by decreeing the fields/ghosts conjugated with the corresponding antifields, $s\cdot=(\cdot,S)$, where $(,)$ signifies the antibracket and S denotes the canonical generator of the BRST symmetry. It is a bosonic functional of ghost number zero involving both the field/ghost and antifield spectra, which obeys the classical master equation

$$(S,S)=0. \tag{14}$$

The classical master equation is equivalent with the second-order nilpotency of s, $s^2=0$, while its solution encodes the entire gauge structure of the associated theory. It is easy to check [13] that the complete solution to the master equation for the free model under study reads as

$$\begin{aligned}
S=S^{\mathrm{L}}\left[t_{\mu_1\cdots\mu_5|\alpha}\right]+\int d^Dx\left[t^{*\mu_1\cdots\mu_5|\alpha}\left(\partial_{[\mu_1}\overset{(1)}{C}_{\mu_2\ldots\mu_5]|\alpha}\right.\right.\\
\left.+\partial_{[\mu_1}\overset{(1)}{\eta}_{\mu_2\ldots\mu_5\alpha]}+6\partial_\alpha\overset{(1)}{\eta}_{\mu_1\ldots\mu_5}\right)\\
+\overset{(4)}{C}{}^{*\mu_1|\alpha}\partial_{(\mu_1}\overset{(5)}{\eta}_{\alpha)}+\sum_{m=1}^{3}\overset{(m)}{C}{}^{*\mu_1\ldots\mu_{5-m}|\alpha}\left(\partial_{[\mu_1}\overset{(m+1)}{C}_{\mu_2\ldots\mu_{5-m}]|\alpha}+\right.\\
\left.+\partial_{[\mu_1}\overset{(m+1)}{\eta}_{\mu_2\ldots\mu_{5-m}\alpha]}+(-)^m(6-m)\,\partial_\alpha\overset{(m+1)}{\eta}_{\mu_1\ldots\mu_{5-m}}\right)
\end{aligned}$$

$$+\sum_{m=1}^{4}\frac{5-m}{7-m}\overset{(m)}{\eta}{}^{*\mu_1\ldots\mu_{6-m}}\partial_{[\mu_1}\overset{(m+1)}{\eta}{}_{\mu_2\ldots\mu_{6-m}]}\Bigg]. \tag{15}$$

The main ingredients of the antifield-BRST symmetry derived here will be useful at the analysis of the BRST cohomology for the free, massless tensor field with the mixed symmetry $(5,1)$.

4 No-go and yes-go results on consistent couplings

In the sequel we expose some results on the consistent couplings of a massless tensor field with the mixed symmetry $(5,1)$ to other theories. For obvious reasons, we consider only analytical, local, Lorentz covariant, and Poincaré invariant deformations (i.e., we do not allow explicit dependence on the spacetime coordinates). The analyticity of deformations refers to the fact that the deformed solution to the master equation, (2), is analytical in the coupling constant λ and reduces to the original solution S in the free limit $\lambda = 0$. In addition, we require that the maximum derivative order of each interaction vertex entering the overall interacting Lagrangian is equal to two.

4.1 No-go selfinteractions

According to the deformation method for constructing consistent interactions in gauge field theories exposed in brief in Sec. 2 and to the supplementary working hypotheses discussed in the beginning of the present section, we search for deformations in the space of local forms in the fields, ghosts, antifields, and their spacetime derivatives at maximum form degree, D. If we take into account the notations from (2), then the local form of Eq. (5), which we have seen that controls the first-order deformation of the solution to the master equation, becomes

$$sa = \partial_\mu j^\mu, \qquad \mathrm{gh}\,(a) = 0, \qquad \varepsilon\,(a) = 0, \tag{16}$$

for some local j^μ, and it shows that the nonintegrated density of the first-order deformation pertains to the local cohomology of s at ghost number zero computed in the space of local forms, $a \in H^0\,(s|d)$, where d denotes the exterior spacetime differential. In order to analyze the above equation, we develop a according to the antighost number

$$a = \sum_{m=0}^{I} a_m, \qquad \mathrm{agh}\,(a_m) = m, \qquad \mathrm{gh}\,(a_m) = 0, \qquad \varepsilon\,(a_m) = 0, \tag{17}$$

and assume, without loss of generality, that the above decomposition stops at some finite value of the antighost number, I. By taking into account the splitting (13) of the BRST differential, Eq. (16) becomes equivalent to a tower of local equations, corresponding to the different decreasing values of the antighost number

$$\gamma a_I = \partial_\mu \overset{(I)}{j}{}^\mu, \tag{18}$$

$$\delta a_I + \gamma a_{I-1} = \partial_\mu \overset{(I-1)}{j}{}^\mu, \tag{19}$$

$$\delta a_m + \gamma a_{m-1} = \partial_\mu \overset{(m-1)}{j}{}^\mu, \qquad I-1 \geq m \geq 1, \tag{20}$$

where $\left(\overset{(m)^{\mu}}{j}\right)_{m=\overline{0,I}}$ are some local currents with $\mathrm{agh}\left(\overset{(m)^{\mu}}{j}\right)=m$. It can be proved using a manner similar to that from [14] that we can replace Eq. (18) at strictly positive antighost numbers with

$$\gamma a_I = 0, \qquad \mathrm{agh}\left(a_I\right) = I > 0. \tag{21}$$

In conclusion, under the assumption that $I > 0$, the representative of highest antighost number from the nonintegrated density of the first-order deformation can always be taken to be γ-closed, such that the computation of the first-order deformation requires the computation of the cohomology of the exterior longitudinal differential in the space of local forms, $H\left(\gamma\right)$.

From the actions of γ on the BRST generators it can be shown that $H(\gamma)$ in the space of local forms is generated on the one hand by $\Theta^{*\Delta}$ and $K_{\mu_1\ldots\mu_6|\alpha\beta}$ as well as by their spacetime derivatives and, on the other hand, by the ghosts

$$\mathcal{F}_{\mu_1\ldots\mu_6} \equiv \partial_{[\mu_1} \overset{(1)}{\eta}_{\mu_2\ldots\mu_6]} \tag{22}$$

and $\overset{(5)}{\eta}_{\mu}$, where $\Theta^{*\Delta}$ is a generic notation for all the antifields and

$$K_{\mu_1\ldots\mu_6|\alpha\beta} = \partial_\alpha \partial_{[\mu_1} t_{\mu_2\ldots\mu_6]|\beta} - \partial_\beta \partial_{[\mu_1} t_{\mu_2\ldots\mu_6]|\alpha} \tag{23}$$

represents the curvature tensor. So, the most general, nontrivial representative from $H\left(\gamma\right)$ for the free theory under study reads as

$$a_I = \alpha_I\left(\left[\Theta^{*\Delta}\right], \left[K_{\mu_1\ldots\mu_6|\alpha\beta}\right]\right)\omega^I\left(\mathcal{F}_{\mu_1\ldots\mu_6}, \overset{(5)}{\eta}_{\mu}\right), \tag{24}$$

where the notation $f([q])$ means that f depends on q and its derivatives up to a finite order, while ω^I denotes the elements of pure ghost number I (and antighost number zero) of a basis in the space of polynomials in the corresponding ghosts and some of their first-order derivatives. The objects α_I (obviously nontrivial in $H^0\left(\gamma\right)$) were taken to have a bounded number of derivatives, and therefore they are polynomials in the antifields $\Theta^{*\Delta}$, in the curvature tensor and their derivatives. Due to their γ-closedness, they are called invariant polynomials. At zero antighost number, the invariant polynomials are polynomials in the curvature tensor and its derivatives.

Replacing the solution (24) into Eq. (19) and taking into account the definitions of γ and δ acting on the BRST generators, we remark that a necessary (but not sufficient) condition for the existence of (nontrivial) solutions a_{I-1} is that the invariant polynomials α_I are (nontrivial) objects from the local cohomology of the Koszul–Tate differential $H\left(\delta|d\right)$ at antighost number $I > 0$ and pure ghost number equal to zero (characteristic cohomology). As the free model under study is a linear gauge theory of Cauchy order equal to 6, the general results from [8, 9] ensure that $H\left(\delta|d\right)$ (at pure ghost number zero) is trivial at antighost numbers strictly greater than its Cauchy order

$$H_I\left(\delta|d\right) = 0, \qquad I > 6. \tag{25}$$

Moreover, if the invariant polynomial α_I, with $\mathrm{agh}\left(\alpha_I\right) = I \geq 6$, is trivial in $H_I\left(\delta|d\right)$, then it can be taken to be trivial also in $H_I^{\mathrm{inv}}\left(\delta|d\right)$. This is important since together with (25) ensures that the entire local cohomology of the Koszul–Tate differential in the space of invariant polynomials (invariant characteristic cohomology) is trivial in antighost number strictly greater than 6

$$H_I^{\mathrm{inv}}\left(\delta|d\right) = 0, \qquad I > 6. \tag{26}$$

We can organize the nontrivial representatives of $H_I(\delta|d)$ (at pure ghost number equal to zero) and $H_I^{\mathrm{inv}}(\delta|d)$ with $6 \geq I \geq 2$ respectively into $\overset{(5)*\mu}{\eta}$ and $\left(\overset{(I+1)*\mu_1\ldots\mu_{4-I}|\alpha}{C}, \overset{(I+1)*\mu_1\ldots\mu_{5-I}}{\eta}\right)_{I=\overline{0,3}}$. We remark that there is no nontrivial element in $(H_I(\delta|d))_{I\geq 2}$ or $\left(H_I^{\mathrm{inv}}(\delta|d)\right)_{I\geq 2}$ that effectively involves the curvature $K_{\mu_1\ldots\mu_6|\alpha\beta}$ and/or its derivatives, and the same stands for the quantities that are more than linear in the antifields and/or depend on their derivatives. In contrast to the groups $(H_I(\delta|d))_{I\geq 2}$ and $\left(H_I^{\mathrm{inv}}(\delta|d)\right)_{I\geq 2}$, which are finite-dimensional, the cohomology $H_1(\delta|d)$ at pure ghost number zero, that is related to global symmetries and ordinary conservation laws, is infinite-dimensional since the theory is free.

The previous results on $H(\delta|d)$ and $H^{\mathrm{inv}}(\delta|d)$ at strictly positive antighost numbers are important because they control the obstructions to removing the antifields from the first-order deformation. Indeed, due to (26), it follows that we can successively eliminate all the pieces with $I > 6$ from the nonintegrated density of the first-order deformation by adding only trivial terms, so we can take, without loss of nontrivial objects, the condition $I \leq 6$ in the decomposition (17). The last representative is of the form (24), where the invariant polynomial is necessarily a nontrivial object from $H_I^{\mathrm{inv}}(\delta|d)$ for $I = \overline{2,6}$ and respectively from $H_1(\delta|d)$ for $I = 1$.

After applying the deformation procedure from Sec. 2 in case of a free massless tensor field with the mixed symmetry $(5,1)$ by means of the cohomological ingredients discussed in the above, we are able to prove the next theorem.

Theorem 1 *Under the assumptions of analyticity in the coupling constant, locality, Lorentz covariance, Poincaré invariance and at most two derivatives in the Lagrangian, there are no consistent selfcouplings that can be added to the free action of a massless tensor field with the mixed symmetry* $(5,1)$.

4.2 No-go couplings to the Pauli–Fierz field

In this situation the starting free model describes a free massless tensor field with the mixed symmetry $(5,1)$ and the standard formulation of a spin-two field via the Pauli–Fierz model

$$S^{\mathrm{L}}[t_{\mu_1\ldots\mu_5|\alpha}, h_{\mu\nu}] = S^{\mathrm{L}}[t_{\mu_1\ldots\mu_5|\alpha}] + S^{\mathrm{L}}[h_{\mu\nu}], \tag{27}$$

where $S^{\mathrm{L}}[t_{\mu_1\ldots\mu_5|\alpha}]$ is given in (7) and the Pauli–Fierz action reads as

$$S^{\mathrm{L}}[h_{\mu\nu}] = \int d^D x \left[-\frac{1}{2}(\partial_\mu h_{\nu\rho})\partial^\mu h^{\nu\rho} + (\partial_\mu h^{\mu\rho})\partial^\nu h_{\nu\rho} - (\partial_\mu h)\partial_\nu h^{\nu\mu} + \frac{1}{2}(\partial_\mu h)\partial^\mu h\right], \tag{28}$$

with $D \geq 7$. A generating set of gauge transformations for action (27) is given by (9) and respectively by

$$\delta_\epsilon h_{\mu\nu} = \partial_{(\mu}\epsilon_{\nu)}, \tag{29}$$

where the symbol $(\mu\nu)$ signifies the operation of plain, full symmetrization, without normalization factors. Given the properties of the generating set of gauge transformations (9) in the $(5,1)$ sector and the observation that the set (29) is Abelian and irreducible, it follows that the free theory described by action (27) reveals an overall Abelian gauge algebra and an entire generating set off-shell reducible of order 4. The BRST algebra of local forms is generated by the fields $t_{\mu_1\ldots\mu_5|\alpha}$ and $h_{\mu\nu}$, by the ghosts (11), the antifields (12) and the following ghosts/antifields corresponding to the Pauli–Fierz sector

$$\left(\eta_\mu, h^{*\mu\nu}, \eta^{*\mu}\right), \tag{30}$$

whose properties can be found in [14]. In agreement with the general line of the antifield-BRST method, it results that the free BRST differential s for theory (27) decomposes again like in (13), the corresponding solution to the master equation $(S,S)=0$ being in this situation

$$S = S^{\mathrm{t}} + S^{\mathrm{h}}, \tag{31}$$

where S^{t} is given by the right-hand side of (15) and

$$S^{\mathrm{h}} = S^{\mathrm{L}}\left[h_{\mu\nu}\right] + \int d^D x\, h^{*\mu\nu}\partial_{(\mu}\eta_{\nu)}. \tag{32}$$

It is understood that the present antibracket takes into account all the BRST generators, including those from the Pauli–Fierz sector.

In order to determine the solution to the local first-order deformation Eq. (16), we proceed like in the previous subsection, namely, we expand the nonintegrated density according to the antighost number as in (17) and solve the equivalent tower of equations, given by (21) and (19)–(20). It is convenient to split the first-order deformation into

$$a = a^{\mathrm{h-h}} + a^{\mathrm{h-t}}, \tag{33}$$

where $a^{\mathrm{h-h}}$ denotes the part responsible for the selfinteractions of the Pauli–Fierz field and $a^{\mathrm{h-t}}$ signifies the component that describes only the cross-interactions between $h_{\mu\nu}$ and $t_{\mu_1\ldots\mu_5|\alpha}$. We have seen in the previous subsection that we can take the component related to the selfinteractions of the tensor field $t_{\mu_1\ldots\mu_5|\alpha}$ to be trivial, so we already discarded it from (33). Then, $a^{\mathrm{h-h}}$ is completely known and includes the cubic vertex of the Einstein–Hilbert Lagrangian plus a cosmological term. It already satisfies an equation of the type $sa^{\mathrm{h-h}} = \partial_\mu u^\mu$, which replaced in (16) leads to the conclusion that $a^{\mathrm{h-t}}$ is subject to the equation

$$sa^{\mathrm{h-t}} = \partial_\mu w^\mu, \tag{34}$$

and hence $a^{\mathrm{h-t}}$ is nothing but a (nontrivial) element of the local BRST cohomology in ghost number zero, $H^0(s|d)$. The computation of the solutions to (34) goes along the same line employed in Subsec. 4.1, namely we develop $a^{\mathrm{h-t}}$ according to the antighost number

$$a^{\mathrm{h-t}} = \sum_{m=0}^{I} a_m^{\mathrm{h-t}},\ \mathrm{agh}\left(a_m^{\mathrm{h-t}}\right) = m,\ \mathrm{gh}\left(a_m^{\mathrm{h-t}}\right) = 0,\ \varepsilon\left(a_m^{\mathrm{h-t}}\right) = 0, \tag{35}$$

such that finally (34) becomes equivalent to the tower of equations

$$\gamma a_I^{\mathrm{h-t}} = 0, \tag{36}$$

$$\delta a_I^{\mathrm{h-t}} + \gamma a_{I-1}^{\mathrm{h-t}} = \partial_\mu \overset{(I-1)^\mu}{w}, \tag{37}$$

$$\delta a_m^{\mathrm{h-t}} + \gamma a_{m-1}^{\mathrm{h-t}} = \partial_\mu \overset{(m-1)^\mu}{w},\ I-1 \geq m \geq 1. \tag{38}$$

Eq. (36) shows that $a_I^{\mathrm{h-t}} \in H(\gamma)$, so we need to compute the cohomology of the overall exterior longitudinal differential in order to reveal the component of maximum antighost number from the first-order deformation. It can be proved that the general solution to (36) is expressed by

$$a_I^{\mathrm{h-t}} = \alpha_I^{\mathrm{h-t}}\left(\left[\pi^{*\Theta}\right], \left[K_{\mu_1\ldots\mu_6|\alpha\beta}\right], \left[\mathcal{K}_{\mu\nu|\alpha\beta}\right]\right) e^I\left(\eta_\mu, \partial_{[\mu}\eta_{\nu]}, \mathcal{F}_{\mu_1\ldots\mu_6}, \overset{(5)}{\eta}_\mu\right), \tag{39}$$

for $I > 0$, where $\pi^{*\Theta}$ is a generic notation for all antifields, $K_{\mu_1\ldots\mu_6|\alpha\beta}$ stands for the curvature tensor (23), $\mathcal{K}_{\mu\nu|\alpha\beta}$ signifies the linearized Riemann tensor, and $\mathcal{F}_{\mu_1\ldots\mu_6}$ is defined in (22). The γ-invariant objects $\alpha_I^{\mathrm{h-t}}$ (of pure ghost number equal to zero) are required to fulfill agh $\left(\alpha_I^{\mathrm{h-t}}\right) = I$, and the notation e^I stands for a generic notation of the elements with pure ghost number equal to I of a basis in the space of polynomials in the corresponding ghosts and antisymmetrized first-order derivatives. In addition, every single term from $a_I^{\mathrm{h-t}}$ must contain at least one generator (field or ghost or antifield) from each of the two theories in order to provide effective cross-interactions. As they have a bounded number of derivatives, the quantities $\alpha_I^{\mathrm{h-t}}$ are in fact polynomials in the antifields, in the curvature tensor (23), in the linearized Riemann tensor, and also in their derivatives. They represent the most general nontrivial elements from $H^0(\gamma)$ and will be called again "invariant polynomials".

Substituting the solution (39) into Eq. (37) and taking into account the definitions δ and γ acting on the BRST generators, we obtain, like in the previously investigated situation, that a necessary condition for Eq. (37) to possess (nontrivial) solutions with respect to $a_{I-1}^{\mathrm{h-t}}$ for all $I > 0$ is that the invariant polynomials $\alpha_I^{\mathrm{h-t}}$ are (nontrivial) objects from the characteristic cohomology at antighost number I, $H_I(\delta|d)$. Since the present model is a linear gauge theory of Cauchy order equal to 6, relation (25) is still valid and, moreover it can be shown that (26) holds again, so the invariant characteristic cohomology is also trivial at antighost numbers $I > 6$. On account of the actions of δ on the BRST generators, we are able to identify the nontrivial representatives of $\left(H_I(\delta|d)\right)_{I=\overline{2,6}}$, as well as of $\left(H_I^{\mathrm{inv}}(\delta|d)\right)_{I=\overline{2,6}}$ under the form: $I = 6 - \overset{(5)}{\eta}{}^{*\mu}$, $I = 3,4,5 - \left(\overset{(I+1)}{C}{}^{*\mu_1\ldots\mu_{4-I}|\alpha}, \overset{(I+1)}{\eta}{}^{*\mu_1\ldots\mu_{5-I}}\right)_{I=\overline{1,3}}$ and $I = 2 - \left(\overset{(1)}{C}{}^{*\mu_1\ldots\mu_4|\alpha}, \overset{(1)}{\eta}{}^{*\mu_1\ldots\mu_5}, \eta^{*\mu}\right)$.

The characteristic cohomology and the invariant characteristic cohomology spaces give us information on the obstructions to remove the antifields from the first-order deformation. As a consequence of the result (26), we can eliminate all the terms with $I > 6$ from expansion (35) by adding to it only trivial pieces, and thus work with $I \leq 6$. The last representative of (35) is of the type (39), with the corresponding invariant polynomials necessarily nontrivial in $H_I^{\mathrm{inv}}(\delta|d)$ for $I = \overline{2,6}$ and respectively in $H_1(\delta|d)$ for $I = 1$.

Under these considerations, we have at hand all the ingredients necessary at finding the solutions to Eqs. (36)–(38). They are provided by the next theorem.

Theorem 2 *Under the assumptions of analyticity in the coupling constant, locality, Lorentz covariance, Poincaré invariance and at most two derivatives in the Lagrangian, there are no consistent cross-interactions between a massless tensor field with the mixed symmetry* $(5,1)$ *and a graviton.*

As a consequence, the only pieces that can be added to action (27) are given by the selfinteractions of the Pauli–Fierz field, so we can write that

$$\overline{S}^{\mathrm{L}}\left[t_{\mu_1\ldots\mu_5|\alpha}, h_{\mu\nu}\right] = S^{\mathrm{L}}\left[t_{\mu_1\ldots\mu_5|\alpha}\right] + S^{\mathrm{EH}}\left[g_{\mu\nu}\right], \tag{40}$$

where $S^{\rm EH}[g_{\mu\nu}]$ is the Einstein–Hilbert action

$$S^{\rm EH}[g_{\mu\nu}] = \frac{2}{\kappa^2}\int d^D x\sqrt{-g}\,(R-2\Lambda)\,,\ g_{\mu\nu} = \sigma_{\mu\nu} + \kappa h_{\mu\nu}, \tag{41}$$

$$\kappa = \lambda \overset{(1)}{\kappa} + \lambda^2 \overset{(2)}{\kappa} + \cdots . \tag{42}$$

Action (41) is known to be invariant under the deformed gauge transformations (diffeomorphisms)

$$\frac{1}{\kappa}\delta_\epsilon g_{\mu\nu} = \epsilon_{(\mu;\nu)}, \tag{43}$$

with $\epsilon_{\mu;\nu}$ the covariant derivative of ϵ_μ from General Relativity. In the above R represents the full scalar curvature for the metric $g_{\mu\nu}$, and Λ denotes the cosmological constant

$$\Lambda = \lambda \overset{(1)}{\Lambda} + \lambda^2 \overset{(2)}{\Lambda} + \cdots . \tag{44}$$

4.3 No-go interactions with matter fields

Next, we to analyze the possible consistent interactions between the tensor field $t_{\mu_1\ldots\mu_5|\alpha}$ and matter fields. In view of this, let us consider the "free" action

$$S^{\rm L}\left[t_{\mu_1\ldots\mu_5|\alpha}, y^i\right] = S^{\rm L}\left[t_{\mu_1\ldots\mu_5|\alpha}\right] + S^{\rm matt}\left[y^i\right], \tag{45}$$

where $S^{\rm matt}$ describes a generic matter theory

$$S^{\rm matt}\left[y^i\right] = \int d^D x \mathcal{L}\left(\left[y^i\right]\right), \quad D \geq 7, \tag{46}$$

which is at most second-order derivative. The fields y^i are assumed to have no nontrivial gauge symmetries, the only "true" gauge transformations of action (45) being those of the field $t_{\mu_1\ldots\mu_5|\alpha}$, namely (9), such that

$$\delta_{\theta,\epsilon} y^i = 0. \tag{47}$$

In the sequel we will denote the Grassmann parity of the field y^i by ε_i. We allow the matter action to include couplings among the fields y^i, but presume that the Cauchy order of this theory alone is equal to one. This is however not a restrictive assumption since all the usual matter theories satisfy it. Accordingly, we obtain that the Cauchy order of the entire theory (45) is equal to 6.

The overall BRST differential of this system decomposes like in (13). The actions of γ and δ on the BRST generators from the $(5,1)$ sector are like in Subsec. 4.1, while on those from the matter sector read as

$$\gamma y^i = 0,\ \gamma y_i^* = 0,\ \delta y^i = 0,\ \delta y_i^* = -\frac{\delta^L \mathcal{L}}{\delta y^i}. \tag{48}$$

where y_i^* denote the antifields of the matter fields, of Grassmann parity $\varepsilon(y_i^*) = (\varepsilon_i + 1) \bmod 2$. The canonical action of the BRST differential, $s\cdot = (\cdot, S)$, where the antibracket is extended such as to include the matter fields and their antifields, is generated by the solution to the master equation $(S,S) = 0$, which takes the simple form

$$S = S^{\rm t} + S^{\rm matt}\left[y^i\right], \tag{49}$$

where S^{t} is given by the right-hand side of (15).

The nonintegrated density of the first-order deformation a for this model can be split like in the previous subsection

$$a = a^{\mathrm{m-m}} + a^{\mathrm{m-t}}, \tag{50}$$

where $a^{\mathrm{m-m}}$ governs the selfinteractions of the matter fields and $a^{\mathrm{m-t}}$ controls the cross-interactions between y^i and $t_{\mu_1\ldots\mu_5|\alpha}$. Naturally, a is subject to the equation and conditions from relation (16). Since the matter theory possesses no non-trivial gauge symmetries, it follows that $a^{\mathrm{m-m}}$ coincides with its antighost number zero component

$$a^{\mathrm{m-m}} = a_0^{\mathrm{m-m}}\left(\left[y^i\right]\right), \tag{51}$$

which is already non-trivial in $H^0(\gamma)$ and, actually, in $H^0(s)$

$$sa^{\mathrm{m-m}} = \gamma a_0^{\mathrm{m-m}}\left(\left[y^i\right]\right) = 0. \tag{52}$$

We will ask in addition that the derivative order of $a_0^{\mathrm{m-m}}$ does not exceed that of the original matter Lagrangian $\mathcal{L}([y^i])$, which was assumed to be at most equal to two. Inserting (50) in (16) and using (52), we find that $a^{\mathrm{m-t}}$ itself must be a non-trivial element of $H^0(s|d)$

$$sa^{\mathrm{m-t}} = \partial_\mu j^\mu. \tag{53}$$

In order to solve the last equation, we proceed in the standard manner, namely, we develop $a^{\mathrm{m-t}}$ according to the antighost number up to a finite value

$$a^{\mathrm{m-t}} = \sum_{m=0}^{I} a_m^{\mathrm{m-t}},\ \mathrm{agh}\left(a_m^{\mathrm{m-t}}\right) = m,\ \mathrm{gh}\left(a_m^{\mathrm{m-t}}\right) = 0,\ \varepsilon\left(a_m^{\mathrm{m-t}}\right) = 0, \tag{54}$$

such that (53) becomes equivalent to the tower of equations

$$\gamma a_I^{\mathrm{m-t}} = 0, \tag{55}$$

$$\delta a_I^{\mathrm{m-t}} + \gamma a_{I-1}^{\mathrm{m-t}} = \partial_\mu \overset{(I-1)^\mu}{j}, \tag{56}$$

$$\delta a_m^{\mathrm{m-t}} + \gamma a_{m-1}^{\mathrm{m-t}} = \partial_\mu \overset{(m-1)^\mu}{j},\ I-1 \geq m \geq 1. \tag{57}$$

From (55) we read that the element of maximum antighost number, $a_I^{\mathrm{m-t}}$, belongs, as usually, to the non-trivial part of the cohomology $H(\gamma)$ of the overall "free" theory (45). The first two definitions in (48) show that the presence of the matter fields simply adds to the cohomology $H(\gamma)$ discussed in Subsec. 4.1 the dependence on the fields y^i, on their antifields y_i^*, as well as on their spacetime derivatives, such that the general solution to (55) is

$$a_I^{\mathrm{m-t}} = \alpha_I^{\mathrm{m-t}}\left(\left[\Theta^{*\Delta}\right], \left[y_i^*\right], \left[K_{\mu_1\ldots\mu_6|\alpha\beta}\right], \left[y^i\right]\right)\omega^I\left(\mathcal{F}_{\mu_1\ldots\mu_6}, \overset{(5)}{\eta}_\mu\right),\quad I > 0. \tag{58}$$

We notice that the matter sector contributes only to the invariant polynomials of the theory (45), while the elements ω^I are entirely due to the part corresponding to the $(5,1)$ sector. Substituting (58) in (56), we conclude, like before, that a necessary condition for the existence of (non-trivial) solutions $a_{I-1}^{\mathrm{m-t}}$ is that $\alpha_I^{\mathrm{m-t}}$ are (nontrivial) objects from the characteristic cohomology at antighost number $I > 0$. Meanwhile, $\alpha_I^{\mathrm{m-t}}$ must involve at least one matter generator in order to provide a corresponding Lagrangian $a_0^{\mathrm{m-t}}$ that gives cross-couplings between y^i and $t_{\mu_1\ldots\mu_5|\alpha}$. Recalling once again the fact that the Cauchy order of the "free" theory (45) is equal to 6, we obtain that (25) is valid also in this case and, moreover, it can be shown that (26) is still checked. Since the Cauchy order of the matter theory alone is by assumption equal to one, it follows that the matter sector cannot be nontrivially involved in $H_I(\delta|d)$ and $H_I^{\mathrm{inv}}(\delta|d) = 0$ at $I > 1$. As a consequence, the nontrivial representatives of $(H_I(\delta|d))_{I=\overline{2,6}}$ and $\left(H_I^{\mathrm{inv}}(\delta|d)\right)_{I=\overline{2,6}}$ are the same like in Subsec. 4.1. Thus, we can always take $I \leq 6$ in (54), but the cases $I = \overline{2,6}$ have already been studied in the Subsec. 4.1, where it was shown that there are no deformations.

It is now clear that the cross-interactions between the tensor field $t_{\mu_1\ldots\mu_5|\alpha}$ and the matter fields y^i at the first order in the coupling constant may be produced just by a first-order deformation of the master equation that stops at antighost number one, $I = 1$.

Under these considerations, it can be shown that the first-order deformation takes in the end the form

$$S_1 = \int d^D x \left(a_0^{\mathrm{m-m}}\left(\left[y^i\right]\right) + y_i^* g^i_{\mu_1\ldots\mu_6}\left(\left[y^i\right]\right)\mathcal{F}^{\mu_1\ldots\mu_6} + \frac{1}{5} j^\rho_{\mu_1\ldots\mu_6}\left(\left[y^i\right]\right)\partial^{[\mu_1} t^{\mu_2\ldots\mu_6]|}{}_\rho \right), \tag{59}$$

and its existence requires that the matter theory possesses some nontrivial, bosonic, rigid gauge invariances with completely antisymmetric parameters $\xi^{\mu_1\ldots\mu_6}$ and generators $g^i_{\mu_1\ldots\mu_6}([y^i])$ displaying the Grassmann parities ε_i; $j^\rho_{\mu_1\ldots\mu_6}([y^i])$ are nothing but the corresponding conserved currents following from Noether's theorem

$$\Delta y^i = g^i_{\mu_1\ldots\mu_6}\left(\left[y^i\right]\right)\xi^{\mu_1\ldots\mu_6},\ \varepsilon\left(\xi^{\mu_1\ldots\mu_6}\right) = 0,\ \varepsilon\left(g^i_{\mu_1\ldots\mu_6}\right) = \varepsilon_i, \tag{60}$$

$$\frac{\delta^R \mathcal{L}}{\delta y^i} g^i_{\mu_1\ldots\mu_6}\left(\left[y^i\right]\right) = \partial_\rho j^\rho_{\mu_1\ldots\mu_6}\left(\left[y^i\right]\right),\ \ \varepsilon\left(j^\rho_{\mu_1\ldots\mu_6}\right) = 0. \tag{61}$$

The consistency of the first-order deformation, namely Eq. (6), imposes the necessary conditions

$$\frac{\delta^R g^i_{\mu_1\ldots\mu_6}}{\delta y^j} g^j_{\nu_1\ldots\nu_6} - \frac{\delta^R g^i_{\nu_1\ldots\nu_6}}{\delta y^j} g^j_{\mu_1\ldots\mu_6} = 0. \tag{62}$$

Conditions (62) can be satisfied only in $D = 8$ spacetime dimensions, where their solutions depend on the spacetime coordinates

$$g^i_{\mu_1\ldots\mu_6} = c\varepsilon_{\mu_1\ldots\mu_6\rho\sigma} x^{[\rho}\partial^{\sigma]} y^i, \tag{63}$$

with c an arbitrary real constant. Replacing (63) in (60) and making the notation

$$\varepsilon_{\mu_1\ldots\mu_6\rho\sigma}\xi^{\mu_1\ldots\mu_6} \equiv \xi_{\rho\sigma},\ \xi_{\rho\sigma} = -\xi_{\rho\sigma}, \tag{64}$$

we get that the corresponding rigid symmetry of the matter action reads as

$$\Delta y^i = c x^{[\rho}\partial^{\sigma]} y^i \xi_{\rho\sigma}. \tag{65}$$

The associated conserved current will also involve the spacetime coordinates, and hence the deformed Lagrangian, namely the term involving $j^\rho_{\mu_1\ldots\mu_6}([y^i])$ would break the required Poincaré invariance of cross-interactions and must be removed from (59). In conclusion, the nonintegrated density of the first-order deformation that is consistent to all orders in the coupling constant reduces to selfinteractions of the matter fields, $a_0^{\mathrm{m-m}}([y^i])$. All these results are contained in the next theorem.

Theorem 3 *Under the assumptions of analyticity in the coupling constant, locality, Lorentz covariance, Poincaré invariance and the preservation of the maximum derivative order of both the Lagrangian and field equations, there are no consistent cross-couplings of a massless tensor field with the mixed symmetry* $(5,1)$ *to a generic matter theory. The relaxation of the Poincaré invariance condition might allow for cross-couplings in* $D = 8$.

4.4 Yes-go couplings to p-form gauge fields for $p=1$

In this case the starting free model pictures a free massless tensor field with the mixed symmetry $(5,1)$ and an Abelian vector gauge field

$$S^{\mathrm{L}}[t_{\mu_1\ldots\mu_5|\alpha}, A_\mu] = S^{\mathrm{L}}[t_{\mu_1\ldots\mu_5|\alpha}] + S^{\mathrm{L}}[A_\mu], \tag{66}$$

where $S^{\mathrm{L}}[t_{\mu_1\ldots\mu_5|\alpha}]$ reads as in (7) and the Maxwell action is given by

$$S^{\mathrm{L}}[A_\mu] = -\frac{1}{4}\int d^D x\, F_{\mu\nu}F^{\mu\nu}, \quad D \geq 7, \tag{67}$$

where the Abelian field strength is defined in the standard manner like

$$F_{\mu\nu} = \partial_{[\mu}A_{\nu]}. \tag{68}$$

A generating set of gauge transformations for action (66) is (9) in the $(5,1)$ sector and respectively the $U(1)$ gauge invariance in the vector sector

$$\delta_\epsilon A_\mu = \partial_\mu \epsilon, \tag{69}$$

where ϵ is an arbitrary scalar field. Given the properties of the generating set of gauge transformations (9) in the $(5,1)$ sector and the observation that the set (69) is Abelian and irreducible, it follows that the free theory described by action (66) reveals an overall Abelian gauge algebra and an entire generating set off-shell reducible of order 4. The BRST algebra of local forms is generated by the fields $t_{\mu_1\ldots\mu_5|\alpha}$ and A_μ, by the ghosts (11), the antifields (12) and the following ghosts/antifields corresponding to the Maxwell sector

$$(\eta, A^{*\mu}, \eta^*)\,. \tag{70}$$

It is easy to see that the free BRST differential s for theory (66) splits as in (13), being canonically generated in the antibracket by a solution to the master equation $(S,S)=0$, given here by

$$S = S^{\mathrm{t}} + S^{\mathrm{A}}, \tag{71}$$

where S^{t} reads as in the right-hand side of (15) and

$$S^{\mathrm{A}} = S^{\mathrm{L}}[A_\mu] + \int d^D x\, A^{*\mu}\partial_\mu \eta. \tag{72}$$

It is understood that the present antibracket takes into account all the BRST generators, including those from the Maxwell sector.

The solution to the local first-order deformation, Eq. (16), is found like in the previous subsections, namely, we expand the nonintegrated density a according to the antighost number as in (17) and solve the equivalent tower of equations, given by (21) and (19)–(20). It is easy to see that the first-order deformation reduces here to

$$a = a^{\mathrm{A-t}}, \tag{73}$$

where $a^{\mathrm{A-t}}$ signifies the component that describes only the cross-interactions between A_μ and $t_{\mu_1\ldots\mu_5|\alpha}$. This is because on the one hand the results of Subsec. 4.1 ensure that the component related to the selfinteractions of the tensor field $t_{\mu_1\ldots\mu_5|\alpha}$ is trivial and, on the other hand, a single massless vector fields allows no consistent selfinteractions in $D \geq 7$.

Therefore, $a^{\mathrm{A-t}}$ decomposes as in (17), its components being subject to Eqs. (21) and (19)–(20), with a replaced everywhere by $a^{\mathrm{A-t}}$, namely,

$$a^{\mathrm{A-t}}=\sum_{m=0}^{I}a_{m}^{\mathrm{A-t}},\ \mathrm{agh}\left(a_{m}^{\mathrm{A-t}}\right)=m,\ \mathrm{gh}\left(a_{m}^{\mathrm{A-t}}\right)=0,\ \varepsilon\left(a_{m}^{\mathrm{A-t}}\right)=0, \tag{74}$$

$$\gamma a_{I}^{\mathrm{A-t}}=0, \tag{75}$$

$$\delta a_{I}^{\mathrm{A-t}}+\gamma a_{I-1}^{\mathrm{A-t}}=\partial_{\mu}\overset{(I-1)^{\mu}}{j}, \tag{76}$$

$$\delta a_{m}^{\mathrm{A-t}}+\gamma a_{m-1}^{\mathrm{A-t}}=\partial_{\mu}\overset{(m-1)^{\mu}}{j},\ I-1\geq m\geq 1. \tag{77}$$

Eq. (75) shows that $a_{I}^{\mathrm{A-t}}\in H\left(\gamma\right)$, so we need to compute $H\left(\gamma\right)$ in order to reveal the component of maximum antighost number from the first-order deformation. It can be proved that the general solution to (75) is expressed by

$$a_{I}^{\mathrm{A-t}}=\alpha_{I}^{\mathrm{A-t}}\left(\left[\pi^{*\Theta}\right],\left[K_{\mu_{1}\ldots\mu_{6}|\alpha\beta}\right],\left[F_{\mu\nu}\right]\right)e^{I}\left(\eta,\mathcal{F}_{\mu_{1}\ldots\mu_{6}},\overset{(5)}{\eta}_{\mu}\right), \tag{78}$$

for $I>0$, where $\pi^{*\Theta}$ is a generic notation for all antifields, $K_{\mu_{1}\ldots\mu_{6}|\alpha\beta}$ stands for the curvature tensor (23), $F_{\mu\nu}$ signifies the Abelian field strength (68), $\mathcal{F}_{\mu_{1}\ldots\mu_{6}}$ is defined in (22), and η represents the fermionic ghost associated with the $U(1)$ gauge invariance of the vector field. The γ-invariant objects $\alpha_{I}^{\mathrm{A-t}}$ (of pure ghost number equal to zero) are required to fulfill $\mathrm{agh}\left(\alpha_{I}^{\mathrm{A-t}}\right)=I$, and the notation e^{I} stands for a generic notation of the elements with pure ghost number equal to I of a basis in the space of polynomials in the corresponding ghosts and combinations of ghosts. In addition, every single term from $a_{I}^{\mathrm{A-t}}$ must contain at least one generator (field or ghost or antifield) from each of the two theories in order to provide effective cross-interactions. The quantities $\alpha_{I}^{\mathrm{A-t}}$ are nothing but the most general, nontrivial elements from $H^{0}\left(\gamma\right)$ and will be called again "invariant polynomials".

Substituting the solution (78) into Eq. (76) and taking into account the definitions δ and γ acting on the BRST generators, we obtain, like in the previously investigated situations, that a necessary condition for Eq. (76) to possess nontrivial solutions with respect to $a_{I-1}^{\mathrm{A-t}}$ for all $I>0$ is that the invariant polynomials $\alpha_{I}^{\mathrm{A-t}}$ are nontrivial objects from $H_{I}\left(\delta|d\right)$. Since the present model is a linear gauge theory of Cauchy order equal to 6, relation (25) is still valid and, moreover it can be shown that (26) holds again. On account of the actions of δ on the BRST generators, we are able to identify the nontrivial representatives of $\left(H_{I}\left(\delta|d\right)\right)_{I=\overline{2,6}}$, as well as of $\left(H_{I}^{\mathrm{inv}}\left(\delta|d\right)\right)_{I=\overline{2,6}}$ under the form: $I=6-\overset{(5)^{*\mu}}{\eta}$, $I=3,4,5-\left(\overset{(I+1)^{*\mu_{1}\ldots\mu_{4-I}|\alpha}}{C},\overset{(I+1)^{*\mu_{1}\ldots\mu_{5-I}}}{\eta}\right)_{I=\overline{1,3}}$ and $I=2-\left(\overset{(1)^{*\mu_{1}\ldots\mu_{4}|\alpha}}{C},\overset{(1)^{*\mu_{1}\ldots\mu_{5}}}{\eta},\eta^{*}\right)$.

The characteristic cohomology and the invariant characteristic cohomology spaces give us information on the obstructions to remove the antifields from the first-order deformation. Due to (26), we can eliminate all the terms with $I>6$ from (74) by trivial terms only, and thus work with $I\leq 6$. The last representative of (74) is of the type (78), with the corresponding invariant polynomials necessarily nontrivial in $H_{I}^{\mathrm{inv}}\left(\delta|d\right)$ for $I=\overline{2,6}$ and respectively in $H_{1}\left(\delta|d\right)$ for $I=1$.

Under these considerations, we have at hand all the ingredients necessary at finding the solutions to Eqs. (75)–(77). They are provided by the next theorem.

Theorem 4 *Under the assumptions of analyticity in the coupling constant, locality, Lorentz covariance, Poincaré invariance and at most two derivatives in the Lagrangian, there appear consistent cross-couplings between a massless tensor field with the mixed symmetry* $(5,1)$ *and an Abelian vector field. They break the PT invariance and hold only in* $D=7$*.*

The coupled Lagrangian action can be organized into

$$\bar{S}^{\mathrm{L}}[t_{\mu_1\ldots\mu_5|\alpha}, A_\mu] = S^{\mathrm{L}}[t_{\mu_1\ldots\mu_5|\alpha}] - \frac{1}{4}\int d^7x\, \bar{F}_{\mu\nu}\bar{F}^{\mu\nu}, \tag{79}$$

in terms of the deformed field strength

$$\bar{F}^{\mu\nu} = F^{\mu\nu} + \frac{6\lambda}{5}\varepsilon^{\mu\nu\mu_1\ldots\mu_5}\partial_{[\rho} t_{\mu_1\ldots\mu_5]|}{}^{\rho}, \tag{80}$$

where $S^{\mathrm{L}}[t_{\mu_1\ldots\mu_5|\alpha}]$ is the Lagrangian action of the massless tensor field $t_{\mu_1\ldots\mu_5|\alpha}$ appearing in (7) in $D=7$. We observe that action (79) contains only mixing-component terms of order one and two in the coupling constant. The deformed gauge transformations can be taken of the form (9) in the $(5,1)$ sector and respectively

$$\bar{\delta}_{\theta,\epsilon}A_\mu = \partial_\mu\epsilon + \lambda\varepsilon_{\mu\mu_1\ldots\mu_6}\partial^{\mu_1}\overset{(1)}{\epsilon}{}^{\mu_2\ldots\mu_6} \tag{81}$$

in the vector sector. It is interesting to note that only the gauge transformations of the vector field are modified during the deformation process. This is enforced at order one in the coupling constant by a term linear in the antisymmetrized first-order derivatives of some gauge parameters from the $(5,1)$ sector. The deformed gauge algebra and associated reducibility structure of the coupled model are not modified during the deformation procedure, being the same like in the case of the starting free action (66). It is easy to see that if we impose the PT-invariance at the level of the coupled model, then we obtain no interactions (we must set $\lambda=0$ in these formulas).

It is important to stress that the problem of obtaining consistent interactions strongly depends on the spacetime dimension. For instance, if one starts with action (66) in $D>7$, then *no* term can be added to either the original Lagrangian or its gauge transformations.

5 Conclusion

The main conclusion of this paper is that a massless tensor field with the mixed symmetry $(5,1)$, which is dual to linearized gravity in $D=8$ space-time dimensions, does not interact with itself, the Pauli–Fierz field and respectively a generic matter theory, but can be consistently coupled to an Abelian vector field. These findings have been inferred within the comprehensive and elegant setting of the deformation procedure of the solution to the classical master equation in the context of the antibracket-antifield BRST symmetry, combined with specific cohomological techniques. All results satisfy standard ‘selection rules’ employed in field theories: analyticity in the coupling constant, locality, Lorentz covariance, Poincaré invariance and the preservation of the maximum derivative order of the deformed field equations with respect to the free ones.

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