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Aspects of the quantization of the massive 4-forms

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Abstract

Massive 4-forms are analyzed from the point of view of the Hamiltonian path integral quantization. More precisely, the quantization procedure is based on the path integral of a first-class system equivalent (at both classical and quantum levels) with the original theory. This first-class system is constructed using the Batalin-Fradkin method. This approach finally outputs the manifestly Lorentz covariant path integral for 4- and $(D-5)$-forms with topological coupling.

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1 Introduction

The covariant quantization of Hamiltonian systems possessing only second-class constraints can be done through construction of an equivalent first-class system and then quantizing the resulting first-class system. The construction of the equivalent first-class system can be achieved using constraints conversion [1]–[4] method. This quantization procedure was applied to various models [5]–[21]. The importance of the models with p-form gauge fields (antisymmetric tensor fields of various orders) are interesting from the point of view of string and superstring theory, supergravity, and the gauge theory of gravity [22]–[27] being well-known the inclusion of these fields within the field spectrum of supergravity in 10 or 11 dimensions [24, 25]. In this paper we present diferent aspects of the quantization of the massive 4-forms. The quantization procedure is based on the construction of an equivalent first-class system using Batalin–Fradkin (BF) methods and then quantizing the resulting first-class system. The BF approach [1]–[4] relies on enlarging the original phase-space and constructing a first-class constraint set and a first-class Hamiltonian, with the property that they coincide with the original second-class constraints and respectively with the starting canonical Hamiltonian if one sets all the extravariables equal to zero.

The present paper is organized in three sections. In Section 2 we do a short review on the BF approach to the problem of constructing a first-class system equivalent with a second-class theory, we exemplify in detail this method on massive 4-forms and construct the path integral associated with the first-class systems associated with this model. Based on an appropriate extension of the phase-space, integrating out the auxiliary fields, and performing some field redefinitions, we find the manifestly Lorentz-covariant path integrals corresponding to the Lagrangian formulation of the first-class systems which reduce to the Lagrangian path-integral for 4- and $(D-5)$-forms with topological coupling. Section 3 ends the paper with the main conclusions.

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2 BF method

The starting point is a bosonic dynamic system with the phase-space locally parameterized by n canonical pairs $z^a = (q^i,\ p_i)$, endowed with the canonical Hamiltonian H_c, and subject to the second-class constraints

$$\chi_{\alpha_0}(z) \approx 0, \qquad \alpha_0 = \overline{1, 2M_0}. \tag{1}$$

In order to construct a first-class system equivalent to the starting second-class one in the framework of the BF approach [1]–[4] we enlarge the original phase-space with the variables $(\zeta^\alpha)_{\alpha=\overline{1,2M}}$, $(M \geq M_0)$ and extend the Poisson bracket to the newly added variables. The next step is to construct a set of independent, smooth, real functions defined on the extended phase-space, $(G_A(z,\zeta))_{A=\overline{1,M_0+M}}$, such that

$$G_{\alpha_0}(z,0) \equiv \chi_{\alpha_0}(z), \qquad G_{\bar{A}}(z,0) \equiv 0, \qquad [G_A, G_B] = 0, \tag{2}$$

where $\bar{A} = \overline{2M_0+1, M_0+M}$. In the last step we generate a smooth, real function $H_{\mathrm{BF}}(z,\zeta)$, defined on the extended phase-space, with the properties

$$H_{\mathrm{BF}}(z,0) \equiv H_c(z), \qquad [H_{\mathrm{BF}}, G_A] = V_A^B G_B. \tag{3}$$

The previous steps unravel a dynamic system subject to the first-class constraints $G_A(z,\zeta) \approx 0$ and whose evolution is governed by the first-class Hamiltonian $H_{\mathrm{BF}}(z,\zeta)$. The first-class system constructed by the BF method is classically equivalent to the original second-class theory since both display the same number of physical degrees of freedom

$$\mathcal{N}_{\mathrm{BF}} = \frac{1}{2}\left[2n + 2M - 2(M_0+M)\right] = \frac{1}{2}(2n - 2M_0) = \mathcal{N}_O \tag{4}$$

and the corresponding algebras of classical observables are isomorphic. Consequently, the two systems become also equivalent at the level of the path integral quantization and we can to replace the Hamiltonian path integral of the original second-class theory with that of the BF first-class system.

Massive 4-forms in D space-time dimensions ($D \geq 5$) are described by the Lagrangian action [8]

$$S_0^L[A_{\mu_1\ldots\mu_4}] = \int d^D x \left(-\frac{1}{2\cdot 5!}F_{\mu_1\ldots\mu_5}F^{\mu_1\ldots\mu_5} - \frac{m^2}{2\cdot 4!}A_{\mu_1\ldots\mu_4}A^{\mu_1\ldots\mu_4}\right), \tag{5}$$

where the field strength of $A_{\mu_1\ldots\mu_4}$ is defined in the standard manner by $F_{\mu_1\ldots\mu_5} \equiv \partial_{[\mu_1}A_{\mu_2\ldots\mu_5]}$. We use the Minkowski metric tensor of 'mostly minus' signature $\sigma_{\mu\nu} = \sigma^{\mu\nu} = \mathrm{diag}(+-\ldots-)$. The canonical analysis of the model described by the Lagrangian action (5) displays the constraints

$$\chi^{(1)i_1i_2i_3} \equiv \pi^{0i_1i_2i_3} \approx 0, \tag{6}$$

$$\chi^{(2)}_{i_1i_2i_3} \equiv 4\partial^j \pi_{ji_1i_2i_3} - \frac{m^2}{3!}A_{0i_1i_2i_3} \approx 0, \tag{7}$$

and the canonical Hamiltonian

$$H_c = \int d^{D-1}x \left(-\frac{4!}{2}\pi_{i_1\ldots i_4}\pi^{i_1\ldots i_4} - 4A_{0i_1i_2i_3}\partial_j\pi^{ji_1i_2i_3}\right.$$

$$+\frac{1}{2\cdot 5!}F_{i_1\ldots i_5}F^{i_1\ldots i_5}+\frac{m^2}{2\cdot 4!}A_{\mu_1\ldots\mu_4}A^{\mu_1\ldots\mu_4}\bigg), \tag{8}$$

where $\pi^{\mu_1\ldots\mu_4}$ are the canonical momenta conjugated to the fields $A_{\mu_1\ldots\mu_4}$. Constraints (6) and (7) are second-class and irreducible [28].

In the sequel we apply the BF method exposed in the above to the case of massive 4-forms. In view of this, we enlarge the original phase-space by adding the bosonic fields/momenta $\{B^{\mu_1\mu_2\mu_3},\ \Pi_{\mu_1\mu_2\mu_3}\}$. The constraints $G_A(z,\zeta)\approx 0$ gain in the case of massive 4-forms the concrete form

$$G^{(1)i_1i_2i_3}\equiv\chi^{(1)i_1i_2i_3}+mB^{i_1i_2i_3}\approx 0, \tag{9}$$

$$G^{(2)}_{i_1i_2i_3}\equiv\chi^{(2)}_{i_1i_2i_3}-\frac{m}{3!}\Pi_{i_1i_2i_3}\approx 0, \tag{10}$$

$$G_{i_1i_2}\equiv\Pi_{0i_1i_2}\approx 0. \tag{11}$$

Constraints (9)–(11) form an Abelian and irreducible first-class constraint set. The first-class Hamiltonian complying with the general requirements (3) is expressed by

$$H_{\mathrm{BF}}=H_c+\int d^{D-1}x\left[\frac{1}{2\cdot 3!}\Pi^{i_1i_2i_3}\Pi_{i_1i_2i_3}\right.$$
$$-\frac{1}{m}\Pi^{i_1i_2i_3}\left(4\partial^j\pi_{ji_1i_2i_3}-\frac{m^2}{3!}A_{0i_1i_2i_3}\right)-\frac{4}{3!}B^{0i_1i_2}\partial^j\left(mA_{0ji_1i_2}+\Pi_{ji_1i_2}\right)$$
$$\left.-\frac{1}{4}\left(mA^{i_1\ldots i_4}-\frac{3!}{2}\partial^{[i_1}B^{i_2i_3i_4]}\right)\partial_{[i_1}B_{i_2i_3i_4]}\right]. \tag{12}$$

In the sequel we show how massive 4-forms get related to $(D-5)$-form gauge fields. In order to do this, we start from the first-class system constructed in the above and subject to the first-class constraints (9)–(11), whose evolution is governed by the first-class Hamiltonian (12). Imposing the canonical gauge conditions

$$C^{(1)}_{j_1j_2j_3}\equiv A_{0j_1j_2j_3}\approx 0,\qquad C^{j_1j_2}\equiv B^{0j_1j_2}\approx 0, \tag{13}$$

we obtain that (9), (11) and (13) generate a second-class constraint subset, while (10) is first-class. Eliminating the second-class constraints (9), (11) and (13) (the coordinates of the reduced phase-space are $\{A_{i_1\ldots i_4},\ B^{i_1i_2i_3},\ \pi^{i_1\ldots i_4},\ \Pi_{i_1i_2i_3}\}$), we are left with a system subject only to the first-class constraints

$$G_{i_1i_2i_3}\equiv 4\partial^j\pi_{ji_1i_2i_3}-\frac{m}{3!}\Pi_{i_1i_2i_3}\approx 0, \tag{14}$$

while the first-class Hamiltonian (12) takes the form

$$H_{\mathrm{BF}}=\int d^{D-1}x\left[-\frac{4!}{2}\pi_{i_1\ldots i_4}\pi^{i_1\ldots i_4}+\frac{1}{2\cdot 5!}F_{i_1\ldots i_5}F^{i_1\ldots i_5}\right.$$
$$+\frac{m^2}{2\cdot 4!}A_{i_1\ldots i_4}A^{i_1\ldots i_4}+\frac{1}{2\cdot 3!}\Pi^{i_1i_2i_3}\Pi_{i_1i_2i_3}-\frac{4}{m}\Pi^{i_1i_2i_3}\partial^j\pi_{ji_1i_2i_3}$$
$$\left.-\frac{1}{4}\left(mA^{i_1\ldots i_4}-3\partial^{[i_1}B^{i_2i_3i_4]}\right)\partial_{[i_1}B_{i_2i_3i_p]}\right]. \tag{15}$$

We consider the quantities

$$\mathcal{F}_{i_1\ldots i_4}=A_{i_1\ldots i_4}-\frac{3!}{m}\partial_{[i_1}B_{i_2i_3i_4]},\qquad \mathcal{F}_{0i_1i_2i_3}=\frac{1}{m}\Pi_{i_1i_2i_3}, \tag{16}$$

which are in (strong) involution with first-class constraints (14)

$$[\mathcal{F}_{i_1\ldots i_4}, G_{j_1j_2j_3}] = [\mathcal{F}_{0i_1i_2i_3}, G_{j_1j_2j_3}] = 0. \tag{17}$$

We define

$$\mathcal{W}_{\mu_1\ldots\mu_5} = \partial_{[\mu_1}\mathcal{F}_{\mu_2\ldots\mu_5]}, \tag{18}$$

where $\mathcal{F}_{\mu_1\ldots\mu_4} \equiv \{\mathcal{F}_{0i_1i_2i_3}, \mathcal{F}_{i_1\ldots i_4}\}$. By direct computation, it follows that

$$\partial^\nu \mathcal{W}_{\nu\mu_1\ldots\mu_4} = m^2 \mathcal{F}_{\mu_1\ldots\mu_4} + \mathcal{O}\left(G_{i_1i_2i_3}\right). \tag{19}$$

From (19) we obtain that the equalities

$$\partial^\nu \mathcal{F}_{\nu\mu_1\mu_2\mu_3} = 0 \tag{20}$$

hold on the first-class surface (14). The solution to (19) is of the type

$$\mathcal{F}_{\mu_1\ldots\mu_4} = -\frac{1}{m}\frac{1}{(D-4)!}\varepsilon_{\mu_1\ldots\mu_4\nu_1\ldots\nu_{D-4}}\partial^{[\nu_1}V^{\nu_2\ldots\nu_{D-4}]}. \tag{21}$$

Consequently, we enlarge the phase-space by adding the bosonic fields/momenta $\{V^{\nu_1\ldots\nu_{D-5}}, P_{\nu_1\ldots\nu_{D-5}}\}$. If we replace (21) in (14), then the constraint set takes the form

$$4\partial^j \pi_{ji_1i_2i_3} + \frac{m}{3!\,(D-4)!}\varepsilon_{0i_1i_2i_3j_1\ldots j_{D-4}}\partial^{[j_1}V^{j_2\ldots j_{D-4}]} \approx 0, \tag{22}$$

remains first-class, and becomes third order reducible. In order to preserve the number of physical degrees of freedom we have to impose the supplementary constraints (we consider the case $D \geq 6$)

$$P_{0i_1\ldots i_{D-6}} \approx 0, \qquad (-)^{D-5}(D-5)\,\partial^j P_{ji_1\ldots i_{D-6}} \approx 0. \tag{23}$$

The constraints (22) and (23) are first-class and reducible of order $D-6$ (or third order reducible if $D-5 \leq 4$). The gauge transformation of the quantity $\partial_{[i_1}B_{i_2i_3i_4]}$ leads to the relation

$$\partial_{[i_1}B_{i_2i_3i_4]} = \frac{(-)^{D-1}}{3!}\varepsilon_{0i_1\ldots i_4j_1\ldots j_{D-5}}P^{j_1\ldots j_{D-5}}. \tag{24}$$

Using (16), (21) and (24), the first-class Hamiltonian takes the form

$$\begin{aligned}
H_{\mathrm{BF}} = \int d^{D-1}x \Bigg[& -\frac{4!}{2}\pi_{i_1\ldots i_4}\pi^{i_1\ldots i_4} + \frac{1}{2\cdot 5!}F_{i_1\ldots i_5}F^{i_1\ldots i_5} \\
& + (-)^D \frac{1}{2\cdot(D-4)!}\partial_{[i_1}V_{i_2\ldots i_{D-4}]}\partial^{[i_1}V^{i_2\ldots i_{D-4}]} + \frac{m^2}{2\cdot 4!}A_{i_1\ldots i_4}A^{i_1\ldots i_4} \\
& + \frac{1}{m}\frac{1}{(D-4)!}\varepsilon_{0i_1i_2i_3j_1\ldots j_{D-4}}\partial^{[j_1}V^{j_2\ldots j_{D-4}]}\times \\
& \times\left(4\partial_j\pi^{ji_1i_2i_3} + \frac{m}{3!\,(D-4)!}\varepsilon^{0i_1i_2i_3k_1\ldots k_{D-4}}\partial_{[k_1}V_{k_2\ldots k_{D-4}]}\right) \\
& + (-)^D \frac{m}{p!}\varepsilon_{0i_1\ldots i_4 p j_1\ldots j_{D-5}}A^{i_1\ldots i_4}P^{j_1\ldots j_{D-5}} \\
& - (-)^D \frac{(D-5)!}{2}P_{i_1\ldots i_{D-5}}P^{i_1\ldots i_{D-5}}\Bigg].
\end{aligned} \tag{25}$$

For each first-class theory we can identify a set of fundamental classical observables such that they are in a one-to-one correspondence and possess the same Poisson brackets (in the case of the first-class theory with the phase-space locally parameterized by $\{A_{i_1\ldots i_4}, \pi^{i_1\ldots i_4}, B_{i_1i_2i_3}, \Pi^{i_1i_2i_3}\}$ and subject to the first-class constraints (14) the fundamental classical observables read as $\{A_{i_1\ldots i_4}-\frac{3!}{m}\partial_{[i_1}B_{i_2i_3i_4]}, \Pi_{i_1i_2i_3}$ and $\pi^{i_1\ldots i_4}\}$, while for the first-class theory with the phase-space locally parameterized by $\{A_{i_1\ldots i_4}, \pi^{i_1\ldots i_4}, V_{\mu_1\ldots\mu_{D-5}}, P^{\mu_1\ldots\mu_{D-5}}\}$ and subject to the first-class constraints (22) and (23) the fundamental classical observables are $\{A_{i_1\ldots i_4}+\frac{(-)^D}{m}\varepsilon_{0i_1\ldots i_4j_1\ldots j_{D-5}}P^{j_1\ldots j_{D-5}}, -\frac{1}{(D-4)!}\varepsilon_{0i_1i_2i_3j_1\ldots j_{D-4}}\partial^{[j_1}V^{j_2\ldots j_{D-4}]}$ and $\pi^{i_1\ldots i_4}\}$). The procedure exposed previously preserves the equivalence between the two first-class theories. As a result, the BF theory and the reducible first-class system remain equivalent also at the level of the Hamiltonian path integral quantization. This further implies that the reducible first-class system is completely equivalent with the original second-class theory. Due to this equivalence one can replace the Hamiltonian path integral of massive 4-forms with that associated with the first-class system reducible of order $D-6$ (or third order reducible if $D-5\leq 4$). The first-class Hamiltonian (25) outputs the argument of the exponential from the Hamiltonian path integral of the reducible first-class system as

$$\begin{aligned} S_{\mathrm{BF}} = \int d^Dx \big[& (\partial_0 A_{i_1\ldots i_4})\,\pi^{i_1\ldots i_4} + \left(\partial_0 V_{0i_1\ldots i_{D-6}}\right) P^{0i_1\ldots i_{D-6}} \\ & + \left(\partial_0 V_{i_1\ldots i_{D-5}}\right) P^{i_1\ldots i_{D-5}} - \mathcal{H}_{\mathrm{BF}} \\ & - \lambda_{i_1i_2i_3}\left(4\partial_j\pi^{ji_1i_2i_3} + \frac{m}{3!\,(D-4)!}\varepsilon^{0i_1i_2i_3k_1\ldots k_{D-4}}\partial_{[k_1}V_{k_2\ldots k_{D-4}]}\right) \\ & -\lambda^{(1)}_{i_1\ldots i_{D-6}}P^{0i_1\ldots i_{D-6}} - (-)^{D-1}(D-5)\,\lambda^{(2)}_{i_1\ldots i_{D-6}}\partial_j P^{ji_1\ldots i_{D-6}}\Big]. \end{aligned} \tag{26}$$

If we perform the transformation

$$\lambda_{i_1i_2i_3} \longrightarrow \bar{\lambda}_{i_1i_2i_3} = \lambda_{i_1\ldots i_{p-1}} + \frac{1}{m}\frac{1}{(D-4)!}\varepsilon_{0i_1i_2i_3j_1\ldots j_{D-4}}\partial^{[j_1}V^{j_2\ldots j_{D-4}]}, \tag{27}$$

in the path integral, the argument of the exponential becomes

$$\begin{aligned} S_{\mathrm{BF}} = \int d^Dx \big[& (\partial_0 A_{i_1\ldots i_4})\,\pi^{i_1\ldots i_4} + \left(\partial_0 V_{0i_1\ldots i_{D-6}}\right) P^{0i_1\ldots i_{D-6}} \\ & + \left(\partial_0 V_{i_1\ldots i_{D-5}}\right) P^{i_1\ldots i_{D-5}} + \frac{4!}{2}\pi_{i_1\ldots i_4}\pi^{i_1\ldots i_4} - \frac{1}{2\cdot 5!}F_{i_1\ldots i_5}F^{i_1\ldots i_5} \\ & - (-)^D\frac{1}{2\cdot(D-4)!}\partial_{[i_1}V_{i_2\ldots i_{D-4}]}\partial^{[i_1}V^{i_2\ldots i_{D-4}]} \\ & - \frac{m^2}{2\cdot 4!}A_{i_1\ldots i_4}A^{i_1\ldots i_4} + (-)^D\frac{(D-5)!}{2}P_{i_1\ldots i_{D-5}}P^{i_1\ldots i_{D-5}} \\ & - (-)^D\frac{m}{4!}\varepsilon_{0i_1\ldots i_4j_1\ldots j_{D-5}}A^{i_1\ldots i_4}P^{j_1\ldots j_{D-5}} \\ & - \bar{\lambda}_{i_1i_2i_3}\left(4\partial_j\pi^{ji_1i_2i_3} + \frac{m}{3!\,(D-4)!}\varepsilon^{0i_1i_2i_3k_1\ldots k_{D-4}}\partial_{[k_1}V_{k_2\ldots k_{D-4}]}\right) \\ & -\lambda^{(1)}_{i_1\ldots i_{D-6}}P^{0i_1\ldots i_{D-6}} - (-)^{D-1}(D-5)\,\lambda^{(2)}_{i_1\ldots i_{D-6}}\partial_j P^{ji_1\ldots i_{D-6}}\Big]. \end{aligned} \tag{28}$$

At this stage, the Hamiltonian path integral of the reducible first-class systems reads

$$Z_{\mathrm{BF}} = \int \mathcal{D}\left(A_{i_1\ldots i_4}, V_{\mu_1\ldots\mu_{D-5}}, \pi^{i_1\ldots i_4}, P^{\mu_1\ldots\mu_{D-5}}, \bar{\lambda}_{i_1i_2i_3}, \lambda^{(1)}_{i_1\ldots i_{D-6}}, \lambda^{(2)}_{i_1\ldots i_{D-6}}\right)$$

$$\times \mu\left([A_{i_1\ldots i_4}],[V_{\mu_1\ldots\mu_{D-5}}]\right)\exp\left(\mathrm{i}S_{\mathrm{BF}}\right), \tag{29}$$

where '$\mu\left([A_{i_1\ldots i_4}],[V_{\mu_1\ldots\mu_{D-5}}]\right)$' represents the integration measure associated with the model subject to the reducible first-class constraints (22) and (23). This measure includes some suitable canonical gauge conditions [10], is independent of the chosen gauge-fixing conditions [29], and is taken such that (29) is convergent [30]. In order to obtain from (28) a path integral exhibiting a manifestly Lorentz-covariant functional in its exponential, we enlarge the original phase-space with the Lagrange multipliers $\{\bar{\lambda}_{i_1i_2i_3},\lambda^{(2)}_{i_1\ldots i_{D-6}}\}$ and their canonical momenta $\{\pi^{i_1i_2i_3},p^{i_1\ldots i_{D-6}}\}$ and add the constraints [28]

$$\pi^{i_1i_2i_3}\approx 0 \qquad p^{i_1\ldots i_{D-6}}\approx 0. \tag{30}$$

The argument of the exponential from the Hamiltonian path integral for the first-class theory subject to the first-class constraints (22), (23) and (30) reads as

$$\begin{aligned}
S_{\mathrm{BF}} = \int d^Dx\,\Big[&\left(\partial_0 A_{i_1\ldots i_4}\right)\pi^{i_1\ldots i_4}+\left(\partial_0 V_{0i_1\ldots i_{D-6}}\right)P^{0i_1\ldots i_{D-6}}\\
&+\left(\partial_0 V_{i_1\ldots i_{D-5}}\right)P^{i_1\ldots i_{D-5}}+\left(\partial_0\bar{\lambda}_{i_1i_2i_3}\right)\pi^{i_1i_2i_3}+\left(\partial_0\lambda^{(2)}_{i_1\ldots i_{D-6}}\right)p^{i_1\ldots i_{D-6}}\\
&+\frac{4!}{2}\pi_{i_1\ldots i_4}\pi^{i_1\ldots i_4}-\frac{1}{2\cdot 5!}F_{i_1\ldots i_5}F^{i_1\ldots i_5}\\
&-(-)^D\frac{1}{2\cdot(D-4)!}\partial_{[i_1}V_{i_2\ldots i_{D-4}]}\partial^{[i_1}V^{i_2\ldots i_{D-4}]}\\
&-\frac{m^2}{2\cdot p!}A_{i_1\ldots i_4}A^{i_1\ldots i_4}+(-)^D\frac{(D-5)!}{2}P_{i_1\ldots i_{D-5}}P^{i_1\ldots i_{D-5}}\\
&-(-)^D\frac{m}{4!}\varepsilon_{0i_1\ldots i_4j_1\ldots j_{D-5}}A^{i_1\ldots i_4}P^{j_1\ldots j_{D-5}}\\
&-\bar{\lambda}_{i_1i_2i_3}\left(4\partial_j\pi^{ji_1i_2i_3}+\frac{m}{3!\,(D-4)!}\varepsilon^{0i_1i_2i_3k_1\ldots k_{D-4}}\partial_{[k_1}V_{k_2\ldots k_{D-4}]}\right)\\
&-\lambda^{(1)}_{i_1\ldots i_{D-6}}P^{0i_1\ldots i_{D-6}}-(-)^{D-1}(D-5)\,\lambda^{(2)}_{i_1\ldots i_{D-6}}\partial_jP^{ji_1\ldots i_{D-6}}\\
&-\Lambda_{i_1i_2i_3}\pi^{i_1i_2i_3}-\Lambda_{i_1\ldots i_{D-6}}p^{i_1\ldots i_{D-6}}\Big].
\end{aligned} \tag{31}$$

Performing in path integral some partial integrations and using the notations

$$\bar{\lambda}_{i_1i_2i_3}\equiv\bar{A}_{i_1i_2i_30},\quad \lambda^{(2)}_{i_1\ldots i_{D-6}}\equiv\bar{V}_{i_1\ldots i_{D-6}0}, \tag{32}$$

the argument of the exponential becomes

$$\begin{aligned}
S_{\mathrm{BF}} = \int d^Dx\,\Big[&-\frac{1}{2\cdot 5!}F_{i_1\ldots i_5}F^{i_1\ldots i_5}-\frac{1}{2\cdot 4!}\bar{F}_{0i_1\ldots i_4}\bar{F}^{0i_1\ldots i_4}\\
&-(-)^D\frac{1}{2\cdot(D-4)!}F_{i_1\ldots i_{D-4}}F^{i_1\ldots i_{D-4}}\\
&-(-)^D\frac{1}{2\cdot(D-5)!}F_{0i_1\ldots i_{D-5}}F^{0i_1\ldots i_{D-5}}\\
&+\frac{m}{3!\,(D-4)!}\varepsilon_{0i_1i_2i_3j_1\ldots j_{D-4}}\bar{A}^{0i_1i_2i_3}F^{j_1\ldots j_{D-4}}\\
&+\frac{m}{4!\,(D-5)!}\varepsilon_{i_1\ldots i_40j_1\ldots j_{D-5}}A^{i_1\ldots i_4}F^{0j_1\ldots j_{D-5}}\Big].
\end{aligned} \tag{33}$$

In the last functional we also used the notations

$$\bar{F}_{0i_1\ldots i_4} = \partial_0 A_{i_1\ldots i_4} + \partial_{[i_1}\bar{A}_{i_2 i_3 i_4]0}, \;\; F_{i_1\ldots i_{D-4}} = \partial_{[i_1}V_{i_2\ldots i_{D-4}]}, \tag{34}$$

$$F_{0i_1\ldots i_{D-5}} = \partial_0 V_{i_1\ldots i_{D-5}} + (-)^{D-5}\,\partial_{[i_1}\bar{V}_{i_2\ldots i_{D-5}]0}. \tag{35}$$

The functional (33) associated with the reducible first-class system takes a manifestly Lorentz-covariant form

$$S_{\mathrm{BF}} = \int d^D x \left[-\frac{1}{2\cdot 5!}\bar{F}_{\mu_1\ldots\mu_5}\bar{F}^{\mu_1\ldots\mu_5} - (-)^D \frac{1}{2\cdot(D-4)!}F_{\mu_1\ldots\mu_{D-4}}F^{\mu_1\ldots\mu_{D-4}} \right.$$
$$\left. + \frac{m}{4!\,(D-4)!}\varepsilon_{\mu_1\ldots\mu_4\nu_1\ldots\nu_{D-4}}\bar{A}^{\mu_1\ldots\mu_p}F^{\nu_1\ldots\nu_{D-4}} \right], \tag{36}$$

and describes a topological coupling between the 4-form $\bar{A}_{\mu_1\ldots\mu_4}$ and the $(D-5)$-form $\bar{V}_{\mu_1\ldots\mu_{D-5}}$ [31, 32].

3 Conclusion

In this paper we performed the path integral quantization of massive 4-forms in the framework of BF method. The strategy involved two steps. First, starting from the original second-class theory we constructed a first-class theory. Second, we built the Hamiltonian path integral corresponding to the first-class system. The BF approach to the issue of constructing a first-class theory starting from a second-class system demands an appropriate extension of the original phase-space and then the construction of a first-class constraint set and of a first-class Hamiltonian. The Hamiltonian path integral of the first-class system takes a manifestly Lorentz-covariant form after integrating out the auxiliary fields and performing some field redefinitions. We identified the Lagrangian path integral for 4- and $(D-5)$-forms with topological coupling.

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References

[1] Batalin I A and Fradkin E S 1986 *Phys. Lett.* **B180** 157

[2] Batalin I A and Fradkin E S 1990 *Phys. Lett.* **B236** 528

[3] Batalin I A and Fradkin E S 1987 *Nucl. Phys.* **B279** 514

[4] Batalin I A and Tyutin I V 1991 *Int. J. Mod. Phys.* **A6** 3255

[5] Vytheeswaran A S 1994 *Annals Phys.* **236** 297

[6] Bizdadea C and Saliu S O 1996 *Phys. Lett.* **B368** 202

[7] Bizdadea C 1996 *Phys. Rev.* **D53** 7138

[8] Bizdadea C 1996 *J. Phys. A: Math. Gen.* **29** 3985

[9] Vytheeswaran A S 1998 *Int. J. Mod. Phys.* **A13** 765

[10] Cioroianu E M, Sararu S C and Balus O 2010 *Int. J. Mod. Phys.* **A25** 185

[11] Sararu S C 2010 *Rom. J. Phys.* **55** 1076

[12] Ruegg R and Ruiz-Altaba M 2004 *Int. J. Mod. Phys.* **A19** 3265

[13] Banerjee N and Banerjee R 1996 Mod. Phys. Lett. **A11** 1919

[14] Sawayanagi H 1995 *Mod. Phys. Lett.* **A10** 813

[15] Kim Y W, Park M I, Park Y J and Yoon S J 1997 *Int. J. Mod. Phys.* **A12** 4217

[16] Banerjee N, Banerjee R and Ghosh S 1995 *Annals Phys.* **241** 237

[17] Hong S T, Kim Y W, Park Y J and Rothe K D 2002 *Mod. Phys. Lett.* **A17** 435

[18] Gaitán R 1999 *Mod. Phys. Lett.* **A14** 1383

[19] Camacaro J, Gaitán R and Leal L 1997 *Mod. Phys. Lett.* **A12** 3081

[20] Amorim R and Barcelos-Neto J 2002 *Braz. J. Phys.* **32** 227

[21] Allen T J, Bowick M J and Lahiri A 1991 *Mod. Phys. Lett.* **A6** 559

[22] Green M B, Schwarz J H and E. Witten 1987 *Superstring Theory* (Cambridge: Cambridge University Press)

[23] Birmingham D, Blau M, Rakowski M and Thompson G 1991 *Phys. Rept.* **209** 129

[24] Salam A and Sezgin E 1989 *Supergravities in Diverse Dimensions* (Amsterdam: North Holland/Singapore: World Scientific)

[25] Bautier K, Deser S, Henneaux M and Seminara D 1997 *Phys. Lett.* B **406** 49

[26] Ne'eman Y and Eizenberg E 1995 *Membranes and Other Extendons (p-branes)* (Singapore: World Scientific)

[27] Cioroianu E M 2011 *Mod. Phys. Lett.* A **26** 589.

[28] Henneaux M and Teitelboim C 1992 *Quantization of Gauge Systems* (Princeton: Princeton University Press).

[29] Batalin I A and Vilkovisky G A 1977 *Phys. Lett.* B **69** 309

[30] Ferraro R, Henneaux H and Puchin M 1993 *J. Math. Phys.* **34** 2757

[31] Cremmer E and Scherk J 1974 *Nucl. Phys.* B **72** 117

[32] Bizdadea C, Cioroianu E M and Saliu S O 1999 *Phys. Scripta* **60** 120