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No cross-couplings between a collection of massless tensors with the mixed symmetry $(2, 2)$ and a Pauli–Fierz field

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Abstract

Under the hypotheses of analyticity, locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the requirement that the interaction vertices contain at most two space-time derivatives of the fields, we investigate the consistent cross-couplings that can be added between a collection of massless tensor fields with the mixed symmetry $(2, 2)$ and a Pauli–Fierz field. The computations are done with the help of the deformation theory based on a cohomological approach, in the context of the antifield-BRST formalism. Our final result is that no cross-couplings are possible.

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In the past years tensor fields in exotic representations of the Lorentz group [1]–[7] have been proved extremely useful in the dual formulation of field theories of spin two or higher [8]–[14], in showing the impossibility of consistent cross-interactions in the dual formulation of linearized gravity [15], or in the derivation of some exotic gravitational interactions [16, 17]. An important matter related to mixed symmetry type tensor fields is the study of their consistent interactions, among themselves as well as with higher-spin gauge theories [18]–[26]. The most efficient approach to this problem is the cohomological one, based on the deformation of the solution to the master equation [27].

The purpose of this paper is to investigate the consistent cross-couplings between a collection of massless tensor gauge fields with the mixed symmetry of the Riemann tensor and a Pauli–Fierz field. Our analysis relies on the deformation of the solution to the master equation by means of cohomological techniques with the help of the local BRST cohomology, whose component for a collection of $(2, 2)$ fields has been considered in [28] and in the Pauli–Fierz sector has been investigated in [29]. Under the hypotheses of analyticity in the coupling constant, locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the preservation of the number of derivatives on each field, we find that no cross-couplings can be added to the original Lagrangian action.

The starting point is given by the Lagrangian action for a finite collection of free, massless tensor fields with the mixed symmetry of the Riemann tensor and for a Pauli–Fierz field in $D \geq 5$

$$S_0 \left[r^a_{\mu\nu|\alpha\beta}, h_{\mu\nu}\right] = S_0^{\mathrm{r}} \left[r^a_{\mu\nu|\alpha\beta}\right] + S_0^{\mathrm{PF}} \left[h_{\mu\nu}\right], \tag{1}$$

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where

$$S_0^{\mathrm{r}}\left[r^a_{\mu\nu|\alpha\beta}\right]=\int\left\{\frac{1}{8}\left[\left(\partial^\lambda r_a^{\mu\nu|\alpha\beta}\right)\left(\partial_\lambda r^a_{\mu\nu|\alpha\beta}\right)+\left(\partial^\lambda r_a\right)\left(\partial_\lambda r^a\right)\right]\right.$$
$$-\frac{1}{2}\left[\left(\partial_\mu r_a^{\mu\nu|\alpha\beta}\right)\left(\partial^\lambda r^a_{\lambda\nu|\alpha\beta}\right)+\left(\partial^\lambda r_a^{\nu\beta}\right)\left(\partial_\lambda r^a_{\nu\beta}\right)+\left(\partial_\nu r_a^{\nu\beta}\right)\left(\partial_\beta r^a\right)\right]$$
$$\left.-\left(\partial_\mu r_a^{\mu\nu|\alpha\beta}\right)\left(\partial_\beta r^a_{\nu\alpha}\right)+\left(\partial_\nu r_a^{\nu\beta}\right)\left(\partial^\lambda r^a_{\lambda\beta}\right)\right\}d^Dx, \tag{2}$$

$$S_0^{\mathrm{PF}}\left[h_{\mu\nu}\right]=\int\left\{-\frac{1}{2}\left[\left(\partial^\rho h^{\mu\nu}\right)\left(\partial_\rho h_{\mu\nu}\right)-\left(\partial^\rho h\right)\left(\partial_\rho h\right)\right]\right.$$
$$\left.\left(\partial_\rho h^{\rho\mu}\right)\left(\partial^\lambda h_{\lambda\mu}\right)-\left(\partial^\rho h\right)\left(\partial^\lambda h_{\lambda\rho}\right)\right\}d^Dx. \tag{3}$$

We employ the flat Minkowski metric of 'mostly plus' signature $\sigma^{\mu\nu}=\sigma_{\mu\nu}=(-+++ +\ldots)$. The lowercase indices a, b, etc. stand for the collection indices and are assumed to take discrete values 1, 2, ..., n. They are lowered with a symmetric, constant and invertible matrix, of elements k_{ab}, and are raised with the help of the elements k^{ab} of its inverse. Each tensor field $r^a_{\mu\nu|\alpha\beta}$ exhibits the mixed symmetry of the Riemann tensor, so it is separately antisymmetric in the pairs $\{\mu,\nu\}$ and $\{\alpha,\beta\}$, is symmetric under their permutation ($\{\mu,\nu\}\longleftrightarrow\{\alpha,\beta\}$), and satisfies the identity $r^a_{[\mu\nu|\alpha]\beta}\equiv 0$. The notations $r^a_{\nu\beta}$ signify the traces of $r^a_{\mu\nu|\alpha\beta}$, $r^a_{\nu\beta}=\sigma^{\mu\alpha}r^a_{\mu\nu|\alpha\beta}$, which are symmetric, $r^a_{\nu\beta}=r^a_{\beta\nu}$, while r^a represent their double traces , $r^a=\sigma^{\nu\beta}r^a_{\nu\beta}$, which are scalars. The Pauli–Fierz field $h_{\mu\nu}$ is symmetric and h denotes its trace. Action (1) admits a generating set of gauge transformations of the form

$$\delta_\xi r^a_{\mu\nu|\alpha\beta}=\partial_\mu\xi^a_{\alpha\beta|\nu}-\partial_\nu\xi^a_{\alpha\beta|\mu}+\partial_\alpha\xi^a_{\mu\nu|\beta}-\partial_\beta\xi^a_{\mu\nu|\alpha},\quad \delta_\epsilon h_{\mu\nu}=\partial_{(\mu}\epsilon_{\nu)}, \tag{4}$$

where the gauge parameters $\xi^a_{\mu\nu|\alpha}$ and ϵ_ν are arbitrary bosonic tensors, with $\xi^a_{\mu\nu|\alpha}$ displaying the mixed symmetry $(2,1)$. The gauge transformations from (4) are Abelian and off-shell, first-order reducible. Consequently, the Cauchy order of this linear gauge theory is equal to three.

Related to the generators of the BRST algebra, the ghost spectrum contains the fermionic ghosts $\mathcal{C}^a_{\alpha\beta|\mu}$ and η_μ associated with the gauge parameters and the bosonic ghosts for ghosts $\mathcal{C}^a_{\mu\nu}$ corresponding to the first-order reducibility. Obviously, we will require that $\mathcal{C}^a_{\alpha\beta|\mu}$ preserve the mixed symmetry $(2,1)$ and the tensors $\mathcal{C}^a_{\mu\nu}$ remain antisymmetric. The antifield spectrum comprises the antifields $r_a^{*\mu\nu|\alpha\beta}$ and $h^{*\mu\nu}$ associated with the original fields and those corresponding to the ghosts, $\mathcal{C}_a^{*\mu\nu|\alpha}$, $\eta^{*\mu}$, and $\mathcal{C}_a^{*\mu\nu}$. The antifields $r_a^{*\mu\nu|\alpha\beta}$ still have the mixed symmetry $(2,2)$, $h^{*\mu\nu}$ are symmetric, $\mathcal{C}_a^{*\mu\nu|\alpha}$ exhibit the mixed symmetry $(2,1)$, and $\mathcal{C}_a^{*\mu\nu}$ are antisymmetric. Related to the traces of $r_a^{*\mu\nu|\alpha\beta}$ and $h^{*\mu\nu}$, we will use the notations $r_a^{*\nu\beta}=\sigma_{\mu\alpha}r_a^{*\mu\nu|\alpha\beta}$, $r_a^*=\sigma_{\nu\beta}r_a^{*\nu\beta}$, and h^*.

The BRST differential decomposes in the sum between the Koszul–Tate differential and the exterior longitudinal differential, $s=\delta+\gamma$, the corresponding degrees of the generators from the BRST complex being valued like

$$\mathrm{pgh}\left(r^a_{\mu\nu|\alpha\beta}\right)=0=\mathrm{pgh}\left(h_{\mu\nu}\right),\quad \mathrm{pgh}\left(\mathcal{C}^a_{\mu\nu|\alpha}\right)=1=\mathrm{pgh}\left(\eta_\mu\right),$$
$$\mathrm{pgh}\left(\mathcal{C}^a_{\mu\nu}\right)=2,\quad \mathrm{pgh}\left(r_a^{*\mu\nu|\alpha\beta}\right)=0=\mathrm{pgh}\left(h^{*\mu\nu}\right),$$
$$\mathrm{pgh}\left(\mathcal{C}_a^{*\mu\nu|\alpha}\right)=\mathrm{pgh}\left(\eta^{*\mu}\right)=\mathrm{pgh}\left(\mathcal{C}_a^{*\mu\nu}\right)=0,$$
$$\mathrm{agh}\left(r^a_{\mu\nu|\alpha\beta}\right)=0=\mathrm{agh}\left(h_{\mu\nu}\right),\quad \mathrm{agh}\left(\mathcal{C}^a_{\mu\nu|\alpha}\right)=0=\mathrm{agh}\left(\eta_\mu\right),$$

$$\mathrm{agh}\left(\mathcal{C}^{a}_{\mu\nu}\right)=0,\quad \mathrm{agh}\left(r^{*\mu\nu|\alpha\beta}_{a}\right)=1=\mathrm{agh}\left(h^{*\mu\nu}\right),$$
$$\mathrm{agh}\left(\mathcal{C}^{*\mu\nu|\alpha}_{a}\right)=2=\mathrm{agh}\left(\eta^{*\mu}\right),\quad \mathrm{agh}\left(\mathcal{C}^{*\mu\nu}_{a}\right)=3.$$

The actions of δ and γ on the generators from the BRST complex, which enforce the standard BRST properties, are given by

$$\gamma r^{a}_{\mu\nu|\alpha\beta}=\mathcal{C}^{a}_{\alpha\beta|[\nu,\mu]}+\mathcal{C}^{a}_{\mu\nu|[\beta,\alpha]},\quad \gamma h_{\mu\nu}=\partial_{(\mu}\eta_{\nu)}, \tag{5}$$

$$\gamma\mathcal{C}^{a}_{\mu\nu|\alpha}=2\partial_{\alpha}\mathcal{C}^{a}_{\mu\nu}-\partial_{[\mu}\mathcal{C}^{a}_{\nu]\alpha},\quad \gamma\eta_{\mu}=0=\gamma\mathcal{C}^{a}_{\mu\nu}, \tag{6}$$

$$\gamma r^{*\mu\nu|\alpha\beta}_{a}=0=\gamma h^{*\mu\nu},\quad \gamma\mathcal{C}^{*\mu\nu|\alpha}_{a}=0=\gamma\eta^{*\mu},\quad \gamma\mathcal{C}^{*\mu\nu}_{a}=0, \tag{7}$$

$$\delta r^{a}_{\mu\nu|\alpha\beta}=0=\delta h_{\mu\nu},\quad \delta\mathcal{C}^{a}_{\mu\nu|\alpha}=0=\delta\eta_{\mu},\quad \delta\mathcal{C}^{a}_{\mu\nu}=0, \tag{8}$$

$$\delta r^{*\mu\nu|\alpha\beta}_{a}=\frac{1}{4}R^{\mu\nu|\alpha\beta}_{a},\quad \delta h^{*\mu\nu}=2H^{\mu\nu},\quad \delta\eta^{*\mu}=-2\partial_{\nu}h^{*\nu\mu}, \tag{9}$$

$$\delta\mathcal{C}^{*\alpha\beta|\nu}_{a}=-4\partial_{\mu}r^{*\mu\nu|\alpha\beta}_{a},\quad \delta\mathcal{C}^{*\mu\nu}_{a}=3\partial_{\alpha}\mathcal{C}^{*\mu\nu|\alpha}_{a}. \tag{10}$$

In the above $R^{\mu\nu|\alpha\beta}_{a}$ is defined by $\delta S^{\mathrm{r}}_{0}/\delta r^{\mu\nu|\alpha\beta}_{a}\equiv-\left(1/4\right)R^{a}_{\mu\nu|\alpha\beta}$ and $H_{\mu\nu}=-(1/2)\delta S^{\mathrm{PF}}_{0}/\delta h^{\mu\nu}$ represents the components of the linearized Einstein tensor

$$H_{\mu\nu}=K_{\mu\nu}-\frac{1}{2}\sigma_{\mu\nu}K,\quad H_{\mu\nu}=H_{\nu\mu}, \tag{11}$$

with $K_{\mu\nu}$ the linearized Ricci tensor and K the linearized scalar curvature, which are defined with the help of the linearized Riemann tensor

$$K_{\mu\nu|\alpha\beta}=-\frac{1}{2}\left(\partial_{\mu}\partial_{\alpha}h_{\nu\beta}-\partial_{\nu}\partial_{\alpha}h_{\mu\beta}-\partial_{\mu}\partial_{\beta}h_{\nu\alpha}+\partial_{\nu}\partial_{\beta}h_{\mu\alpha}\right) \tag{12}$$

via its trace and respectively its second-order contraction $K_{\mu\nu}=K^{\alpha}{}_{\mu|\alpha\nu}$, $K=K^{\mu}{}_{\mu}$.

The solution to the classical master equation for the free model under study reduces to the sum between the solutions in the two sectors

$$S=S^{\mathrm{r}}+S^{\mathrm{h}}, \tag{13}$$

where

$$\begin{aligned}S^{\mathrm{r}}=S^{\mathrm{r}}_{0}\left[r^{a}_{\mu\nu|\alpha\beta}\right]+\int&\left[r^{*\mu\nu|\alpha\beta}_{a}\left(\partial_{\mu}\mathcal{C}^{a}_{\alpha\beta|\nu}-\partial_{\nu}\mathcal{C}^{a}_{\alpha\beta|\mu}+\partial_{\alpha}\mathcal{C}^{a}_{\mu\nu|\beta}-\partial_{\beta}\mathcal{C}^{a}_{\mu\nu|\alpha}\right)\right.\\&\left.+\mathcal{C}^{*\mu\nu|\alpha}_{a}\left(2\partial_{\alpha}\mathcal{C}^{a}_{\mu\nu}-\partial_{[\mu}\mathcal{C}^{a}_{\nu]\alpha}\right)\right]d^{D}x,\end{aligned} \tag{14}$$

$$S^{\mathrm{h}}=S^{\mathrm{PF}}_{0}\left[h_{\mu\nu}\right]+\int h^{*\mu\nu}\partial_{(\mu}\eta_{\nu)}d^{D}x. \tag{15}$$

The reformulation of the problem of consistent deformations of a given action and of its gauge symmetries in the antifield-BRST setting is based on the observation that if a deformation of the classical theory can be consistently constructed, then the solution S to the master equation for the initial theory can be deformed into the solution $\bar{S}$ of the master equation for the interacting theory

$$S\longrightarrow\bar{S}=S+gS_{1}+g^{2}S_{2}+g^{3}S_{3}+g^{4}S_{4}+\cdots, \tag{16}$$

$$\left(S,S\right)=0\longrightarrow\left(\bar{S},\bar{S}\right)=0. \tag{17}$$

The projection of (17) for $\bar{S}$ on the various powers of the coupling constant induces the following tower of equations:

$$g^0 : (S, S) = 0, \tag{18}$$
$$g^1 : (S_1, S) = 0, \tag{19}$$
$$g^2 : (S_2, S) + \frac{1}{2}(S_1, S_1) = 0, \tag{20}$$
$$g^3 : (S_3, S) + (S_1, S_2) = 0, \tag{21}$$
$$\vdots$$

In the sequel we compute all consistent interactions that can be added to the free action (1) by solving the deformation equations (19)–(21), etc., by means of specific cohomological techniques, under the general hypotheses mentioned in the introductory paragraph.

In order to analyze equation (19) satisfied by the non-integrated density of the first-order deformation a ($S_1 = \int a d^D x$), $sa = \partial_\mu m^\mu$, it is convenient to split the first-order deformation into

$$a = a^{\mathrm{h}} + a^{\mathrm{r}} + a^{\mathrm{int}}, \tag{22}$$

where a^{h} denotes the part responsible for the self-interactions of the Pauli–Fierz field, a^{r} is related to the deformations of the tensor fields $r^a_{\mu\nu|\alpha\beta}$, and a^{int} signifies the component that describes only the cross-interactions between $h_{\mu\nu}$ and $r^a_{\mu\nu|\alpha\beta}$. Then, a^{h} is completely known (for a detailed analysis, see for instance [29])

$$a^{\mathrm{h}} = a_0^{\mathrm{h}} + a_1^{\mathrm{h}} + a_2^{\mathrm{h}}, \tag{23}$$

where

$$a_2^{\mathrm{h}} = \eta^{*\mu}\eta^\alpha \partial_\mu \eta_\alpha, \tag{24}$$
$$a_1^{\mathrm{h}} = -h^{*\mu\nu}\eta^\alpha \left(\partial_\mu h_{\nu\alpha} + \partial_\nu h_{\mu\alpha} - \partial_\alpha h_{\mu\nu}\right), \tag{25}$$

and a_0^{h} is the cubic vertex of the Einstein–Hilbert Lagrangian plus a cosmological term. The piece a^{r} has been computed in [28] and is given by

$$a^{\mathrm{r}} = \sum_{a=1}^{n} c_a r^a, \tag{26}$$

with c_a some real, arbitrary constants.

We ensure the space-time locality of the deformations by working in the algebra of local differential forms with coefficients that are polynomial functions in the fields, ghosts, antifields, and their space-time derivatives (algebra of local forms), meaning that the non-integrated density of the first-order deformation, a, is a polynomial function in all these variables (algebra of local functions). Inserting (22) into the equation $sa = \partial_\mu m^\mu$ and using the fact that the first two components already obey the equations $sa^{\mathrm{h}} = \partial_\mu m^\mu_{\mathrm{h}}$ and $sa^{\mathrm{r}} = \partial_\mu m^\mu_{\mathrm{r}}$, it follows that only a^{int} is unknown, being subject to the equation

$$sa^{\mathrm{int}} = \partial_\mu m^\mu_{\mathrm{int}}. \tag{27}$$

Next, we develop a^{int} according to the antighost number and assume that this expansion contains a finite number of terms, with the maximum value of the antighost number equal to I. Due to the decomposition $s = \delta + \gamma$, this equation becomes equivalent to the chain

$$\gamma a_I^{\mathrm{int}} = 0, \quad I > 0, \tag{28}$$

$$\delta a_I^{\text{int}} + \gamma a_{I-1}^{\text{int}} = \partial_\mu \overset{(I-1)^\mu}{m}_{\text{int}}, \tag{29}$$

$$\delta a_k^{\text{int}} + \gamma a_{k-1}^{\text{int}} = \partial_\mu \overset{(k-1)^\mu}{m}_{\text{int}}, \quad I-1 \geq k \geq 1, \tag{30}$$

where $\left(\overset{(k)^\mu}{m}_{\text{int}}\right)_{k=\overline{0,I}}$ are some local currents, with agh $\left(\overset{(k)^\mu}{m}_{\text{int}}\right) = k$. In conclusion, for $I > 0$ we have that $a_I^{\text{int}} \in H^*(\gamma)$.

Initially, we compute the cohomology $H^*(\gamma)$ in the algebra of local functions. Due to the fact that the exterior longitudinal differential γ splits as

$$\gamma = \gamma_{\text{r}} + \gamma_{\text{h}}, \tag{31}$$

where γ_{r} acts non-trivially only in the $(2,2)$ sector and γ_{h} does the same, but in the Pauli–Fierz sector, Künneth's Theorem for cohomologies ensure that

$$H^*(\gamma) = H^*(\gamma_{\text{r}}) \otimes H^*(\gamma_{\text{h}}). \tag{32}$$

Combining the results from [28] and [29] on $H^*(\gamma_{\text{r}})$ and respectively on $H^*(\gamma_{\text{h}})$, it follows that the general solution to (28) reads

$$a_I^{\text{int}} = \alpha_I\left(\left[\omega^{*\Theta}\right], \left[F^a_{\mu\nu\lambda|\alpha\beta\gamma}\right], \left[K_{\mu\nu|\alpha\beta}\right]\right)\omega^I\left(\eta_\mu, \partial_{[\mu}\eta_{\nu]}, \mathcal{C}^a_{\mu\nu}, \partial_{[\mu}\mathcal{C}^a_{\nu\alpha]}\right), \tag{33}$$

where

$$\omega^{*\Theta} = \left(r_a^{*\mu\nu|\alpha\beta}, h^{*\mu\nu}, \mathcal{C}_a^{*\mu\nu|\alpha}, \eta^{*\mu}, \mathcal{C}_a^{*\mu\nu}\right), \tag{34}$$

$F^a_{\mu\nu\lambda|\alpha\beta\gamma}$ stand for the curvature tensors in the $(2,2)$ sector

$$\begin{aligned} F^a_{\mu\nu\lambda|\alpha\beta\gamma} = {} & \partial_\lambda\partial_\gamma r^a_{\mu\nu|\alpha\beta} + \partial_\mu\partial_\gamma r^a_{\nu\lambda|\alpha\beta} + \partial_\nu\partial_\gamma r^a_{\lambda\mu|\alpha\beta} \\ & + \partial_\lambda\partial_\alpha r^a_{\mu\nu|\beta\gamma} + \partial_\mu\partial_\alpha r^a_{\nu\lambda|\beta\gamma} + \partial_\nu\partial_\alpha r^a_{\lambda\mu|\beta\gamma} \\ & + \partial_\lambda\partial_\beta r^a_{\mu\nu|\gamma\alpha} + \partial_\mu\partial_\beta r^a_{\nu\lambda|\gamma\alpha} + \partial_\nu\partial_\beta r^a_{\lambda\mu|\gamma\alpha}, \end{aligned} \tag{35}$$

and $K_{\mu\nu|\alpha\beta}$ is the linearized Riemann tensor (12). The notation $f([q])$ means that f depends on q and its subsequent derivatives.

In fact, the coefficients $\alpha_I\left(\left[\omega^{*\Theta}\right], \left[F^a_{\mu\nu\lambda|\alpha\beta\gamma}\right], \left[K_{\mu\nu|\alpha\beta}\right]\right)$ are nothing but the invariant polynomials (in form degree zero) of the theory (1). The notation ω^I signifies the elements of pure ghost number equal to I of a basis in the space of polynomials in η_μ, $\partial_{[\mu}\eta_{\nu]}$, $\mathcal{C}^a_{\mu\nu}$, and $\partial_{[\mu}\mathcal{C}^a_{\nu\alpha]}$.

Substituting solution (33) into the next equation, (29), we obtain that the existence of non-trivial solutions a_{I-1}^{int} to equation (29) for $I > 0$ is that the invariant polynomials α_I appearing in (33) generate non-trivial elements from $H_I^D(\delta|d)$. Taking into account the fact that the maximum Cauchy order of the free gauge theory (1) is equal to three, we have that [30]

$$H_k^D(\delta|d) = 0, \quad k > 3. \tag{36}$$

Meantime, it can be proven that

$$H_k^{\text{inv}D}(\delta|d) = 0, \quad k > 3, \tag{37}$$

where $H_k^{\text{inv}D}(\delta|d)$ denotes the invariant characteristic cohomology in antighost number k. On account of the general results from [28] and [29] on the invariant characteristic cohomology, we are able to identify the non-trivial representatives of $\left(H_k^D\left(\delta|d\right)\right)_{k\geq 2}$, as well as of $\left(H_k^{\mathrm{inv}D}\left(\delta|d\right)\right)_{k\geq 2}$, under the form

$$\begin{array}{cc} \text{agh} & \begin{array}{c}\text{non}-\text{trivial representatives}\\ \text{spanning } H_k^D\left(\delta|d\right) \text{ and } H_k^{\mathrm{inv}D}\left(\delta|d\right)\end{array} \\ k>3 & \text{none} \\ k=3 & \mathcal{C}_a^{*\mu\nu} \\ k=2 & \mathcal{C}_a^{*\mu\nu|\alpha}, \eta^{*\mu} \end{array} . \tag{38}$$

The previous results on $H_I^D\left(\delta|d\right)$ and $H_I^{\mathrm{inv}D}\left(\delta|d\right)$ allow us to eliminate successively all the terms of antighost number $k>3$ from the non-integrated density of the first-order deformation. The last representative is of the form (33), where the invariant polynomials necessarily define non-trivial elements from $H_I^{\mathrm{inv}D}\left(\delta|d\right)$ if $I=2,3$ or respectively from $H_1^D\left(\delta|d\right)$ if $I=1$.

In view of the above considerations we can assume that the first-order deformation stops at $I=3$

$$a^{\mathrm{int}} = a_0^{\mathrm{int}} + a_1^{\mathrm{int}} + a_2^{\mathrm{int}} + a_3^{\mathrm{int}}, \tag{39}$$

where a_3^{int} is of the form (33) for $I=3$. At this point we enforce the assumption on the maximum derivative order of the corresponding a_0^{int} to be equal to two. Using the result that the most general representative of $H_3^{\mathrm{inv}D}\left(\delta|d\right)$ are the undifferentiated antifields $\mathcal{C}_a^{*\alpha\beta}$ (see (38) for $k=3$) and that the elements of pure ghost number three that fulfill the condition on the maximum derivative order are given by

$$\left(\eta_\mu\eta_\nu\eta_\rho, \eta_\mu\eta_\nu\partial_{[\rho}\eta_{\lambda]}, \mathcal{C}_{\mu\nu}^b\eta_\rho, \mathcal{C}_{\mu\nu}^b\partial_{[\rho}\eta_{\lambda]}, \partial_{[\mu}\mathcal{C}_{\nu\rho]}^b\eta_\lambda\right), \tag{40}$$

we can write down that the general solution to equation (28) for $I=3$ like

$$\begin{aligned} a_3^{\mathrm{int}} = {} & \mathcal{C}_a^{*\alpha\beta}\left(f_{1\alpha\beta}^{a\mu\nu\rho}\eta_\mu\eta_\nu\eta_\rho + f_{2\alpha\beta}^{a\mu\nu\rho\lambda}\eta_\mu\eta_\nu\partial_{[\rho}\eta_{\lambda]} + g_{1b\alpha\beta}^{a\mu\nu\rho}\mathcal{C}_{\mu\nu}^b\eta_\rho\right. \\ & \left. + g_{2b\alpha\beta}^{a\mu\nu\rho\lambda}\mathcal{C}_{\mu\nu}^b\partial_{[\rho}\eta_{\lambda]} + g_{3b\alpha\beta}^{a\mu\nu\rho\lambda}\partial_{[\mu}\mathcal{C}_{\nu\rho]}^b\eta_\lambda\right) + \gamma b_3, \end{aligned} \tag{41}$$

where all the coefficients of the type f and g are required to be non-derivative constants. Combining this result with the symmetries of the various coefficients due to the corresponding symmetries of the antifield and of the ghosts, we remain with the following independent possibilities in $D\geq 5$ space-time dimensions:

$$a_3^{\mathrm{int}} = a_3^{(1)\mathrm{int}} + a_3^{(2)\mathrm{int}} + a_3^{(3)\mathrm{int}}, \tag{42}$$

where

$$D=5, \quad a_3^{(1)\mathrm{int}} = \varepsilon^{\alpha\beta\mu\nu\rho}\mathcal{C}_{\alpha\beta}^{*a}\left(c_{1a}\eta_\mu\eta_\nu\eta_\rho + d_{1ab}\mathcal{C}_{\mu\nu}^b\eta_\rho\right) + \gamma b_3^{(1)}, \tag{43}$$

$$\begin{aligned} D=6, \quad a_3^{(2)\mathrm{int}} = {} & \varepsilon^{\alpha\beta\mu\nu\rho\lambda}\mathcal{C}_{\alpha\beta}^{*a}\left(c_{2a}\eta_\mu\eta_\nu\partial_{[\rho}\eta_{\lambda]}\right. \\ & \left. + d_{2ab}\mathcal{C}_{\mu\nu}^b\partial_{[\rho}\eta_{\lambda]} + d_{3ab}\partial_{[\mu}\mathcal{C}_{\nu\rho]}^b\eta_\lambda\right) + \gamma b_3^{(2)}, \end{aligned} \tag{44}$$

$$\begin{aligned} D\geq 5, \quad a_3^{(3)\mathrm{int}} = {} & \mathcal{C}_a^{*\alpha\beta}\left(c_3^a\eta_\alpha\eta^\rho\partial_{[\beta}\eta_{\rho]} + d_{4b}^a\mathcal{C}_\alpha^{b\ \rho}\partial_{[\rho}\eta_{\beta]}\right. \\ & \left. + d_{5b}^a\partial_{[\alpha}\mathcal{C}_{\beta\rho]}^b\eta^\rho + d_{6a}^b\partial_{[\rho}\mathcal{C}_{\alpha\beta]}^b\eta^\rho\right) + \gamma b_3^{(3)}. \end{aligned} \tag{45}$$

In the above all quantities of the type c or d are real constants. Obviously, since $a_3^{\rm int}$ is subject to equation (29) for $I=3$ and components (43)–(45) are mutually independent, it follows that each of them must separately fulfill such an equation, i.e.,

$$\delta a_3^{(i){\rm int}} = -\gamma a_2^{(i){\rm int}} + \partial_\mu m_{\rm int}^{(i)\mu}, \quad i=1,2,3. \tag{46}$$

By computing the action of δ on $\left(a_3^{(i){\rm int}}\right)_{i=1,2,3}$ and using definitions (5)–(10), we infer that none of them can be written like in the right-hand side of (46), no matter what $\left(b_3^{(i)}\right)_{i=1,2,3}$ we take in the right-hand side of (43)–(45), such that we must set all the nine types constants equal to zero

$$c_{ma}=0, \quad m=1,2,3, \quad d_{nab}=0, \quad n=1,2,3,4,5,6, \tag{47}$$

and so $a_3^{\rm int}=0$.

We pass to the next eligible value ($I=2$) and write

$$a^{\rm int} = a_0^{\rm int} + a_1^{\rm int} + a_2^{\rm int}. \tag{48}$$

Repeating the reasoning developed in the above, we obtain that $a_2^{\rm int}$ is, up to trivial, γ-exact contributions, of the form (33) for $I=2$, with the elements of pure ghost number two obeying the assumption on the maximum number of derivatives from the corresponding $a_0^{\rm int}$ being equal to two expressed by

$$\left(\eta_\mu\eta_\nu, \eta_\mu\partial_{[\nu}\eta_{\rho]}, \mathcal{C}^a_{\mu\nu}, \partial_{[\mu}\mathcal{C}^a_{\nu\rho]}\right). \tag{49}$$

Using the fact that the general representative of $H_2^{{\rm inv}D}(\delta|d)$ is spanned in this situation by the undifferentiated antifields $\mathcal{C}_a^{*\alpha\beta|\gamma}$ and $\eta^{*\alpha}$ (see (38) for $k=2$), to which we add the requirement that $a_2^{\rm int}$ comprises only terms that effectively mix the ghost/antifield sectors of the starting free theories, and combining these with , we obtain that

$$\begin{aligned} a_2^{\rm int} = {} & \mathcal{C}_a^{*\alpha\beta|\gamma}\left(g_{1\alpha\beta\gamma}^{a\mu\nu}\eta_\mu\eta_\nu + g_{2\alpha\beta\gamma}^{a\mu\nu\rho}\eta_\mu\partial_{[\nu}\eta_{\rho]}\right) \\ & + \eta^{*\alpha}\left(g_{3b\alpha}^{\mu\nu}\mathcal{C}^b_{\mu\nu} + g_{4b\alpha}^{\mu\nu\rho}\partial_{[\mu}\mathcal{C}^b_{\nu\rho]}\right) + \gamma b_2, \end{aligned} \tag{50}$$

where the coefficients denoted by g are imposed to be non-derivative constants. Taking into account the identity $\mathcal{C}^{*[\alpha\beta|\gamma]}\equiv 0$ and the hypothesis that we work only in $D\geq 5$ space-time dimensions, we arrive at

$$a_2^{\rm int} = \frac{c'^a}{2}\mathcal{C}_a^{*\alpha\beta|\mu}\partial_{[\alpha}\eta_{\beta]}\eta_\mu + \frac{c''^a}{2}\mathcal{C}_a^{*\alpha\beta|}{}_\beta\partial_{[\alpha}\eta_{\mu]}\eta^\mu + \gamma b_2. \tag{51}$$

We will analyze these terms separately. The first one leads to non-vanishing components of antighost number one and respectively zero as solutions to the equations

$$\delta a_2'^{\rm int} + \gamma a_1'^{\rm int} = \partial_\mu \overset{(1)}{m}{}'^{\mu}_{\rm int}, \quad \delta a_1'^{\rm int} + \gamma a_0'^{\rm int} = \partial_\mu \overset{(0)}{m}{}'^{\mu}_{\rm int}, \tag{52}$$

where we made the notation

$$a_2'^{\rm int} = \frac{c'^a}{2}\mathcal{C}_a^{*\alpha\beta|\mu}\partial_{[\alpha}\eta_{\beta]}\eta_\mu. \tag{53}$$

Indeed, straightforward calculations output

$$a_1'^{\rm int} = \frac{c'^a}{2}r_a^{*\mu\nu|\alpha\beta}\left[\left(\partial_\mu h_{\nu\alpha} - \partial_\nu h_{\mu\alpha}\right)\eta_\beta + \left(\partial_\alpha h_{\beta\mu} - \partial_\beta h_{\alpha\mu}\right)\eta_\nu\right.$$

$$-\left(\partial_{\mu}h_{\nu\beta}-\partial_{\nu}h_{\mu\beta}\right)\eta_{\alpha}-\left(\partial_{\alpha}h_{\beta\nu}-\partial_{\beta}h_{\alpha\nu}\right)\eta_{\mu}\Big], \tag{54}$$

$$a_{0}^{\prime\mathrm{int}}=\frac{c^{\prime a}}{8}R_{a}^{\mu\nu|\alpha\beta}\left(h_{\mu\alpha}h_{\nu\beta}-h_{\mu\beta}h_{\nu\alpha}\right). \tag{55}$$

In consequence, we obtained a possible form of the first-order deformation for the cross-interactions between the Pauli–Fierz theory and the tensor fields $r_{\mu\nu|\alpha\beta}^{a}$ like

$$a^{\prime\mathrm{int}}=a_{0}^{\prime\mathrm{int}}+a_{1}^{\prime\mathrm{int}}+a_{2}^{\prime\mathrm{int}}, \tag{56}$$

where the quantities in the right-hand side of (56) are expressed by (53)–(55). However, $a^{\prime\mathrm{int}}$ is trivial in the context of the overall non-integrated density a^{int} of the first-order deformation in the sense that it is in a trivial class of the local cohomology of the free BRST differential $H^{0,D}\left(s|d\right)$. Indeed, one can check that it can be put in a s-exact modulo d form

$$a^{\prime\mathrm{int}}=c^{\prime a}s\left[\frac{1}{3}\mathcal{C}_{a}^{*\mu\nu}\eta_{\mu}\eta_{\nu}-\frac{1}{2}\mathcal{C}_{a}^{*\alpha\beta|\mu}\left(h_{\alpha\mu}\eta_{\beta}-h_{\beta\mu}\eta_{\alpha}\right)\right.$$
$$\left.+\frac{1}{2}r_{a}^{*\mu\nu|\alpha\beta}\left(h_{\mu\alpha}h_{\nu\beta}-h_{\mu\beta}h_{\nu\alpha}\right)\right]+\partial_{\mu}l^{\mu}, \tag{57}$$

and so it can be eliminated from a^{int} by setting

$$c^{\prime a}=0. \tag{58}$$

The second piece in (51), which is clearly non-trivial, appears to be more interesting. Indeed, let us fix the trivial (γ-exact) contribution from the right-hand side of (51) to

$$b_{2}=\frac{c^{\prime\prime a}}{2}\mathcal{C}_{a}^{*\alpha\beta|}{}_{\beta}h_{\alpha\gamma}\eta^{\gamma}, \tag{59}$$

which is equivalent to starting from

$$a_{2}^{\prime\prime\mathrm{int}}=c^{\prime\prime a}\mathcal{C}_{a}^{*\alpha\beta|}{}_{\beta}\left(\partial_{\alpha}\eta_{\mu}\right)\eta^{\mu}. \tag{60}$$

Then, it yields the component of antighost one as solution to the equation $\delta a_{2}^{\prime\prime\mathrm{int}}+\gamma a_{1}^{\prime\prime\mathrm{int}}=\partial_{\mu}\overset{(1)}{m}{}_{\mathrm{int}}^{\prime\prime\mu}$ in the form

$$a_{1}^{\prime\prime\mathrm{int}}=2c^{\prime\prime a}r_{a}^{*\mu\alpha}\left(\partial_{\mu}h_{\alpha\lambda}+\partial_{\alpha}h_{\mu\lambda}-\partial_{\lambda}h_{\mu\alpha}\right)\eta^{\lambda}. \tag{61}$$

Next, we pass to the equation

$$\delta a_{1}^{\prime\prime\mathrm{int}}+\gamma a_{0}^{\prime\prime\mathrm{int}}=\partial_{\mu}\overset{(0)}{m}{}_{\mathrm{int}}^{\prime\prime\mu}, \tag{62}$$

where

$$\delta a_{1}^{\prime\prime\mathrm{int}}=-\frac{c^{\prime\prime a}}{2}R_{a}^{\mu\alpha}\left(\partial_{\mu}h_{\alpha\lambda}+\partial_{\alpha}h_{\mu\lambda}-\partial_{\lambda}h_{\mu\alpha}\right)\eta^{\lambda}. \tag{63}$$

In the sequel we will show that there are no solutions to (62). Our procedure goes as follows. Suppose that there exist solutions $a_{0}^{\prime\prime\mathrm{int}}$ to equation (62). Using formula (63), it follows that such an $a_{0}^{\prime\prime\mathrm{int}}$ must be linear in the tensor fields $r_{\mu\nu|\alpha\beta}^{a}$, quadratic in the Pauli–Fierz field, and second-order in the derivatives. Integrating by parts in the corresponding functional constructed from $a_0^{\prime\prime\mathrm{int}}$ allows us to move the derivatives such as to act only on the Pauli–Fierz fields, and therefore to work with

$$a_0^{\prime\prime\mathrm{int}} = c^{\prime\prime a} r_a^{\mu\nu|\alpha\beta} a_{\mu\nu|\alpha\beta}^{\mathrm{lin}} \left(h\partial\partial h, \partial h\partial h\right), \tag{64}$$

where the above notation signifies that $a_{\mu\nu|\alpha\beta}^{\mathrm{lin}}$ is a linear combination of the generic polynomials between parentheses (with the mixed symmetry of the tensor fields $r_{\mu\nu|\alpha\beta}^{a}$). By direct computation we get that

$$\gamma a_0^{\prime\prime\mathrm{int}} = \partial^\mu \left(4c^{\prime\prime a} \mathcal{C}_a^{\alpha\beta|\nu} a_{\mu\nu|\alpha\beta}^{\mathrm{lin}}\right) - 4c^{\prime\prime a} \mathcal{C}_a^{\alpha\beta|\nu} \partial^\mu a_{\mu\nu|\alpha\beta}^{\mathrm{lin}} + c^{\prime\prime a} r_a^{\mu\nu|\alpha\beta} \gamma a_{\mu\nu|\alpha\beta}^{\mathrm{lin}}, \tag{65}$$

where

$$\gamma a_{\mu\nu|\alpha\beta}^{\mathrm{lin}} = \bar{a}_{\mu\nu|\alpha\beta}^{\mathrm{lin}} \left(h\partial\partial\eta, \partial h\partial\eta, \partial\partial h\partial\eta\right), \tag{66}$$

with η a generic notation for the Pauli–Fierz ghost η_μ. As $\delta a_1^{\prime\prime\mathrm{int}}$ contains no ghosts from the $r_{\mu\nu|\alpha\beta}^{a}$-sector, we require that $\gamma a_0^{\prime\prime\mathrm{int}}$ obeys the property

$$\partial^\mu a_{\mu\nu|\alpha\beta}^{\mathrm{lin}} \left(h\partial\partial h, \partial h\partial h\right) = 0, \tag{67}$$

such that

$$\gamma a_0^{\prime\prime\mathrm{int}} = \partial^\mu \left(4c^{\prime\prime a} \mathcal{C}_a^{\alpha\beta|\nu} a_{\mu\nu|\alpha\beta}^{\mathrm{lin}}\right) + c^{\prime\prime a} r_a^{\mu\nu|\alpha\beta} \gamma a_{\mu\nu|\alpha\beta}^{\mathrm{lin}}. \tag{68}$$

Simple calculations in (63) give

$$\delta a_1^{\prime\prime\mathrm{int}} = \partial_\mu p^\mu + c^{\prime\prime a} r_a^{\mu\nu|\alpha\beta} b_{\mu\nu|\alpha\beta}^{\mathrm{lin}} \left(\partial h\partial\partial\eta, \partial\partial h\partial\eta, \eta\partial\partial\partial h\right). \tag{69}$$

Inserting (68)–(69) in (62) and observing that only $b_{\mu\nu|\alpha\beta}^{\mathrm{lin}}$ contains terms that are third-order in the derivatives of the Pauli–Fierz fields, we conclude that the existence of $a_0^{\prime\prime\mathrm{int}}$ is completely dictated by the behavior of $b_{\mu\nu|\alpha\beta}^{\mathrm{lin}}$. More precisely, $a_0^{\prime\prime\mathrm{int}}$ exists if and only if the part of the type $\eta\partial\partial\partial h$ from $b_{\mu\nu|\alpha\beta}^{\mathrm{lin}}$ vanishes identically and/or can be written like the δ-variation of something like $\partial h^* r\eta$. Direct computation produces the part from $b_{\mu\nu|\alpha\beta}^{\mathrm{lin}}$ of order three in the derivatives of the Pauli–Fierz fields in the form

$$\begin{aligned} b_{\mu\nu|\alpha\beta}^{\mathrm{lin}} \left(\eta\partial\partial\partial h\right) &\sim \eta^\lambda \partial_\lambda \left[\sigma_{\beta\nu} \left(\partial_\mu \partial^\rho h_{\rho\alpha} + \partial_\alpha \partial^\rho h_{\rho\mu} - \Box h_{\alpha\mu} - \partial_\alpha \partial_\mu h\right)\right. \\ &- \frac{1}{2} \sigma_{\beta\nu} \sigma_{\alpha\mu} \left(\partial^\rho \partial^\gamma h_{\rho\gamma} - \Box h\right) + \partial_\beta \partial_\nu h_{\alpha\mu} + \left(\alpha \longleftrightarrow \beta, \mu \longleftrightarrow \nu\right) \\ &- \left(\beta \longleftrightarrow \alpha, \mu \to \mu, \nu \to \nu\right) - \left.\left(\mu \longleftrightarrow \nu, \alpha \to \alpha, \beta \to \beta\right)\right], \end{aligned} \tag{70}$$

and it neither vanishes identically nor is proportional with $\delta\left(\partial_\lambda h_{\alpha\mu}^*\right)$, as it can be observed from expression (11) of the functions that define the field equations for the Pauli–Fierz field. The rest of the terms from (70) are obtained from the first ones by making the indicated index-changes. In conclusion, we must also take

$$c^{\prime\prime a} = 0 \tag{71}$$

in (60), so $a_2^{\mathrm{int}} = 0$.

Now, we analyze the next possibility, namely $I = 1$

$$a^{\mathrm{int}} = a_0^{\mathrm{int}} + a_1^{\mathrm{int}}, \tag{72}$$

where a_1^{int} must be searched among the non-trivial solutions to the equation $\gamma a_1^{\mathrm{int}} = 0$, which are offered by

$$a_1^{\mathrm{int}} = \alpha_1 \left(\left[r_a^{*\mu\nu|\alpha\beta}\right], \left[h^{*\mu\nu}\right], \left[F_{\mu\nu\lambda|\alpha\beta\gamma}^{a}\right], \left[K_{\mu\nu|\alpha\beta}\right]\right) \omega^1 \left(\eta_\mu, \partial_{[\mu} \eta_{\nu]}\right), \tag{73}$$

where the elements of pure ghost number one are

$$\left(\eta_\mu, \partial_{[\mu}\eta_{\nu]}\right). \tag{74}$$

On the one hand, the assumption on the maximum derivative order of the interacting Lagrangian being equal to two prevents the coefficients α_1 to depend on either the curvature tensors or their space-time derivatives. On the other hand, $a_1^{\rm int}$ can involve only the antifields $r_a^{*\mu\nu|\alpha\beta}$ and their space-time derivatives, because otherwise, as ω^1 includes only the Pauli–Fierz ghosts, it would not lead to cross-interactions between the fields $r^a_{\mu\nu|\alpha\beta}$ and $h_{\mu\nu}$. Moving in addition the derivatives from these antifields such as to act only on the elements (74) from $a_1^{\rm int}$ and relying again on the assumption on the maximum derivative order, we eventually remain with one possibility (up to γ-exact quantities)

$$\begin{aligned} a_1^{\rm int} &\sim k^a r_a^{*\mu\nu|\alpha\beta}\left(\sigma_{\mu\alpha}\partial_{[\nu}\eta_{\beta]} - \sigma_{\mu\beta}\partial_{[\nu}\eta_{\alpha]} + \sigma_{\nu\beta}\partial_{[\mu}\eta_{\alpha]} - \sigma_{\nu\alpha}\partial_{[\mu}\eta_{\beta]}\right) \\ &= 4k^a r_a^{*\nu\beta}\partial_{[\nu}\eta_{\beta]} \equiv 0, \end{aligned} \tag{75}$$

which vanishes identically due to the symmetry of the trace of the antifields $r_a^{*\mu\nu|\alpha\beta}$.

As $a_1^{\rm int}$ in (75) vanishes, we remain with one more case, namely where $a^{\rm int}$ reduces to its antighost number zero piece

$$a^{\rm int} = a_0^{\rm int}\left(\left[r^a_{\mu\nu|\alpha\beta}\right], \left[h_{\mu\nu}\right]\right), \tag{76}$$

which is subject to the equation

$$\gamma a_0^{\rm int} = \partial_\mu \overset{(0)}{m}{}^\mu_{\rm int}. \tag{77}$$

There are two types of solutions to (77). The first one corresponds to $\overset{(0)}{m}{}^\mu_{\rm int} = 0$ and is given by arbitrary polynomials that mix the curvature tensors (35) and their space-time derivatives with the linearized Riemann tensor (12) and its derivatives, which are however excluded from the condition on the maximum derivative order of $a_0^{\rm int}$ (their derivative order is at least four). The second one is associated with $\overset{(0)}{m}{}^\mu_{\rm int} \neq 0$, being understood that we discard the divergence-like solutions $a_0^{\rm int} = \partial_\mu z^\mu$ and preserve the maximum derivative-order restriction. Denoting the Euler–Lagrange derivatives of $a_0^{\rm int}$ by $B_a^{\mu\nu|\alpha\beta} \equiv \delta a_0^{\rm int}/\delta r^a_{\mu\nu|\alpha\beta}$ and respectively by $D^{\mu\nu} = \delta a_0^{\rm int}/\delta h_{\mu\nu}$, we get that equation (77) implies

$$\partial_\mu B_a^{\mu\nu|\alpha\beta} = 0, \quad \partial_\mu D^{\mu\nu} = 0. \tag{78}$$

The tensors $B_a^{\mu\nu|\alpha\beta}$ and $D^{\mu\nu}$ are imposed to contain at most two derivatives and to have the mixed symmetry of $r^a_{\mu\nu|\alpha\beta}$ and respectively of $h_{\mu\nu}$. Meanwhile, they must yield a Lagrangian density $a_0^{\rm int}$ that effectively couples the two sorts of fields, so $B_a^{\mu\nu|\alpha\beta}$ and $D^{\mu\nu}$ effectively depend on $h_{\mu\nu}$ and respectively on $r^a_{\mu\nu|\alpha\beta}$. The solutions to equations (78) are of the type

$$\frac{\delta a_0^{\rm int}}{\delta r^a_{\mu\nu|\alpha\beta}} \equiv B_a^{\mu\nu|\alpha\beta} = \partial_\rho\partial_\gamma \hat{\Phi}_a^{\mu\nu\rho|\alpha\beta\gamma}, \quad \frac{\delta a_0^{\rm int}}{\delta h_{\mu\nu}} \equiv D^{\mu\nu} = \partial_\alpha\partial_\beta \tilde{\Phi}^{\mu\alpha|\nu\beta}, \tag{79}$$

where $\hat{\Phi}_a^{\mu\nu\rho|\alpha\beta\gamma}$ and $\tilde{\Phi}^{\mu\alpha|\nu\beta}$ depend only on the undifferentiated fields $h_{\mu\nu}$ and $r^a_{\mu\nu|\alpha\beta}$ (otherwise, the corresponding $a_0^{\rm int}$ would be more than second-order in the derivatives), with $\hat{\Phi}_a^{\mu\nu\rho|\alpha\beta\gamma}$ having the mixed symmetry of the curvature tensors $F_a^{\mu\nu\rho|\alpha\beta\gamma}$ and $\tilde{\Phi}^{\mu\alpha|\nu\beta}$ that of the linearized Riemann tensor. We introduce a derivation on the algebra of non-integrated densities depending on $r^a_{\mu\nu|\alpha\beta}$, $h_{\mu\nu}$ and on their derivatives, that counts the powers of the fields and their derivatives

$$\bar{N} = \sum_{n\geq 0} \left[\left(\partial_{\mu_1\ldots\mu_n} r^a_{\mu\nu|\alpha\beta}\right) \frac{\partial}{\partial\left(\partial_{\mu_1\ldots\mu_n} r^a_{\mu\nu|\alpha\beta}\right)} + \left(\partial_{\mu_1\ldots\mu_n} h_{\mu\nu}\right) \frac{\partial}{\partial\left(\partial_{\mu_1\ldots\mu_n} h_{\mu\nu}\right)} \right], \tag{80}$$

and observe that the action of $\bar{N}$ on an arbitrary non-integrated density $\bar{u}\left(\left[r^a_{\mu\nu|\alpha\beta}\right], [h_{\mu\nu}]\right)$ is

$$\bar{N}\bar{u} = r^a_{\mu\nu|\alpha\beta} \frac{\delta \bar{u}}{\delta r^a_{\mu\nu|\alpha\beta}} + h_{\mu\nu} \frac{\delta \bar{u}}{\delta h_{\mu\nu}} + \partial_\mu r^\mu, \tag{81}$$

where $\delta\bar{u}/\delta r^a_{\mu\nu|\alpha\beta}$ and $\delta\bar{u}/\delta h_{\mu\nu}$ denote the variational derivatives of $\bar{u}$. In the case where $\bar{u}$ is a homogeneous polynomial of order $p > 0$ in the fields and their derivatives, we have that $\bar{N}\bar{u} = p\bar{u}$, and so

$$\bar{u} = \frac{1}{p}\left(r^a_{\mu\nu|\alpha\beta} \frac{\delta \bar{u}}{\delta r^a_{\mu\nu|\alpha\beta}} + h_{\mu\nu} \frac{\delta \bar{u}}{\delta h_{\mu\nu}} \right) + \partial_\mu \left(\frac{1}{p} r^\mu\right). \tag{82}$$

As a_0^{int} can always be decomposed as a sum of homogeneous polynomials of various orders, it is enough to analyze the equation (77) for a fixed value of p. Putting $\bar{u} = a_0^{\mathrm{int}}$ in (82) and inserting (79) in the associated relation, we can write

$$a_0^{\mathrm{int}} = \frac{1}{p}\left(r^a_{\mu\nu|\alpha\beta} \partial_\rho \partial_\gamma \hat{\Phi}_a^{\mu\nu\rho|\alpha\beta\gamma} + h_{\mu\nu} \partial_\alpha \partial_\beta \tilde{\Phi}^{\mu\alpha|\nu\beta} \right) + \partial_\mu \bar{r}^\mu. \tag{83}$$

Integrating twice by parts in (83) and recalling the mixed symmetries of $\hat{\Phi}_a^{\mu\nu\rho|\alpha\beta\gamma}$ and $\tilde{\Phi}^{\mu\alpha|\nu\beta}$, we infer that

$$a_0^{\mathrm{int}} = k_1 F^a_{\mu\nu\rho|\alpha\beta\gamma} \hat{\Phi}_a^{\mu\nu\rho|\alpha\beta\gamma} + k_2 K_{\mu\alpha|\nu\beta} \tilde{\Phi}^{\mu\alpha|\nu\beta} + \partial_\mu \bar{l}^\mu, \tag{84}$$

with $k_1 = 1/9p$ and $k_2 = -1/2p$. By computing the action of γ on (84), we obtain that $p = 2$ and

$$a_0^{\mathrm{int}} = k'^a R_a^{\mu\alpha} h_{\mu\alpha}. \tag{85}$$

As the above a_0^{int} vanishes on the stationary surface of field equations for $r^a_{\mu\nu|\alpha\beta}$, it is trivial in $H^{0,D}(s|d)$, so it can be removed from the first-order deformation by choosing

$$k' = 0. \tag{86}$$

Putting together the results obtained so far, we can state that $S_1^{\mathrm{int}} = 0$ and so

$$S_1 = S_1^{\mathrm{h}} + S_1^{\mathrm{r}}, \tag{87}$$

where S_1^{h} is the first-order deformation of the solution to the master equation for the Pauli–Fierz theory and S_1^{r} is given in the right-hand side of (26). The consistency of S_1 at the second order in the coupling constant is governed by equation (20), where $\left(S_1^{\mathrm{h}}, S_1^{\mathrm{r}}\right) = 0 = \left(S_1^{\mathrm{r}}, S_1^{\mathrm{r}}\right)$, and thus we have that $S_2^{\mathrm{r}} = 0 = S_2^{\mathrm{int}}$, while S_2^{h} is highly non-trivial and is known to describe the quartic vertex of the Einstein–Hilbert action, as well as the second-order contributions to the gauge transformations and to the associated non-Abelian gauge algebra. The vanishing of S_1^{int} and S_2^{int} further leads, via the equations that stipulate the higher-order deformation equations, to the result that actually

$$S_k^{\mathrm{int}} = 0, \quad k \geq 1. \tag{88}$$

The main conclusion of this paper is that, under the general conditions of analyticity in the coupling constant, space-time locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the requirement that the interacting Lagrangian is at most second-order derivative, there are no consistent, non-trivial cross-couplings between the Pauli–Fierz field and a collection of massless tensor fields with the mixed symmetry of the Riemann tensor. The only pieces that can be added to action (1) are given by some cosmological terms for the tensors $r^a_{\mu\nu|\alpha\beta}$ and, naturally, by the self-interactions of the Pauli–Fierz field, which produce the Einstein–Hilbert action, invariant under diffeomorphisms.

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