# Interactions in theories with many massless tensors with the mixed symmetry $(3,1)$. Case of couplings with a vector field 

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#### Abstract

Under the hypotheses of smoothness in the coupling constant, locality, Lorentz covariance, and Poincare invariance of the deformations, combined with the preservation of the number of derivatives on each field, the consistent interactions between a collection of free massless tensor gauge fields with the mixed symmetry of a two-column Young diagram of the type ( 3,1 ) and one Abelian vector field are addressed. The main result is that a single mixed symmetry tensor field from the collection gets coupled to the vector field. Our final result resembles to the well known fact from General Relativity according to wich there is one graviton in a given world.


## 1 Introduction

Tensor fields in "exotic" representations of the Lorentz group, characterized by a mixed Young symmetry type $[1,2,3,4,5,6,7]$, held the attention lately on some important issues, like the dual formulation of field theories of spin two or higher $[8,9,10,11,12,13,14]$, the impossibility of consistent cross-interactions in the dual formulation of linearized gravity [15], a Lagrangian first-order approach $[16,17]$ to some classes of massless or partially massive mixed symmetry type tensor gauge fields, suggestively resembling to the tetrad formalism of General Relativity, or the derivation of some exotic gravitational interactions [18, 19]. An important matter related to mixed symmetry type tensor fields is the study of their consistent interactions, among themselves as well as with higher-spin gauge theories $[20,21,22,23,24,25,26,27,28]$. The most efficient approach to this problem is the cohomological one, based on the deformation of the solution to the master equation [29]. The purpose of this paper is to expose the main result on the consistent interactions between a collection of massless tensor gauge fields, each with the mixed symmetry of a two-column Young diagram of the type $(3,1)$ and one vector field [30]. It is worth mentioning the duality of a free massless tensor gauge field with the mixed symmetry $(3,1)$ to the Pauli-Fierz theory in $D=6$ dimensions and, in this respect, some developments concerning the dual formulations of linearized gravity from the perspective of $M$-theory [31, 32, 33]. Our analysis relies on the deformation of the solution to the master equation by means of cohomological techniques with the help of the local BRST cohomology, whose component in a single $(3,1)$ sector has been reported in detail in [34]. This paper generalizes our results from [35] regarding the cross-interactions between a single massless $(3,1)$ field and a vector field. Under the hypotheses of analiticity in the coupling constant, locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the preservation of the number of derivatives on each field, we find a deformation of the solution to the master equation that provides nontrivial cross-couplings. This case corresponds to a 5 -dimensional spacetime and is described by a deformed solution that stops at order two in the coupling constant. The interacting Lagrangian action contains only mixing-component terms of order one and two in the coupling constant, but only one mixed

[^0]symmetry tensor field from the collection gets coupled to the vector field, while the others remain free. At the level of the gauge transformations, only those of the vector field are modified at order one in the coupling constant with a term linear in the antisymmetrized first-order derivatives of a single gauge parameter from the $(3,1)$ sector such that the gauge algebra and the reducibility structure of the coupled model are not modified during the deformation procedure, being the same like in the case of the starting free action. Our result is interesting since it exhibits strong similarities to the Einstein gravitons from General Relativity, in the sense that no nontrivial cross-couplings between different fields with the mixed symmetry $(3,1)$ are allowed, neither direct nor intermediated by a vector field.

## 2 Free model. BRST symmetry

We begin with the Lagrangian action

$$
\begin{gather*}
S_{0}\left[t_{\lambda \mu \nu \mid \alpha}^{A}, V_{\mu}\right]=\int d^{D} x\left\{\frac{1}{2}\left[\left(\partial^{\rho} t_{A}^{\lambda \mu \nu \mid \alpha}\right)\left(\partial_{\rho} t_{\lambda \mu \nu \mid \alpha}^{A}\right)-\left(\partial_{\alpha} t_{A}^{\lambda \mu \nu \mid \alpha}\right)\left(\partial^{\beta} t_{\lambda \mu \nu \mid \beta}^{A}\right)\right]\right. \\
-\frac{3}{2}\left[\left(\partial_{\lambda} t_{A}^{\lambda \mu \nu \mid \alpha}\right)\left(\partial^{\rho} t_{\rho \mu \nu \mid \alpha}^{A}\right)+\left(\partial^{\rho} t_{A}^{\lambda \mu}\right)\left(\partial_{\rho} t_{\lambda \mu}^{A}\right)\right]+3\left(\partial_{\alpha} t_{A}^{\lambda \mu \nu \mid \alpha}\right)\left(\partial_{\lambda} t_{\mu \nu}^{A}\right) \\
\left.+3\left(\partial_{\rho} t_{A}^{\rho \mu}\right)\left(\partial^{\lambda} t_{\lambda \mu}^{A}\right)-\frac{1}{4} F_{\mu \nu} F^{\mu \nu}\right\} \equiv S_{0}^{t}\left[t_{\lambda \mu \nu \mid \alpha}^{A}\right]+S_{0}^{V}\left[V_{\mu}\right] \tag{1}
\end{gather*}
$$

in $\mathrm{D} \geq 5$ spacetime dimensions, with $\mathrm{A}=\overline{1, n}$ and $n \geq 2$. Each massless tensor field $t_{\lambda \mu \nu \mid \alpha}^{A}$ has the mixt symmetry $(3,1)$ and hence transforms according to an irreducible representation of $G L(D, R)$ correponding to a 4 -cell Young diagram with two columns and three rows. It is thus completly antisymmetric in its first three indices and satisfies the identity $t_{[\lambda \mu \nu \mid \alpha]}^{A} \equiv 0$. The collection of indices $A, B$, etc., are raised and lowered with a quadratic form $k_{A B}$ that defines a positively-defined metric in the internal space. It can always be normalized to $\delta_{A B}$ by a simple linear field redefinition, so one can take $k_{A B}=\delta_{A B}$ and re-write (1) as

$$
\begin{equation*}
S_{0}\left[t_{\lambda \mu \nu \mid \alpha}^{A}, V_{\mu}\right]=\int d^{D} x\left[\sum_{A=1}^{n} \mathcal{L}_{0}^{t}\left(t_{\lambda \mu \nu \mid \alpha}^{A}, \partial_{\rho} t_{\lambda \mu \nu \mid \alpha}^{A}\right)+\mathcal{L}_{0}^{V}\left(V_{\mu}, \partial_{\nu} V_{\mu}\right)\right] \tag{2}
\end{equation*}
$$

where $\mathcal{L}_{0}^{t}\left(t_{\lambda \mu \nu \mid \alpha}^{A}, \partial_{\rho} t_{\lambda \mu \nu \mid \alpha}^{A}\right)$ is the Lagrangian density for the field $A$. The field strength of the vector field $V_{\mu}$ is defined in the standard manner by

$$
\begin{equation*}
F_{\mu \nu}=\partial_{\mu} V_{\nu}-\partial_{\nu} V_{\mu} \tag{3}
\end{equation*}
$$

The notation $[\lambda \ldots \alpha]$ signifies complete antisimmetry with respect to the (Lorentz) indices between brackets, with the convetions that the minimum number of terms is always used and the result is never divided by the number of terms. The trace of $t_{\lambda \mu \nu \mid \alpha}^{A}$ is defined by $t_{\lambda \mu}^{A}=\sigma^{\nu \alpha} t_{\lambda \mu \nu \mid \alpha}^{A}$ and it is obviously an antisymmetric tensor. Everywhere in this papaer we employ the flat Minkowski metric of 'mostly plus' signature $\sigma^{\mu \nu}=\sigma_{\mu \nu}=(-,+,+,+,+\cdots)$

A generating set of gauge transformations for the action (1) can be taken of the form

$$
\begin{gather*}
\delta_{\varepsilon, \chi} t_{\lambda \mu \nu \mid \alpha}^{A}=-3 \partial_{[\lambda} \varepsilon_{\mu \nu \alpha]}^{A}+4 \partial_{[\lambda} \varepsilon_{\mu \nu] \alpha}^{A}+\partial_{[\lambda} \chi_{\mu \nu] \mid \alpha}^{A}  \tag{4}\\
\delta_{\varepsilon} V_{\mu}=\partial_{\mu} \varepsilon \tag{5}
\end{gather*}
$$

where the gauge parameters $\varepsilon_{\mu \nu \alpha}^{A}$ determine $n$ completely antisymmetric tensors, the other set of gauge parameters displays the mixed symmetry $(2,1)$, such that they are antisymmetric in the first two indices and satisfy the identity $\chi_{[\mu \nu \mid \alpha]}^{A}=0$, and the gauge parameter $\varepsilon$ is a scalar. The generating set of gauge transformations $(4-5)$ is off-shell, second-stage reducible, the accompanying gauge algebra being obviously Abelian.

The construction of the antifield-BRST symmetry for this free theory debuts with the identification of the algebra on which the BRST differentia $s$ acts. The generators of the BRST algebra are of two
kinds: fields/ghosts and antifields. The ghost spectrum for the model under study comprises the fermionic ghosts $\left\{\eta_{\lambda \mu \nu}^{A}, \mathcal{G}_{\mu \nu \mid \alpha}^{A}, \eta\right\}$ associated with the gauge parameters $\left\{\varepsilon_{\lambda \mu \nu}^{A}, \chi_{\mu \nu \mid \alpha}^{A}, \varepsilon\right\}$ from $(4-5)$, the bosonic ghosts for ghosts $\left\{C_{\mu \nu}^{A}, \mathcal{C}_{\nu \alpha}^{A}\right\}$ due to the first-stage reducibility relations, and also the fermionic ghosts for ghosts $C_{\nu}^{A}$ corresponding to the second-stage reducibility relations. We ask that $\eta_{\lambda \mu \nu}^{A}$ and $C_{\mu \nu}^{A}$ are completely antisymmetric, $\mathcal{G}_{\mu \nu \mid \alpha}^{A}$ display the mixed symmetry $(2,1)$, and $\mathcal{C}_{\nu \alpha}^{A}$ are symmetric. The antifield spectrum is organized into the antifields $\left\{t_{A}^{* \lambda \mu \nu \mid \alpha}, V^{* \mu}\right\}$ of the original tensor fields, together with those of the ghosts, $\left\{\eta_{A}^{* \lambda \mu \nu}, \mathcal{G}_{A}^{* \mu \nu \mid \alpha}, \eta^{*}\right\},\left\{C_{A}^{* \mu \nu}, \mathcal{C}_{A}^{* \nu \alpha}\right\}$, and respectively $C_{A}^{* \nu}$, of statistics opposite to that of the associated fields/ghosts. It is understood that $t_{A}^{* \lambda \mu \nu \mid \alpha}$ exhibit the same mixed-symmetry properties like $t_{\lambda \mu \nu \mid \alpha}^{A}$ and similarly with respect to $\eta_{A}^{* \lambda \mu \nu}, \mathcal{G}_{A}^{* \mu \nu \mid \alpha}, C_{A}^{* \mu \nu}$, and $\mathcal{C}_{A}^{* \nu \alpha}$. For subsequent purpose, we denote the trace of $t_{A}^{* \lambda \mu \nu \mid \alpha}$ by $t_{A}^{* \lambda \mu}$, being understood that it is antisymmetric.

Since both the gauge generators and reducibility functions for this model are field-independent, it follows that the BRST differential $s$ simply reduces to

$$
\begin{equation*}
s=\delta+\gamma \tag{6}
\end{equation*}
$$

where $\delta$ represents the Koszul-Tate differential, graded by the antighost number agh (agh $(\delta)=-1)$ and $\gamma$ stands for the exterior derivative along the gauge orbits, whose degree is named pure ghost number pgh $(\mathrm{pgh}(\gamma)=1)$. The overall degree that grades the BRST complex is known as the ghost number (gh) and is defined like the difference between the pure ghost number and the antighost number, such that $g h(s)=g h(\delta)=g h(\gamma)=1$. According to the standard rules of the BRST method, the corresponding degrees of the generators from the BRST complex are valued like

$$
\begin{gathered}
\operatorname{pgh}\left(\eta_{\lambda \mu \nu}^{A}\right)=\operatorname{pgh}\left(\mathcal{G}_{\mu \nu \mid \alpha}^{A}\right)=\operatorname{pgh}(\eta)=1 \\
\operatorname{pgh}\left(C_{\mu \nu}^{A}\right)=2=\operatorname{pgh}\left(\mathcal{C}_{\nu \alpha}^{A}\right), \operatorname{pgh}\left(C_{\nu}^{A}\right)=3 \\
\operatorname{agh}\left(t_{A}^{* \lambda \mu \nu \mid \alpha}\right)=1=\operatorname{agh}\left(V^{* \mu}\right) \\
\operatorname{agh}\left(\eta_{A}^{* \lambda \mu \nu}\right)=\operatorname{agh}\left(\mathcal{G}_{A}^{* \mu \nu \mid \alpha}\right)=\operatorname{agh}\left(\eta^{*}\right)=2 \\
\operatorname{agh}\left(C_{A}^{* \mu \nu}\right)=3=\operatorname{agh}\left(\mathcal{C}_{A}^{* \nu \alpha}\right), \operatorname{agh}\left(C_{A}^{* \nu}\right)=4
\end{gathered}
$$

plus the usual rules that the degrees of the original fields, the antighost number of the ghosts and the pure ghost number of the antifields all vanish. The actions of $\delta$ and $\gamma$ on the generators from the BRST complex are given by

$$
\begin{gather*}
\gamma t_{\lambda \mu \nu \mid \alpha}^{A}=-3 \partial_{[\lambda} \eta_{\mu \nu \alpha]}^{A}+4 \partial_{[\lambda} \eta_{\mu \nu] \alpha}^{A}+\partial_{[\lambda} \mathcal{G}_{\mu \nu] \mid \alpha}^{A}, \gamma V_{\mu}=\partial_{\mu} \eta  \tag{7}\\
\gamma \eta_{\lambda \mu \nu}^{A}=-\frac{1}{2} \partial_{[\lambda} C_{\mu \nu]}^{A}, \gamma \eta=0  \tag{8}\\
\gamma \mathcal{G}_{\mu \nu \mid \alpha}^{A}=2 \partial_{[\mu} C_{\nu \alpha]}^{A}-3 \partial_{[\mu} C_{\nu] \alpha}^{A}+\partial_{[\mu} \mathcal{C}_{\nu] \alpha}^{A}  \tag{9}\\
\gamma C_{\mu \nu}^{A}=\partial_{[\mu} C_{\nu]}^{A}, \gamma \mathcal{C}_{\nu \alpha}^{A}=-3 \partial_{(\nu} C_{\alpha)}^{A}, \gamma C_{\nu}^{A}=0  \tag{10}\\
\gamma t_{A}^{* \lambda \mu \nu \mid \alpha}=\gamma V^{* \mu}=\gamma \eta_{A}^{* \lambda \mu \nu}=\gamma \mathcal{G}_{A}^{* \mu \nu \mid \alpha}=\gamma \eta^{*}=0  \tag{11}\\
\gamma C_{A}^{* \mu \nu}=\gamma \mathcal{C}_{A}^{* \nu \alpha}=\gamma C_{A}^{* \nu}=0  \tag{12}\\
\delta t_{\lambda \mu \nu \mid \alpha}^{A}=\delta V_{\mu}=\delta \eta_{\lambda \mu \nu}^{A}=\delta \mathcal{G}_{\mu \nu \mid \alpha}^{A}=\delta \eta=0  \tag{13}\\
\delta t_{A}^{* \lambda \mu \nu \mid \alpha}=T_{A}^{\lambda \mu \nu \mid \alpha}, \delta V^{* \mu}=-\partial_{\nu} F^{\nu \mu}, \delta \eta_{A}^{* \lambda \mu \nu}=-4 \partial_{\alpha} t_{A}^{* \lambda \mu \nu \mid \alpha}  \tag{14}\\
\delta \mathcal{G}_{A}^{* \mu \nu \mid \alpha}=-\partial_{\lambda}\left(3 t_{A}^{* \lambda \mu \nu \mid \alpha}-t_{A}^{* \mu \nu \alpha \mid \lambda}\right), \delta \eta^{*}=-\partial_{\mu} V^{* \mu} \tag{15}
\end{gather*}
$$

$$
\begin{gather*}
\delta C_{A}^{* \mu \nu}=3 \partial_{\lambda}\left(\mathcal{G}_{A}^{* \mu \nu \mid \lambda}-\frac{1}{2} \eta_{A}^{* \lambda \mu \nu}\right), \delta \mathcal{C}_{A}^{* \nu \alpha}=\partial_{\mu} \mathcal{G}_{A}^{* \mu(\nu \mid \alpha)}  \tag{16}\\
\delta C_{A}^{* \nu}=6 \partial_{\mu}\left(\mathcal{C}_{A}^{* \mu \nu}-\frac{1}{3} C_{A}^{* \mu \nu}\right) \tag{17}
\end{gather*}
$$

where $T_{A}^{\lambda \mu \nu \mid \alpha}$ are minus the Euler-Lagrange derivatives of action (1) with respect to the field $t_{\lambda \mu \nu \mid \alpha}^{A}$. The Lagrangian Brst differential admits a canonical action in a structure named antibracket and defined by decreeing the fields/ghosts conjugated with the corresponding antifields, $s \cdot=(\cdot, S)$, where (, ) signifies the antibracket and $S$ denotes the canonical generator of the BRST symmetry. It is a bosonic functional of ghost number zero(involving both field/ghost and antifield spectra) that obeys the master equation $(S, S)=0$. The master equation is equivalent with the second-order nilpotency of $s$, where its solution $S$ encodes the entire gauge structure of the associated theory. The complete solution to the master equation for the free model under study is given by

$$
\begin{gather*}
S=S_{0}\left[t_{\lambda \mu \nu \mid \alpha}^{A}, V_{\mu}\right]+\int d^{D} x\left[t_{A}^{* \lambda \mu \nu \mid \alpha}\left(3 \partial_{\alpha} \eta_{\lambda \mu \nu}^{A}+\partial_{[\lambda} \eta_{\mu \nu] \alpha}^{A}+\partial_{[\lambda} \mathcal{G}_{\mu \nu] \mid \alpha}^{A}\right)\right. \\
-\frac{1}{2} \eta_{A}^{* \lambda \mu \nu} \partial_{[\lambda} C_{\mu \nu]}^{A}+\mathcal{G}_{A}^{* \mu \nu \mid \alpha}\left(2 \partial_{\alpha} C_{\mu \nu}^{A}-\partial_{[\mu} C_{\nu] \alpha}^{A}+\partial_{[\mu} \mathcal{C}_{\nu] \alpha}^{A}\right) \\
\left.+C_{A}^{* \mu \nu} \partial_{[\mu} C_{\nu]}^{A}-3 \mathcal{C}_{A}^{* \nu \alpha} \partial_{(\nu} C_{\alpha)}^{A}+V^{* \mu} \partial_{\mu} \eta\right] \tag{18}
\end{gather*}
$$

## 3 Brief review of the deformation procedure

The reformulation of the problem of consistent deformations of a given action and of its gauge symmetries in the antifield-BRST setting is based on the observation that if a deformation of the classical theory can be consistently constructed, then the solution to the master equation for the initial theory can be deformed into the solution of the master equation for the interacting theory

$$
\begin{equation*}
\bar{S}=S+g S_{1}+g^{2} S_{2}+O\left(g^{3}\right), \varepsilon(\bar{S})=0, g h(\bar{S})=0 \tag{19}
\end{equation*}
$$

such that

$$
\begin{equation*}
(\bar{S}, \bar{S})=0 \tag{20}
\end{equation*}
$$

The projection of (20) on the various powers of the coupling constant induces the following tower of equations:

$$
\begin{gather*}
g^{0}:(S, S)=0  \tag{21}\\
g^{1}:\left(S_{1}, S\right)=0  \tag{22}\\
g^{2}: \frac{1}{2}\left(S_{1}, S_{1}\right)+\left(S_{2}, S\right)=0, \tag{23}
\end{gather*}
$$

The first equation is satisfed by hypothesis. The second governs the first-order deformation of the solution to the master equation, $S_{1}$ and shoes that $S_{1}$ is a BRST co-cycle, $s S_{1}=0$. This means that $S_{1}$ pertains to the ghost number zero cohomological space of $s, H^{0}(s)$, which is generically nonempty because it is isomorphic to the space of physical observables of the free theory. The remaining equations are responsible for the higher-order deformations of the solution to the master equation. Once that the deformation equations (21-23), etc., have been solved by means of specific cohomological techniques, from the consistent nontrivial deformed solution to the master equation one can extract all the information on the gauge structure of the resulting interacting theory.

## 4 Main results

The aim of the papaer is to investigate the consistent interactions that can be added to the initial action without modifying either the field spectrum or the number of independent gauge symmetries.

We consider only smooth, local, and manifestly covariant deformations and restrict to Poincareinvariant quantities, i.e. we do not allow explicit dependence on the spacetime coordinates.

We ask that the deformed gauge theory preserves the Cauchy order of the uncoupled model, which enforces the requirement that the interacting Lagrangian is of maximum order equal to two in the spacetime derivatives of the fields at each order in the coupling constant.

There appear two distinct solutions that exclude each other.
The first type of solution stops at order one in the coupling constant and reads as

$$
\begin{equation*}
\bar{S}=S+\frac{g}{3 \cdot 4!} \int d^{5} x \varepsilon^{\lambda \mu \nu \rho \kappa} F_{\lambda \mu} F_{\nu \rho} V_{\kappa} \tag{24}
\end{equation*}
$$

where $S$ is given in (1) in $D=5$.
This case is not interesting since it provides no cross-couplings between the vector field and the tensor field. It simply restricts the free Lagrangian action to evolve on a five-dimensional spacetime and adds to it a generalized Abelian Chern-Simons term, without changing the original gauge transformations and, in consequence, neither the original Abelian gauge algebra nor the reducibility structure.

The second type of full deformed solution to the master equation ends at order two in the coupling constant and is given by

$$
\begin{align*}
\bar{S} & =S+g \sum_{A=1}^{n}\left[y^{A} \int d^{5} x \varepsilon^{\lambda \mu \nu \rho \kappa}\left(V_{\lambda}^{*} \mathcal{F}_{\mu \nu \rho \kappa}^{A}-\frac{2}{3} F_{\lambda \mu} \partial_{[\xi} t_{\nu \rho \kappa] \mid \theta}^{A} \sigma^{\theta \xi}\right)\right] \\
& +\frac{16 g^{2}}{3} \sum_{A, B=1}^{n}\left[y^{A} y^{B} \int d^{5} x\left(\partial_{[\xi} t_{\nu \rho \kappa] \mid \theta^{A}}^{A} \sigma^{\theta \xi}\right) \partial^{\left[\xi^{\prime}\right.} t^{B \nu \rho \kappa] \mid \theta^{\prime}} \sigma_{\theta^{\prime} \xi^{\prime}}\right] \tag{25}
\end{align*}
$$

where all $\mathcal{F}_{\mu \nu \rho \kappa}^{A}$ have the pure ghost number equal to one and are defined like the antisymmetrized first-order derivatives of the ghosts from the sector $(3,1)$

$$
\begin{equation*}
\mathcal{F}_{\mu \nu \rho \kappa}^{A} \equiv \partial_{[\mu} \eta_{\nu \rho \kappa]}^{A} \tag{26}
\end{equation*}
$$

These are in fact the only non-trivial elements with the pure ghost number equal to one from the cohomology of the exterior derivative along the gauge orbits, $H(\gamma)$. The quantities $y^{A}$ are $n$ arbitrary, real numbers and $\varepsilon^{\lambda \mu \nu \rho \kappa}$ is the Levi-Civita symbol in $D=5$. This solution 'lives' also in a five-dimensional space-time. From (25) we read all the information on the gauge structure of the coupled theory. The terms of antighost number zero in (25) provide the Lagrangian action. They can be equivalently organized as

$$
\begin{equation*}
\bar{S}_{0}\left[t_{\lambda \mu \nu \mid \alpha}^{A}, V_{\mu}\right]=S_{0}^{t}\left[t_{\lambda \mu \nu \mid \alpha}^{A}\right]-\frac{1}{4} \int d^{5} x \bar{F}_{\mu \nu} \bar{F}^{\mu \nu} \tag{27}
\end{equation*}
$$

in terms of the deformed field strength

$$
\begin{equation*}
\bar{F}^{\mu \nu}=F^{\mu \nu}+\frac{4 g}{3} \varepsilon^{\mu \nu \alpha \beta \gamma} \sum_{A=1}^{n}\left(y^{A} \partial_{[\rho} t_{\alpha \beta \gamma] \mid}^{A}{ }^{\rho}\right), \tag{28}
\end{equation*}
$$

where $S_{0}^{t}\left[t_{\lambda \mu \nu \mid \alpha}^{A}\right]$ is the Lagrangian action of the massless tensor fileds $t_{\lambda \mu \nu \mid \alpha}^{A}$ appearing in (1) in $D=5$. We observe that the action (27) contains only mixing-component terms of order one and two in the coupling constant. The piece of antighost number one appearing in (25) gives the deformed gauge transformations in the form

$$
\begin{gather*}
\bar{\delta}_{\varepsilon, \chi} t_{\lambda \mu \nu \mid \alpha}^{A}=-3 \partial_{[\lambda} \varepsilon_{\mu \nu \alpha]}^{A}+4 \partial_{[\lambda} \varepsilon_{\mu \nu] \alpha}^{A}+\partial_{[\lambda} \chi_{\mu \nu] \mid \alpha}^{A}  \tag{29}\\
\bar{\delta}_{\varepsilon, \chi} V^{\mu}=\partial^{\mu} \varepsilon+4 g \varepsilon^{\mu \alpha \beta \gamma \delta} \sum_{A=1}^{n}\left(y^{A} \partial_{\alpha} \varepsilon_{\beta \gamma \delta}^{A}\right)  \tag{30}\\
\widehat{Y}=M^{T} Y M \tag{31}
\end{gather*}
$$

with $M^{T}$ the transposed of $M$, such that $\widehat{Y}$ is diagonalized and a single diagonal element(for definiteness, we take the first) is non-vanishing

$$
\begin{equation*}
\widehat{Y}^{11}=\sum_{A=1}^{n}\left(y^{A}\right)^{2} \equiv y^{2}, \quad \widehat{Y}^{1 A^{\prime}}=\widehat{Y}^{B^{\prime} 1}=\widehat{Y}^{A^{\prime} B^{\prime}}=0, \quad A^{\prime}, B^{\prime}=\overline{2, n} \tag{32}
\end{equation*}
$$

If we make the notation

$$
\begin{equation*}
\widehat{y}^{A}=M^{A C} y^{C} \tag{33}
\end{equation*}
$$

then relation (32) implies

$$
\begin{equation*}
\widehat{y}^{A}=y \delta_{1}^{A} \tag{34}
\end{equation*}
$$

Now, we make the linear field redefinition

$$
\begin{equation*}
t_{\lambda \mu \nu \mid \alpha}^{A}=M^{A C} \widehat{t}_{\lambda \mu \nu \mid \alpha}^{C} \tag{35}
\end{equation*}
$$

with $M^{A C}$ the elements of $M$. It is easy to see that this transformation leaves $S_{0}^{t}\left[t_{\lambda \mu \nu \mid \alpha}^{A}\right]$ invariant (it remains equal to a sum of free actions, one for every transformed field $\widehat{t}_{\lambda \mu \nu \mid \alpha}^{A}$ from the collection) and, moreover, the deformed action (27) becomes

$$
\begin{equation*}
\bar{S}_{0}\left[t_{\lambda \mu \nu \mid \alpha}^{A}, V_{\mu}\right]=S_{0}^{t}\left[\widehat{t}_{\lambda \mu \nu \mid \alpha}^{A}\right]-\frac{1}{4} \int d^{5} x \bar{F}_{\mu \nu}^{\prime} \bar{F}^{\prime \mu \nu} \tag{36}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{F}^{\prime \mu \nu}=F^{\mu \nu}+\frac{4 g}{3} y \varepsilon^{\mu \nu \alpha \beta \gamma} \partial_{[\rho} t_{\alpha \beta \gamma] \mid}^{1} \tag{37}
\end{equation*}
$$

Action (36) is invariant under the gauge transformations

$$
\begin{gather*}
\bar{\delta}_{\widehat{\varepsilon}, \widehat{\chi}} \widehat{t}_{\lambda \mu \nu \mid \alpha}^{A}=-3 \partial_{[\lambda} \widehat{\varepsilon}_{\mu \nu \alpha]}^{A}+4 \partial_{[\lambda} \widehat{\varepsilon}_{\mu \nu] \alpha}^{A}+\partial_{[\lambda} \widehat{\chi}_{\mu \nu] \mid \alpha}^{A}  \tag{38}\\
\bar{\delta}_{\widehat{\varepsilon}, \widehat{\chi}} V^{\mu}=\partial^{\mu} \varepsilon+4 g y \varepsilon^{\mu \alpha \beta \gamma \delta} \partial_{\alpha} \widehat{\varepsilon}_{\beta \gamma \delta}^{1} \tag{39}
\end{gather*}
$$

where now the new gauge parameters are

$$
\begin{equation*}
\widehat{\varepsilon}_{\mu \nu \alpha}^{A}=\varepsilon_{\mu \nu \alpha}^{B} M^{B A}, \quad \widehat{\chi}_{\mu \nu \mid \alpha}^{A}=\chi_{\mu \nu \mid \alpha}^{B} M^{B A} \tag{40}
\end{equation*}
$$

In conclusion, one cannot couple different fields with the mixed symmetry $(3,1)$ through a vector field. A single field of this kind may be non-trivially coupled in $D=5$, while the others remain free.

## 5 Conclusion

In this paper we have shown the rigidity of the couplings of a collection of tensor fields with the mixed symmetry $(3,1)$ to a vector field. Our final result resembles to the well known fact from the General Relativity according to which there is one graviton in a given world. This is not a surprise since the action of a free tensor field with the mixed symmetry $(3,1)$ is dual to the linearized gravity (in $D=6$ ).

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