## 11-dimensional Supergravity as an interacting theory on Minkowski space

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#### Abstract

Under the hypotheses of smoothness of the interactions in the coupling constant, locality, Poincaré invariance, Lorentz covariance, and the preservation of the number of derivatives on each field in the Lagrangian of the interacting theory (the same number of derivatives like in the free Lagrangian), we derive the consistent interactions in D = 11 among massless gravitini, a graviton, and a 3-form.

### 1 Introduction

It is well known that the field spectrum of D = 11, N = 1 SUGRA consists in a massless spin-2 field, a massless spin-3/2 field and a 3-form gauge field [1]-[2]. In the free limit the action of simple SUGRA in D = 11 reduces to the sum between Pauli-Fierz, Rarita-Schwinger, and a standard abelian 3-form actions

$$S_0^{\rm L}[h_{\mu\nu},\psi_{\mu},A_{\mu\nu\rho}] = S_0^{\rm PF}[h_{\mu\nu}] + S_0^{\rm RS}[\psi_{\mu}] + S_0^{\rm 3F}[A_{\mu\nu\rho}].$$
(1)

In order to determine the consistent interactions that can be added to action (1) we must study, beside the self-interactions, which are known from the literature, also the cross-couplings. The latter problem can be solved in two steps: firstly, we determine the interaction vertices containing only two of the three types of fields, and then the vertices including all the three kinds.

The plan of the talk :

- the derivation of consistent interactions between an abelian three-form gauge field and a Pauli-Fierz graviton;
- the investigation of the consistent couplings between an abelian three-form gauge field and a massless spin-3/2 field;
- the analysis of the cross-couplings between a Pauli-Fierz graviton and a massless Rarita-Schwinger field;
- the determination of all possible interactions among a graviton, a massless spin-3/2 field, and a three-form gauge field;

We investigate all these problems in the framework of the deformation theory [3] based on local BRST cohomology [4].

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# 2 First problem: the derivation of consistent interactions among an abelian three-form gauge field and a Pauli-Fierz graviton

#### 2.1 Free model

Our starting point is the Lagrangian action represented by the sum between Pauli-Fierz and an abelian three-form actions in eleven space-time dimensions

$$S_{0}^{h,A}[h_{\mu\nu}, A_{\mu\nu\rho}] = S_{0}^{PF}[h_{\mu\nu}] + S_{0}^{3F}[A_{\mu\nu\rho}] = \int d^{11}x \left[ -\frac{1}{2} \left( \partial_{\mu}h_{\nu\rho} \right) \left( \partial^{\mu}h^{\nu\rho} \right) + \left( \partial_{\mu}h^{\mu\rho} \right) \left( \partial^{\nu}h_{\nu\rho} \right) - \left( \partial_{\mu}h \right) \left( \partial_{\nu}h^{\nu\mu} \right) + \frac{1}{2} \left( \partial_{\mu}h \right) \left( \partial^{\mu}h \right) - \frac{1}{2 \cdot 4!} F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\lambda} \right].$$
(2)

The theory (2) is invariant under the gauge transformations

$$\delta_{\epsilon} h_{\mu\nu} = \partial_{(\mu} \epsilon_{\nu)}, \qquad \delta_{\varepsilon} A_{\mu\nu\rho} = \partial_{[\mu} \varepsilon_{\nu\rho]}. \tag{3}$$

The gauge parameters  $\epsilon_{\mu}$  and  $\varepsilon_{\mu\nu}$  are bosonic, the last set being completely antisymmetric. The gauge algebra of the free theory (2) is Abelian.

We observe that if in (3) we make the transformations

$$\varepsilon_{\mu\nu} \to \varepsilon_{\mu\nu}^{(\theta)} = \partial_{[\mu}\theta_{\nu]},$$
(4)

then the gauge variation of the 3-form identically vanishes

$$\delta_{\varepsilon^{(\theta)}} A_{\mu\nu\rho} \equiv 0. \tag{5}$$

Moreover, if in (4) we perform the changes

$$\theta_{\mu} \to \theta_{\mu}^{(\phi)} = \partial_{\mu}\phi, \tag{6}$$

with  $\phi$  an arbitrary scalar field, then the transformed gauge parameters (4) identically vanish

$$\varepsilon_{\mu\nu}^{\left(\theta^{(\phi)}\right)} \equiv 0. \tag{7}$$

Meanwhile, there is no non-vanishing local transformation of  $\phi$  that annihilates  $\theta_{\mu}^{(\phi)}$  of the form (6), and hence no further local reducibility identity. All these allow us to conclude that the generating set of gauge transformations (3) is off-shell second-stage reducible.

The structure of the reducibility relations is important from the point of view of the BRST symmetry as it requires the introduction of a tower of ghosts for ghosts, as well as of their antifields.

In order to construct the BRST symmetry for (2) we introduce the field, ghost, and antifield spectra

$$\Phi^{\Delta_0} = (h_{\mu\nu}, A_{\mu\nu\rho}), \qquad \Phi^*_{\Delta_0} = (h^{*\mu\nu}, A^{*\mu\nu\rho})$$
(8)

$$\eta^{\Delta_1} = (\eta_{\mu}, C_{\mu\nu}), \qquad \eta^*_{\Delta_1} = (\eta^{*\mu}, C^{*\mu\nu}), \qquad (9)$$

$$\eta^{\Delta_2} = (C_{\mu}), \qquad \eta^*_{\Delta_2} = (C^{*\mu}), \qquad (10)$$

$$\eta^{\Delta_3} = (C), \qquad \eta^*_{\Delta_3} = (C^*).$$
 (11)

In this case the anticanonical action of the BRST symmetry,  $s^{h,A} \cdot = (\cdot, S^{h,A})$ , is realized via a solution to the master equation  $(S^{h,A}, S^{h,A}) = 0$  that reads as

$$S^{h,A} = S_0^{h,A}[h_{\mu\nu}, A_{\mu\nu\rho}] + \int d^{11}x \left( h^{*\mu\nu} \partial_{(\mu}\eta_{\nu)} + A^{*\mu\nu\rho} \partial_{[\mu}C_{\nu\rho]} + C^{*\mu\nu} \partial_{[\mu}C_{\nu]} + C^{*\mu} \partial_{\mu}C \right).$$
(12)

#### 2.2 Construction of consistent interactions

We will associate with (12) a deformed solution

$$S^{h,A} \rightarrow \bar{S}^{h,A} = S^{h,A} + \lambda S_1^{h,A} + \lambda^2 S_2^{h,A} + \cdots$$
$$= S^{h,A} + \lambda \int d^{11}x \, a^{h,A} + \lambda^2 \int d^{11}x \, b^{h,A} + \cdots, \qquad (13)$$

which is the BRST generator of the interacting theory,  $(\bar{S}^{h,A}, \bar{S}^{h,A}) = 0$ , such that the components of  $\bar{S}^{h,A}$  are restricted to satisfy the tower of equations:

$$\left(S^{\mathrm{h},\mathrm{A}},S^{\mathrm{h},\mathrm{A}}\right) = 0, \tag{14}$$

$$2\left(S_{1}^{h,A},S^{h,A}\right) = 0,$$
 (15)

$$2\left(S_{2}^{h,A}, S^{h,A}\right) + \left(S_{1}^{h,A}, S_{1}^{h,A}\right) = 0,$$
:
(16)

The interactions are obtained under the following (reasonable) assumptions: smoothness, locality, Lorentz covariance, Poincaré invariance, and preservation of the number of derivatives on each field (derivative order assumption).

By direct computation we obtain the first-order deformation in the interacting sector like

$$S_{1}^{h,A} = S_{1}^{h-A} + S_{1}^{A} + S_{1}^{h} =$$

$$= \int d^{11}x \left\{ k \left[ -C^{*} \left( \partial^{\mu}C \right) \eta_{\mu} - \frac{1}{2} C_{\mu}^{*} \left( C_{\nu} \partial^{[\mu} \eta^{\nu]} - \left( \partial_{\nu}C \right) h^{\mu\nu} \right. \right. \\ \left. + 2 \left( \partial^{\nu}C^{\mu} \right) \eta_{\nu} \right) + C_{\mu\nu}^{*} \left( h_{\rho}^{\mu} \partial^{\rho}C^{\nu} - \left( \partial^{\rho}C^{\mu\nu} \right) \eta_{\rho} - \frac{1}{2} C_{\rho} \partial^{[\mu} h^{\nu]\rho} \right. \\ \left. + C_{\rho}^{\nu} \partial^{[\mu} \eta^{\rho]} \right) - A_{\mu\nu\rho}^{*} \left( \eta_{\lambda} \partial^{\lambda} A^{\mu\nu\rho} + \frac{3}{2} A^{\nu\rho}{}_{\lambda} \partial^{[\mu} \eta^{\lambda]} - \frac{3}{2} \left( \partial^{\lambda}C^{\nu\rho} \right) h_{\lambda}^{\mu} \right. \\ \left. - \frac{3}{2} C^{\rho\lambda} \partial^{[\mu} h_{\lambda}^{\nu]} \right) + \frac{1}{4} F_{\mu\nu\rho\lambda} \left( \partial^{\mu} \left( A^{\nu\rho\sigma} h_{\sigma}^{\lambda} \right) + \frac{1}{4!} F^{\mu\nu\rho\lambda} h \right. \\ \left. - \frac{1}{3} F^{\mu\nu\rho\sigma} h_{\sigma}^{\lambda} \right) \right] + q \varepsilon^{\mu_{1} \cdots \mu_{11}} A_{\mu_{1}\mu_{2}\mu_{3}} F_{\mu_{4} \cdots \mu_{7}} F_{\mu_{8} \cdots \mu_{11}} \right\} + S_{1}^{h},$$

$$(17)$$

where k and q are arbitrary real constants and  $S_1^{\rm h}$  reads as

$$S_{1}^{h} = \int d^{11}x \left\{ \frac{1}{2} \eta^{*\mu} \eta^{\nu} \partial_{[\mu} \eta_{\nu]} + h^{*\mu\rho} \left( (\partial_{\rho} \eta^{\nu}) h_{\mu\nu} - \eta^{\nu} \partial_{[\mu} h_{\nu]\rho} \right) + \mathcal{L}_{1}^{H-E} - 2\Lambda h \right\}.$$
(18)

In (18) we used the notations  $\mathcal{L}_1^{H-E}$  and  $\Lambda$  for the cubic vertex of the Einstein-Hilbert Lagrangian and respectively the cosmological constant.

The consistency of the first-order deformation (the existence of the second-order deformation) requires that the real constant k satisfies the equation

$$k(k+1) = 0, (19)$$

with the non-trivial solution

$$k = -1, \tag{20}$$

and q arbitrary constant.

The previous results can be summarized in the following theorem.

**Theorem 1** Under the assumptions of: i) space-time locality, ii) smoothness of the deformations in the coupling constant, iii) (background) Lorentz invariance, iv) Poincaré invariance (i.e. we do not allow explicit dependence on the space-time coordinates), v) the maximum number of derivatives in the interacting Lagrangian is two, the only consistent deformation of (2) involving a spin-2 field and an abelian 3-form gauge field reads as

$$\bar{S}_{0}^{\text{h,A}}[h_{\mu\nu}, A_{\mu\nu\rho}] = S_{0}^{\text{PF}}[h_{\mu\nu}] + S_{0}^{3\text{F}}[A_{\mu\nu\rho}] + \\
+\lambda \int d^{11}x \left[ \mathcal{L}_{1}^{\text{H-E}} - 2\Lambda h - \frac{1}{4 \cdot 4!} F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\lambda} h + \\
\frac{1}{2 \cdot 3!} F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\sigma} h_{\sigma}^{\lambda} - \frac{1}{4} F_{\mu\nu\rho\lambda} \partial^{\mu} \left( A^{\nu\rho\sigma} h_{\sigma}^{\lambda} \right) + \\
+q \varepsilon^{\mu_{1} \cdots \mu_{11}} A_{\mu_{1}\mu_{2}\mu_{3}} F_{\mu_{4} \cdots \mu_{7}} F_{\mu_{8} \cdots \mu_{11}} \right] + O\left(\lambda^{2}\right),$$
(21)

and it is invariant under the gauge transformations

$$\bar{\delta}_{\epsilon}h_{\mu\nu} = \partial_{(\mu}\epsilon_{\nu)} + \lambda \left(\frac{1}{2} \left(h_{\rho(\mu}\partial_{\nu)}\epsilon^{\rho} - \epsilon^{\rho}\partial_{(\mu}h_{\nu)\rho}\right) + \epsilon^{\rho}\partial_{\rho}h_{\mu\nu}\right) + O\left(\lambda^{2}\right),\tag{22}$$

$$\overline{\delta}_{\epsilon,\varepsilon}A_{\mu\nu\rho} = \partial_{[\mu}\varepsilon_{\nu\rho]} + \lambda \left[\epsilon^{\lambda}\partial_{\lambda}A_{\mu\nu\rho} + \frac{1}{2}A^{\lambda}{}_{[\mu\nu}\delta^{\sigma}{}_{\rho]}\partial_{[\sigma}\epsilon_{\lambda]} - \frac{1}{2}\left(\partial^{\lambda}\varepsilon_{[\mu\nu}\right)h_{\rho]\lambda} + \frac{1}{2}\varepsilon^{\lambda}{}_{[\mu}\partial_{\nu}h_{\rho]\lambda}\right] + O\left(\lambda^{2}\right),$$
(23)

which remain second-order reducible, with the first-order reducibility given by

$$\varepsilon_{\mu\nu} \to \varepsilon_{\mu\nu}^{(\theta)} = \partial_{[\mu}\theta_{\nu]} + \frac{\lambda}{2} \left[ \left( \partial^{\rho}\theta_{[\mu} \right) h_{\nu]\rho} + \theta^{\rho}\partial_{[\mu}h_{\nu]\rho} \right] + O\left(\lambda^2\right), \tag{24}$$

and the second-order redundancy expresses by

$$\theta_{\mu} \to \theta_{\mu}^{(\phi)} = \partial_{\mu}\phi - \frac{\lambda}{2}h_{\mu\nu}\partial^{\nu}\phi + O\left(\lambda^{2}\right).$$
(25)

The results summarized here are presented in detail (including the computation of the second order deformation for the solution to the classical master equation) in [5]. In this way we obtain that the first two orders of the interacting Lagrangian resulting from our setting originate in the development of the full interacting Lagrangian (in eleven space-time dimensions)

$$\tilde{\mathcal{L}} = \frac{2}{\lambda^2} \sqrt{g} \left( R - 2\lambda^2 \Lambda \right) + \mathcal{L}^{\mathrm{h-A}},$$

where the cross-coupling part reads as

$$\mathcal{L}^{h-A} = -\frac{1}{2\cdot 4!} \sqrt{g} \bar{F}_{\mu\nu\rho\lambda} \bar{F}^{\mu\nu\rho\lambda} + \lambda q \epsilon^{\mu_1\dots\mu_{11}} \bar{A}_{\mu_1\mu_2\mu_3} \bar{F}_{\mu_4\dots\mu_7} \bar{F}_{\mu_8\dots\mu_{11}}$$

with  $g = \det g_{\mu\nu}$ ,  $\Lambda$  the cosmological constant,  $\lambda$  the coupling constant, and q an arbitrary, real constant. Consequently, we showed [5] the uniqueness of interactions described by  $\tilde{\mathcal{L}}$ . The above interacting Lagrangian for  $\Lambda = 0$  is a part of D = 11, N = 1 SUGRA Lagrangian. We note that the graviton sector is allowed at this stage to include a cosmological term, unlike D = 11, N = 1 SUGRA. This is not a surprise since it is the simultaneous presence of all fields (supplemented with massless gravitini) that ensures the annihilation of the cosmological constant, as it will be seen in the last part of this talk.

## 3 Second problem: the investigation of the consistent couplings between an abelian three-form gauge field and a massless spin-3/2 field

#### 3.1 Free model

The starting point is the free theory whose Lagrangian action is written as the sum between an abelian three-form and a non-massive Rarita-Schwinger actions in eleven space-time dimensions

$$S_0^{\mathcal{A},\psi} \left[ A_{\mu\nu\rho}, \psi_{\mu} \right] = S_0^{3F} \left[ A_{\mu\nu\rho} \right] + S_0^{RS} \left[ \psi_{\mu} \right] =$$
$$= \int d^{11}x \left( -\frac{1}{2 \cdot 4!} F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\lambda} - \frac{\mathrm{i}}{2} \bar{\psi}_{\mu} \gamma^{\mu\nu\rho} \partial_{\nu} \psi_{\rho} \right).$$
(26)

In the above the fermionic fields  $\psi_{\mu}$  are considered to be real (Majorana) spinors ( $\bar{\psi} = \psi^c$ ). We work with the metric  $\sigma_{\mu\nu} = \text{diag}(+, -, \dots, -)$ , the gamma matrices ( $\gamma_{\mu}$ ) are all purely imaginary. We take the matrix  $\gamma^0$  to be antisymmetric and Hermitian,  $\gamma^i$  ( $i = \overline{1, 10}$ ) are symmetric and anti-Hermitian.

The theory described by action (26) possesses an Abelian, off-shell, second-order reducible generating set of gauge transformations

$$\delta_{\varepsilon} A_{\mu\nu\rho} = \partial_{[\mu} \varepsilon_{\nu\rho]}, \qquad \delta_{\varepsilon} \psi_{\mu} = \partial_{\mu} \varepsilon.$$
<sup>(27)</sup>

Related to the gauge parameters,  $\varepsilon_{\mu\nu}$  are bosonic and completely antisymmetric and  $\varepsilon$  is a fermionic Majorana spinor. The redundancy of the generating set of gauge transformations for the three-form gauge field has been presented in the above [in the part that solves the problem of constructing consistent interactions among an abelian three-form gauge field and a Pauli-Fierz graviton].

The structure of the reducibility relations is important from the point of view of the BRST symmetry as it requires the introduction of a tower of ghosts for ghosts, as well as of their antifields.

In order to construct the BRST symmetry for (26) we introduce the field, ghost, and antifield spectra

$$\Phi^{\Delta_0} = (A_{\mu\nu\rho}, \psi_{\mu}), \qquad \Phi^*_{\Delta_0} = (A^{*\mu\nu\rho}, \psi^{*\mu}), \qquad (28)$$

$$\eta^{\Delta_1} = (C_{\mu\nu}, \xi), \qquad \eta^*_{\Delta_1} = (C^{*\mu\nu}, \xi^*), \qquad (29)$$

$$\eta^{\Delta_2} = (C_{\mu}), \qquad \eta^*_{\Delta_2} = (C^{*\mu}), \qquad (30)$$

$$\eta^{\Delta_3} = (C), \qquad \eta^*_{\Delta_3} = (C^*).$$
 (31)

In this case the anticanonical action of the BRST symmetry,  $s^{A,\psi} \cdot = (\cdot, S^{A,\psi})$ , is realized via a solution to the master equation  $(S^{A,\psi}, S^{A,\psi}) = 0$  that reads as

$$S^{A,\psi} = S_0^{A,\psi} [A_{\mu\nu\rho}, \psi_{\mu}] + \int d^{11}x \left( \psi^{*\mu} \partial_{\mu} \xi + A^{*\mu\nu\rho} \partial_{[\mu} C_{\nu\rho]} + C^{*\mu\nu} \partial_{[\mu} C_{\nu]} + C^{*\mu} \partial_{\mu} C \right).$$
(32)

#### **3.2** Construction of consistent interactions

We will associate with (32) a deformed solution

$$S^{A,\psi} \rightarrow \bar{S}^{A,\psi} = S^{A,\psi} + \lambda S_1^{A,\psi} + \lambda^2 S_2^{A,\psi} + \cdots$$
$$= S^{A,\psi} + \lambda \int d^{11}x \, a^{A,\psi} + \lambda^2 \int d^{11}x \, b^{A,\psi} + \cdots, \qquad (33)$$

which is the BRST generator of the interacting theory,  $(\bar{S}^{A,\psi}, \bar{S}^{A,\psi}) = 0$ , such that the components of  $\bar{S}^{A,\psi}$  are restricted to satisfy the tower of equations:

$$\left(S^{\mathcal{A},\psi},S^{\mathcal{A},\psi}\right) = 0, \tag{34}$$

$$2\left(S_1^{\mathcal{A},\psi},S^{\mathcal{A},\psi}\right) = 0, \qquad (35)$$

$$2\left(S_{2}^{A,\psi},S^{A,\psi}\right) + \left(S_{1}^{A,\psi},S_{1}^{A,\psi}\right) = 0,$$
:
(36)

The interactions are obtained under the following (reasonable) assumptions: smoothness, locality, Lorentz covariance, Poincaré invariance, and preservation of the number of derivatives on each field (derivative order assumption).

By direct computation we obtain the first-order deformation

$$S_{1}^{A,\psi} = S_{1}^{A-\psi} + S_{1}^{A} + S_{1}^{\psi} =$$

$$= \int d^{11}x \left[ \tilde{k} \left( \frac{1}{2} C^{*\mu\nu} \bar{\xi} \gamma_{\mu\nu} \xi - 3A^{*\mu\nu\rho} \bar{\xi} \gamma_{\mu\nu} \psi_{\rho} + \frac{1}{9} \psi^{*\mu} F_{\mu\nu\rho\lambda} \gamma^{\nu\rho\lambda} \xi - \frac{1}{3 \cdot 4!} \psi^{*\mu} F^{\nu\rho\lambda\sigma} \gamma_{\mu\nu\rho\lambda\sigma} \xi + \frac{1}{4} \bar{\psi}_{\mu} \gamma_{\nu\rho} \psi_{\lambda} F^{\mu\nu\rho\lambda} - \frac{1}{2 \cdot 4!} \bar{\psi}^{\alpha} \gamma_{\alpha\beta\mu\nu\rho\lambda} \psi^{\beta} F^{\mu\nu\rho\lambda} \right) +$$

$$+ m \left( \psi_{\mu}^{*} \gamma^{\mu} \xi + \frac{9i}{2} \psi_{\mu} \gamma^{\mu\nu} \psi_{\nu} \right) + \frac{1}{4} q \varepsilon^{\mu_{1} \cdots \mu_{11}} A_{\mu_{1} \mu_{2} \mu_{3}} F_{\mu_{4} \cdots \mu_{7}} F_{\mu_{8} \cdots \mu_{11}} \right], \qquad (37)$$

where k, m and q are arbitrary constants. In the above the terms proportional with m contain only fields/antifields from the Rarita-Schwinger sector, those proportional with q include only the three-form gauge field and the other pieces mix both sectors.

The consistency of the first-order deformation (the existence of the second-order deformation) requires

$$\tilde{k} = 0 \qquad and \qquad m = 0, \tag{38}$$

and q remains an arbitrary constant.

The previous results can be summarized in the following theorem.

**Theorem 2** Under the assumptions of: i) space-time locality, ii) smoothness of the deformations in the coupling constant, iii) (background) Lorentz invariance, iv) Poincaré invariance (i.e. we do not allow explicit dependence on the space-time coordinates), v) the maximum number of derivatives in the interacting Lagrangian is two, the only consistent deformation of (26) involving an abelian three-form gauge field and a massless spin-3/2 field reads as

$$\bar{S}_{0}^{\mathrm{A},\psi}[A_{\mu\nu\rho},\psi_{\mu}] = S_{0}^{\mathrm{3F}}[A_{\mu\nu\rho}] + S_{0}^{\mathrm{RS}}[\psi_{\mu}] +q\lambda \int d^{11}x \varepsilon^{\mu_{1}\cdots\mu_{11}} A_{\mu_{1}\mu_{2}\mu_{3}}F_{\mu_{4}\cdots\mu_{7}}F_{\mu_{8}\cdots\mu_{11}},$$
(39)

and it is invariant under the original gauge transformations.

This result does not contradict the presence in the Lagrangian of D = 11, N = 1 SUGRA of a quartic vertex expressing self-interactions among the gravitini. We prove in [7] and [8] that this vertex, which appears at order two in the coupling constant, is due to the *simultaneous* presence of gravitini, three-form, *and* graviton.

The results synthesized in this part are developed in [6].

## 4 Third problem: the analysis of the cross-couplings among a Pauli-Fierz graviton and a massless Rarita-Schwinger field

#### 4.1 Free model

The Lagrangian action for the free theory is represented by the sum between Pauli-Fierz and a nonmassive Rarita-Schwinger actions in eleven space-time dimensions

$$S_{0}^{\mathrm{h},\psi} [h_{\mu\nu},\psi_{\mu}] = S_{0}^{\mathrm{PF}} [h_{\mu\nu}] + S_{0}^{\mathrm{RS}} [\psi_{\mu}] =$$

$$= \int d^{11}x \left[ -\frac{1}{2} \left( \partial_{\mu}h_{\nu\rho} \right) \left( \partial^{\mu}h^{\nu\rho} \right) + \left( \partial_{\mu}h^{\mu\rho} \right) \left( \partial^{\nu}h_{\nu\rho} \right) - \left( \partial_{\mu}h \right) \left( \partial_{\nu}h^{\nu\mu} \right) + \frac{1}{2} \left( \partial_{\mu}h \right) \left( \partial^{\mu}h \right) - \frac{\mathrm{i}}{2} \bar{\psi}_{\mu} \gamma^{\mu\nu\rho} \partial_{\nu}\psi_{\rho} \right].$$
(40)

Action (40) possesses an irreducible and Abelian generating set of gauge transformations

$$\delta_{\varepsilon}h_{\mu\nu} = \partial_{(\mu}\epsilon_{\nu)}, \qquad \delta_{\varepsilon}\psi_{\mu} = \partial_{\mu}\varepsilon, \tag{41}$$

with  $\epsilon_{\mu}$  bosonic and  $\varepsilon$  fermionic gauge parameters.

In order to construct the BRST symmetry for (26) we introduce the field, ghost, and antifield spectra

$$\Phi^{\Delta_0} = (h_{\mu\nu}, \psi_{\mu}), \qquad \Phi^*_{\Delta_0} = \left(h^{*\mu\nu}, \psi^*_{\mu}\right), \tag{42}$$

$$\eta^{\Delta_1} = (\eta_{\mu}, \xi), \qquad \eta^*_{\Delta_1} = (\eta^{*\mu}, \xi^*), \qquad (43)$$

In this case the anticanonical action of the BRST symmetry,  $s^{h,\psi} = (\cdot, S^{h,\psi})$ , is realized via a solution to the master equation  $(S^{h,\psi}, S^{h,\psi}) = 0$  that reads as

$$S^{h,\psi} = S_0^{h,\psi} \left[ h_{\mu\nu}, \psi_{\mu} \right] + \int d^{11}x \left( h^{*\mu\nu} \partial_{(\mu} \eta_{\nu)} + \psi^{*\mu} \partial_{\mu} \xi \right).$$
(44)

#### 4.2 Construction of consistent interactions

We will associate with (44) a deformed solution

$$S^{\mathbf{h},\psi} \rightarrow \bar{S}^{\mathbf{h},\psi} = S^{\mathbf{h},\psi} + \lambda S_1^{\mathbf{h},\psi} + \lambda^2 S_2^{\mathbf{h},\psi} + \cdots$$
$$= S^{\mathbf{h},\psi} + \lambda \int d^{11}x \, a^{\mathbf{h},\psi} + \lambda^2 \int d^{11}x \, b^{\mathbf{h},\psi} + \cdots, \qquad (45)$$

which is the BRST generator of the interacting theory,  $(\bar{S}^{h,\psi}, \bar{S}^{h,\psi}) = 0$ , such that the components of  $\bar{S}^{h,\psi}$  are restricted to satisfy the tower of equations:

$$\left(S^{\mathbf{h},\psi},S^{\mathbf{h},\psi}\right) = 0, \tag{46}$$

$$2\left(S_{1}^{h,\psi},S^{h,\psi}\right) = 0,$$
 (47)

$$2\left(S_{2}^{h,\psi},S^{h,\psi}\right) + \left(S_{1}^{h,\psi},S_{1}^{h,\psi}\right) = 0,$$
:
(48)

The interactions are obtained under the following (reasonable) assumptions: smoothness, locality, Lorentz covariance, Poincaré invariance, and preservation of the number of derivatives on each field (derivative order assumption). By direct computation we obtain the first-order deformation

$$S_{1}^{\mathrm{h},\psi} = S_{1}^{\mathrm{h}-\psi} + S_{1}^{\psi} + S_{1}^{\mathrm{h}} =$$

$$= \int d^{11}x \left\{ \bar{k} \left[ \frac{\mathrm{i}}{4} \left( \frac{1}{2} \bar{\psi}^{\mu} \left( \gamma^{\rho} \psi^{\nu} - 2\sigma^{\nu\rho} \gamma_{\lambda} \psi^{\lambda} \right) \partial_{[\mu} h_{\nu]\rho} - h \bar{\psi}_{\mu} \gamma^{\mu\nu\rho} \partial_{\nu} \psi_{\rho} \right. \right. \\ \left. + \bar{\psi}_{\mu} \gamma^{\mu\nu\rho} \left( \partial^{\lambda} \psi_{\rho} \right) h_{\nu\lambda} - \frac{1}{2} \bar{\psi}_{\mu} \gamma^{\mu\nu\rho} \psi^{\lambda} \partial_{[\nu} h_{\rho]\lambda} \right) + \frac{1}{4} \left( \mathrm{i} h^{*\mu\nu} \bar{\xi} \gamma_{\mu} \psi_{\nu} \right. \\ \left. + \frac{1}{2} \psi^{*\mu} \gamma^{\alpha\beta} \left( \psi_{\mu} \partial_{[\alpha} \eta_{\beta]} - \xi \partial_{[\alpha} h_{\beta]\mu} \right) + 4 \psi^{*\mu} \left( \partial_{\nu} \psi_{\mu} \right) \eta^{\nu} + 2 \psi^{*\mu} \psi^{\nu} \partial_{[\mu} \eta_{\nu]} \right. \\ \left. - 2 \psi^{*\mu} \left( \partial^{\nu} \xi \right) h_{\mu\nu} \right) + \xi^{*} \left( \partial_{\mu} \xi \right) \eta^{\mu} - \frac{1}{8} \left( \frac{\mathrm{i}}{2} \eta^{*\mu} \bar{\xi} \gamma_{\mu} \xi - \xi^{*} \gamma^{\mu\nu} \xi \partial_{[\mu} \eta_{\nu]} \right) \right] \\ \left. + \mathrm{i} m \psi_{\mu}^{*} \gamma^{\mu} \xi - \frac{9m}{2} \bar{\psi}_{\mu} \gamma^{\mu\nu} \psi_{\nu} \right\} + S_{1}^{\mathrm{h}},$$

$$(49)$$

with  $\bar{k}$  and m some arbitrary constants.

The consistency of the first-order deformation (the existence of the second-order deformation) imposes

$$\bar{k} = 0 \qquad and \qquad m = 0. \tag{50}$$

The previous results can be summarized in the following theorem.

**Theorem 3** Under the assumptions of: i) space-time locality, ii) smoothness of the deformations in the coupling constant, iii) (background) Lorentz invariance, iv) Poincaré invariance (i.e. we do not allow explicit dependence on the space-time coordinates), v) the maximum number of derivatives in the interacting Lagrangian is two, the only consistent deformation of (40) involving a spin-2 field and a massless spin-3/2 field reads as

$$\bar{S}_{0}^{\mathrm{h},\psi}[h_{\mu\nu},\psi_{\mu}] = S_{0}^{\mathrm{PF}}[h_{\mu\nu}] + S_{0}^{\mathrm{RS}}[\psi_{\mu}] +\lambda \int d^{11}x \left(\mathcal{L}^{\mathrm{H-E}} - 2\Lambda h\right) + O\left(\lambda^{2}\right),$$
(51)

and it is invariant under the gauge transformations

$$\overline{\delta}_{\epsilon}h_{\mu\nu} = \partial_{(\mu}\epsilon_{\nu)} + \lambda \left(\frac{1}{2} \left(h_{\rho(\mu}\partial_{\nu)}\epsilon^{\rho} - \epsilon^{\rho}\partial_{(\mu}h_{\nu)\rho}\right) + \epsilon^{\rho}\partial_{\rho}h_{\mu\nu}\right) + O\left(\lambda^{2}\right),\tag{52}$$

$$\overline{\delta}_{\varepsilon}\psi_{\mu} = \delta_{\varepsilon}\psi_{\mu} = \partial_{\mu}\varepsilon. \tag{53}$$

As in the above,  $\mathcal{L}^{H-E}$  represents the cubic vertex of the Einstein-Hilbert Lagrangian.

The absence of self-interactions among the gravitini in D = 11 at this level does not contradict the presence in the Lagrangian of D = 11, N = 1 SUGRA of a quartic gravitini vertex.

The results shortly addressed here are presented in detail in [7]. In the same paper we also make the comparison with the case D = 4, where gravitini are known to allow self-interactions in the presence of a graviton, such that their 'mass' constant becomes related to the cosmological one.

## 5 Fourth problem: all possible interactions among a graviton, a massless spin-3/2 field, and a three-form gauge field

The starting point of the last problem is represented by the free model with the Lagrangian action given by the sum between Pauli-Fierz, an abelian three-form and a non-massive Rarita-Schwinger actions in eleven space-time dimensions

$$S_{0}^{\mathrm{h},\mathrm{A},\psi}[h_{\mu\nu},A_{\mu\nu\rho},\psi_{\mu}] = S_{0}^{\mathrm{PF}}[h_{\mu\nu}] + S_{0}^{\mathrm{3F}}[A_{\mu\nu\rho}] + S_{0}^{\mathrm{RS}}[\psi_{\mu}] = \int d^{11}x \left[ -\frac{1}{2} \left( \partial_{\mu}h_{\nu\rho} \right) \left( \partial^{\mu}h^{\nu\rho} \right) + \left( \partial_{\mu}h^{\mu\rho} \right) \left( \partial^{\nu}h_{\nu\rho} \right) - \left( \partial_{\mu}h \right) \left( \partial_{\nu}h^{\nu\mu} \right) \right. \\ \left. + \frac{1}{2} \left( \partial_{\mu}h \right) \left( \partial^{\mu}h \right) - \frac{1}{2 \cdot 4!} F_{\mu\nu\rho\lambda}F^{\mu\nu\rho\lambda} - \frac{\mathrm{i}}{2}\bar{\psi}_{\mu}\gamma^{\mu\nu\rho}\partial_{\nu}\psi_{\rho} \right].$$

$$(54)$$

The theory described by action (54) possesses an Abelian, off-shell, second-order reducible generating set of gauge transformations

$$\delta_{\varepsilon} h_{\mu\nu} = \partial_{(\mu} \epsilon_{\nu)}, \qquad \delta_{\varepsilon} A_{\mu\nu\rho} = \partial_{[\mu} \varepsilon_{\nu\rho]}, \qquad \delta_{\varepsilon} \psi_{\mu} = \partial_{\mu} \varepsilon.$$
(55)

In order to construct the BRST symmetry for (54) we introduce the field, ghost, and antifield spectra

$$\Phi^{\Delta_0} = (h_{\mu\nu}, A_{\mu\nu\rho}, \psi_{\mu}), \qquad \Phi^*_{\Delta_0} = \left(h^{*\mu\nu}, A^{*\mu\nu\rho}, \psi^*_{\mu}\right)$$
(56)

$$\Phi_{\Delta_{0}}^{-1} = (h_{\mu\nu}, A_{\mu\nu\rho}, \psi_{\mu}), \qquad \Phi_{\Delta_{0}}^{-1} = (h^{\mu\nu}, A^{\mu\nu\rho}, \psi_{\mu}^{+})$$

$$\eta^{\Delta_{1}} = (\eta_{\mu}, C_{\mu\nu}, \xi), \qquad \eta^{*}_{\Delta_{1}} = (\eta^{*\mu}, C^{*\mu\nu}, \xi^{*}), \qquad (57)$$

$$\eta^{\Delta_{2}} = (C_{\mu}), \qquad \eta^{*}_{\Delta_{2}} = (C^{*\mu}), \qquad (58)$$

$$\eta^{\Delta_2} = (C_{\mu}), \qquad \eta^*_{\Delta_2} = (C^{*\mu}), \qquad (58)$$

$$\eta^{\Delta_3} = (C), \qquad \eta^*_{\Delta_3} = (C^*).$$
(59)

In this case the anticanonical action of the BRST symmetry,  $s^{h,A,\psi} = (\cdot, S^{h,A,\psi})$ , is realized via a solution to the master equation  $\left(S^{\mathrm{h},\mathrm{A},\psi},S^{\mathrm{h},\mathrm{A},\psi}\right)=0$  that reads as

$$S^{h,A,\psi} = S_0^{h,A,\psi} [h_{\mu\nu}, A_{\mu\nu\rho}, \psi_{\mu}] + \int d^{11}x \left( h^{*\mu\nu} \partial_{(\mu} \eta_{\nu)} + \psi^{*\mu} \partial_{\mu} \xi + A^{*\mu\nu\rho} \partial_{[\mu} C_{\nu\rho]} + C^{*\mu\nu} \partial_{[\mu} C_{\nu]} + C^{*\mu} \partial_{\mu} C \right).$$
(60)

#### Construction of consistent interactions 5.1

We will associate with (60) a deformed solution

$$S^{\mathbf{h},\mathbf{A},\psi} \rightarrow \bar{S}^{\mathbf{h},\mathbf{A},\psi} = S^{\mathbf{h},\mathbf{A},\psi} + \lambda S_1^{\mathbf{h},\mathbf{A},\psi} + \lambda^2 S_2^{\mathbf{h},\mathbf{A},\psi} + \cdots$$
$$= S^{\mathbf{h},\mathbf{A},\psi} + \lambda \int d^{11}x \, a^{\mathbf{h},\mathbf{A},\psi} + \lambda^2 \int d^{11}x \, b^{\mathbf{h},\mathbf{A},\psi} + \cdots, \qquad (61)$$

which is the BRST generator of the interacting theory,  $(\bar{S}^{h,A,\psi}, \bar{S}^{h,A,\psi}) = 0$ , such that the components of  $\bar{S}^{h,A,\psi}$  are restricted to satisfy the tower of equations:

$$\left(S^{\mathbf{h},\mathbf{A},\psi},S^{\mathbf{h},\mathbf{A},\psi}\right) = 0, \tag{62}$$

$$2\left(S_1^{\mathbf{h},\mathbf{A},\psi},S^{\mathbf{h},\mathbf{A},\psi}\right) = 0, \tag{63}$$

$$2\left(S_{2}^{h,A,\psi},S^{h,A,\psi}\right) + \left(S_{1}^{h,A,\psi},S_{1}^{h,A,\psi}\right) = 0,$$
:
(64)

The interactions are obtained under the following (reasonable) assumptions: smoothness, locality, Lorentz covariance, Poincaré invariance, and preservation of the number of derivatives on each field (derivative order assumption).

For this situation, the first-order deformation of the solution to the classical master equation can be written as

$$S_1^{\mathbf{h},\mathbf{A},\psi} = S_1^{\mathbf{h}-\mathbf{A}} + S_1^{\mathbf{h}-\psi} + S_1^{\mathbf{A}-\psi} + S_1^{\psi} + S_1^{\mathbf{h}} + S_1^{\mathbf{A}}.$$
(65)

The consistency of the first-order deformation (the existence of the second-order deformation) requires that the constants  $k, \bar{k}, q, m$  and  $\Lambda$  to satisfy the following algebraic equations

$$\tilde{k}^2 + \frac{k^2}{32} = 0, \qquad 180m^2 - \bar{k}\Lambda = 0,$$
(66)

$$k(k+1) = 0, \qquad \tilde{k}^2 - \frac{kk}{32} = 0,$$
(67)

$$m\tilde{k} = 0, \qquad \tilde{k}\left(q + \frac{\tilde{k}}{3\cdot(12)^3}\right) = 0,$$
(68)

$$\bar{k}(\bar{k}-1) = 0, \qquad \tilde{k}(k+\bar{k}) = 0.$$
 (69)

There are two types of nontrivial solutions, namely

$$k = -1 \text{ or } k = 0, \qquad \tilde{k} = \bar{k} = m = 0, \qquad \Lambda, q = \text{arbitrary},$$
(70)

$$k = -\bar{k} = -1, \qquad \tilde{k}_{1,2} = \pm \frac{i\sqrt{2}}{8}, \qquad q_{1,2} = -\frac{4\bar{k}_{1,2}}{(12)^4}, \qquad m = 0 = \Lambda.$$
 (71)

The former type is less interesting from the point of view of interactions since it maximally allows the graviton to be coupled to the 3-form (if k = -1).

For this reason in the sequel we will extensively focus on the latter solution, (71), which forbids both the presence of the cosmological term for the spin-2 field and the appearance of gravitini 'mass' constant.

The previous results can be summarized in the following theorem.

**Theorem 4** Under the assumptions of: i) space-time locality, ii) smoothness of the deformations in the coupling constant, iii) (background) Lorentz invariance, iv) Poincaré invariance (i.e. we do not allow explicit dependence on the space-time coordinates), v) the maximum number of derivatives in the interacting Lagrangian is two, the only consistent deformation of (54) involving a spin-2 field, an abelian three-form gauge field and a massless spin-3/2 field reads as

$$\bar{S}_{0}^{\mathrm{h},\mathrm{A},\psi}[h_{\mu\nu},A_{\mu\nu\rho},\psi_{\mu}] = S_{0}^{\mathrm{PF}}[h_{\mu\nu}] + S_{0}^{3\mathrm{F}}[A_{\mu\nu\rho}] + S_{0}^{\mathrm{RS}}[\psi_{\mu}] \\
+\lambda \int d^{11}x \left[ \mathcal{L}^{\mathrm{H}-\mathrm{E}} - \frac{1}{4\cdot4!} F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\lambda} h + \frac{1}{2\cdot3!} F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\sigma} h_{\sigma}^{\lambda} - \frac{1}{4} F_{\mu\nu\rho\lambda} \partial^{\mu} \left( A^{\nu\rho\sigma} h_{\sigma}^{\lambda} \right) + \frac{1}{q\epsilon^{\mu_{1}\cdots\mu_{11}}} A_{\mu_{1}\mu_{2}\mu_{3}} F_{\mu_{4}\cdots\mu_{7}} F_{\mu_{8}\cdots\mu_{11}} \\
-\tilde{k}_{i} \left( \frac{1}{4} \bar{\psi}_{\mu} \gamma_{\nu\rho} \psi_{\lambda} F^{\mu\nu\rho\lambda} + \frac{1}{2\cdot4!} \bar{\psi}^{\alpha} \gamma_{\alpha\beta\mu\nu\rho\lambda} \psi^{\beta} F^{\mu\nu\rho\lambda} \right) \\
+ \frac{1}{4} \left( \frac{1}{2} \bar{\psi}^{\mu} \left( \gamma^{\rho} \psi^{\nu} - 2\sigma^{\nu\rho} \gamma_{\lambda} \psi^{\lambda} \right) \partial_{[\mu} h_{\nu]\rho} - h \bar{\psi}_{\mu} \gamma^{\mu\nu\rho} \partial_{\nu} \psi_{\rho} \\
+ \bar{\psi}_{\mu} \gamma^{\mu\nu\rho} \left( \partial^{\lambda} \psi_{\rho} \right) h_{\nu\lambda} - \frac{1}{2} \bar{\psi}_{\mu} \gamma^{\mu\nu\rho} \psi^{\lambda} \partial_{[\nu} h_{\rho]\lambda} \right] + O \left( \lambda^{2} \right)$$
(72)

and it is invariant under the gauge transformations

$$\bar{\delta}_{\epsilon,\varepsilon}h_{\mu\nu} = \partial_{(\mu}\epsilon_{\nu)} + \lambda \left[\frac{1}{2}h_{\rho(\mu}\partial_{\nu)}\epsilon^{\rho} - \frac{1}{2}\epsilon^{\rho}\partial_{(\mu}h_{\nu)\rho} + \epsilon^{\rho}\partial_{\rho}h_{\mu\nu} + \frac{i}{8}\bar{\varepsilon}\gamma_{(\mu}\psi_{\nu)}\right] + ...,$$
(73)  
$$\bar{\delta}_{\epsilon,\varepsilon}\psi_{\mu} = \partial_{\mu}\varepsilon + \lambda \left[-\frac{1}{2}h_{\mu}^{\nu}\partial_{\nu}\varepsilon + (\partial_{\alpha}\psi_{\mu})\epsilon^{\alpha} + \frac{1}{2}\psi^{\nu}\partial_{[\mu}\epsilon_{\nu]} + \frac{1}{8}\gamma^{\alpha\beta}\psi_{\mu}\partial_{[\alpha}\epsilon_{\beta]} \right]$$
$$\frac{1}{2}\alpha\beta \partial_{\mu}\mu_{\mu\nu} + \frac{i\tilde{k}_{i}}{2}\left(-\mu\rho\lambda - \mu_{\mu\nu}\right)\left[-\frac{1}{2}h_{\mu\nu}^{\nu}\partial_{\mu}\varepsilon + (\partial_{\mu}\psi_{\mu})\partial_{\mu}\phi_{\mu}\right] + ...,$$
(73)

$$-\frac{1}{8}\gamma^{\alpha\beta}\varepsilon\partial_{[\alpha}h_{\beta]\mu} + \frac{1\kappa_i}{9}\left(\gamma^{\nu\rho\lambda}\varepsilon F_{\mu\nu\rho\lambda} - \frac{1}{8}\gamma_{\mu\nu\rho\lambda\sigma}\varepsilon F^{\nu\rho\lambda\sigma}\right)\right] + \dots,$$
(74)

$$\bar{\delta}_{\epsilon,\varepsilon}A_{\alpha\beta\gamma} = \partial_{[\alpha}\varepsilon_{\beta\gamma]} + \lambda \left[\epsilon^{\delta}\partial_{\delta}A_{\alpha\beta\gamma} + \frac{1}{2}A^{\delta}{}_{[\alpha\beta}\delta^{\sigma}_{\gamma]}\partial_{[\sigma}\epsilon_{\delta]} - \frac{1}{2}\left(\partial^{\delta}\varepsilon_{[\alpha\beta]}\right)h_{\gamma]\delta} + \frac{1}{2}\varepsilon^{\delta}{}_{[\alpha}\partial_{\beta}h_{\gamma]\delta} - \tilde{k}_{i}\bar{\xi}\gamma_{[\alpha\beta}\psi_{\gamma]}\right] + \dots,$$
(75)

which remain second-order reducible.

The effective expression of the second-order deformation and the interpretation of the interacting theory are exposed in [8].

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### References

- [1] E. Cremmer, B. Julia and J. Scherk, Phys. Lett. B76, 409 (1978)
- B. de Wit, Introduction to supergravity, in Supersymmetry and Supergravity '84: Proceedings of the Trieste Spring School 4-14 April, 1984, eds. B. de Wit, P. Fayet and P. van Nieuwenhuizen, p. 49 (World Scientific, Singapore, 1984)
- [3] G. Barnich and M. Henneaux, Phys. Lett. B311, 123 (1993)
- [4] G. Barnich, F. Brandt and M. Henneaux, Phys. Rept. 338, 439 (2000)
- [5] E. M. Cioroianu, E. Diaconu and S. C. Sararu, On the uniqueness of D = 11 interactions among a graviton, a massless gravitino and a three-form. I: Pauli-Fierz and three-form, To appear in Int. J. Mod. Phys. A (2008)
- [6] E. M. Cioroianu, E. Diaconu and S. C. Sararu, On the uniqueness of D = 11 interactions among a graviton, a massless gravitino and a three-form. II: Three-form and gravitini, To appear in Int. J. Mod. Phys. A (2008)
- [7] E. M. Cioroianu, E. Diaconu and S. C. Sararu, On the uniqueness of D = 11 interactions among a graviton, a massless gravitino and a three-form. III: Graviton and gravitini, *To appear in Int. J. Mod. Phys.* **A** (2008)
- [8] E. M. Cioroianu, E. Diaconu and S. C. Sararu, On the uniqueness of D = 11 interactions among a graviton, a massless gravitino and a three-form. IV: Putting things together, To appear in Int. J. Mod. Phys. A (2008)