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The spectrum of the generators in the $sp(3)$ BRST Lagrangian approach for 1-reducible theories

Carmen Ionescu
Dept. of Theor. Phys., Univ. of Craiova, 13 A. I. Cuza Str. 200585

Abstract

A consistent $sp(3)$ BRST description of the 1-reducible gauge theories in a Lagrangian form is possible using for variables and operators a bi-graduation (gh, lev). It has to be done in an extended space generated by the fields (real and ghost type) and antifields. The complete spectrum of these generators will be done in the paper.

1 Introduction

Many models of interesting field theories have gauge invariance properties which are expressed by using some linear dependent generators. Such examples of gauge reducible theories are offered by the two forms models [1], [2] by the models from gravitation and supergravitation in spaces with $d \neq 4$ [3], or by superstrings [4], [5].

An important technique in studying these models is represented by the BRST approach [6], [7]. It allows to include all the gauge invariances of the model in a more general and global symmetry, s, called the BRST symmetry. Moreover, it is well known that a more general symmetry has been defined, the BRST-antiBRST symmetry [8], [9] and extended formalism has beed developed. This $sp(2)$ symmetry solved a lot of practical and principial problems in constructing and in understanding the BRST technique: a consistent approach to anomalies, the correct understanding of the non-minimal sector in the BRST setting. Despite that, other problems are still remaining and a more general approach has been necessary. This is why general $sp(n \geq 3)$ BRST theories has been formulated [10].

A complete and consistent Hamiltonian description has been done using new graduation rules, based on spliting the generators on many levels, both for irreducible and reducible theories. A $sp(3)$ Lagrangian description following this approach has been presented for irreducible case only [11]. To end "the circle" the development of the formalism for reducible theories is needed. After this, the equivalence between the sp(3) BRST Hamiltonian and Lagrangian approaches needs to be done. The present paper will do a first step from the sp(3) BRST Lagrangian formalism by presenting how the extended space (with ghost-fields and antifields) for 1-reducible theories can be constructed.

The paper has the following structure: after this introductive part, in the section 2, general ideas on the $sp(3)$ BRST Lagrangian theory will be recalled. In the section 3, the construction of the exterior longitudinal tricomplex (generated by fields: real and ghost-type) will be done. The Koszul-Tate tricomplex (generated by antifields) will be built in the section 4. Some concluding remarks will end the paper.

2 General ideas on the sp(3) BRST Lagrangian theory

Let us consider a theory described by the Lagrangian action $S_0[q]$ which are invariant at the gauge transformations:

$$\delta_\varepsilon q^i = R^i_{\alpha_0}(q)\varepsilon^{\alpha_0},\ i = 1, \cdots, n;\ \alpha_0 = 1, \cdots, m_0 \tag{1}$$

where real variables $q \equiv \{q^1, \cdots, q^n\}$ have the Grassmann parities $\varepsilon(q^i) = \varepsilon_i$. The gauge parameters ε^{α_0} have the Grassmann parities $\varepsilon(\varepsilon^{\alpha_0}) = \varepsilon_{\alpha_0}$ and the generators of the gauge transformations $R^i_{\alpha_0} =$

$R^i_{\alpha_0}(q)$ have $\varepsilon(R^i_{\alpha_0}) = \varepsilon_i + \varepsilon_{\alpha_0} \pmod 2$. The gauge algebra is given by

$$\frac{\delta^R R^i_{\alpha_0}}{\delta q^j} R^j_{\beta_0} - (-)^{\varepsilon_{\alpha_0}\varepsilon_{\beta_0}} \frac{\delta^R R^i_{\beta_0}}{\delta q^j} R^j_{\alpha_0} = R^i_{\gamma_0} c^{\gamma_0}_{\alpha_0\beta_0} - \frac{\delta^R S_0}{\delta q^j} M^{ji}_{\alpha_0\beta_0} \tag{2}$$

where the structure functions $c^{\gamma_0}_{\alpha_0\beta_0}$ and $M^{ji}_{\alpha_0\beta_0}$ can depend by the real fields and satisfy the symmetry properties:

$$c^{\gamma_0}_{\alpha_0\beta_0} = -(-)^{\varepsilon_{\alpha_0}\varepsilon_{\beta_0}} c^{\gamma_0}_{\beta_0\alpha_0},\ M^{ji}_{\alpha_0\beta_0} = -(-)^{\varepsilon_{\alpha_0}\varepsilon_{\beta_0}} M^{ji}_{\beta_0\alpha_0} = -(-)^{\varepsilon_{\alpha_0}\varepsilon_{\beta_0}} M^{ij}_{\alpha_0\beta_0}. \tag{3}$$

For simplicity reasons, we will consider the case when the gauge generators satisfy a Lie type algebra ($M^{ji}_{\alpha_0\beta_0} = 0$).

The invariance of the action at the previous gauge transformation leads to the Noether identities:

$$\frac{\delta^R S_0}{\delta q^i} R^i_{\alpha_0} = 0. \tag{4}$$

In the previous relations, the upper index R signifies the right derivative.

We will suppose that the gauge transformations (1) are reducibile, that is not all gauge generators $R^i_{\alpha_0}$ are independent. Nontrivial functions $Z^{\alpha_0}_{\alpha_1} = Z^{\alpha_0}_{\alpha_1}(q)$ exist so that:

$$R^i_{\alpha_0} Z^{\alpha_0}_{\alpha_1} = M^{ij}_{\alpha_1} \frac{\delta S_0}{\delta q^i},\ \alpha_1 = 1, \cdots, m_1 \tag{5}$$

$$\varepsilon(Z^{\alpha_0}_{\alpha_1}) = \varepsilon_{\alpha_0} + \varepsilon_{\alpha_1} \pmod 2,\ \varepsilon(M^{ij}_{\alpha_1}) = \varepsilon_{\alpha_1}. \tag{6}$$

Again for simplicity we will restrict to a 1-reducible theory, where all $Z^{\alpha_0}_{\alpha_1}$ functions are independent, and we will consider the real fields as being bosonic ones, $\varepsilon_i = 0$. The extentions to more sophisticated cases are quite direct.

The $sp(3)$ BRST algebra is defined by:

$$s_a s_b + s_b s_a = 0,\ a, b = 1, 2, 3 \tag{7}$$

where s_1, s_2 and s_3 represent different items of the total BRST operator s:

$$s = s_1 + s_2 + s_3. \tag{8}$$

Moreover, their cohomological groups of order zero ($gh = 0, lev = 0$) have to give the set of all observables of the theory:

$$H_{(0,0)}(s_a) = \{observables\},\ a = 1, 2, 3. \tag{9}$$

Each differential $s_a, a = 1, 2, 3$ can be decomposed as in the standard case [13]:

$$s_a = \delta_a + d_a + \cdots,\ a = 1, 2, 3 \tag{10}$$

where $\{\delta_a, a = 1, 2, 3\}$ represent *the Koszul-Tate differentials* with non-trivial action on the antifields and $\{d_a, a = 1, 2, 3\}$ are *the exterior longitudinale derivatives* acting in the ghosts sector. On the basis of (7) and (8) we obtain that $s^2 = 0$.

As we mentioned, we will develope a "many-levels" approach using a graduation (gh, lev) [12]. The *ghost number* (gh) has the same significance as in the standard BRST theory [13] and the extended space will be generated by a set of ghost-fields and by another set of antifields. In our approach, all these generators will be placed on many levels. Depending on this, each generator will be characterised by a *level number* (lev), degree which will allow to differentiate among the generators with the same ghost number. We will extend for the previous operators the graduation (gh, lev). It will allow to make a distinction between s_1, s_2 and s_3 and to well-define their action on different generators of the extended space.

In conclusion, the extended space of the fields (real and ghost-type) and of the antifields will be structured on many levels $L^{(l)}, l \in Z$, the variables and the operators being double graduated by (gh, lev). As in the standard case [13] we will have $gh = pgh > 0$ for ghosts and $gh = -antigh < 0$ for antifields. The level number is an integer, positive for ghosts ($lev \geq 0$), negative for antifields ($lev \leq 0$) and zero for the original fields or any function of these ($lev = 0$).

The same graduation will be used for operators, too:

$$gh(\delta_a) = -antigh(\delta_a) = 1, lev(\delta_a) = a - 1, \ a = 1, 2, 3 \tag{11}$$

$$gh(d_a) = pgh(d_a) = 1, lev(d_a) = a - 1, \ a = 1, 2, 3. \tag{12}$$

For the BRST operators we will define:

$$gh(s_a) = 1, lev(s_a) = a - 1, \ a = 1, 2, 3. \tag{13}$$

The main problem we intend to solve consists in the construction of a special differential complex (tricomplex), (K, s_1, s_2, s_3), graduated in terms of (gh, lev). The decomposition (10) is made following the ideas: (i) the three diferentials $\delta_a, a = 1, 2, 3$ have to define a differential tricomplex of the form $(K', \delta_1, \delta_2, \delta_3)$, graduated in terms of $(antigh, lev)$ with $antigh \geq 0$ and $lev \leq 0$, s.t. to achieve a triresolution of $C^\infty(\Sigma)$ (Σ is the stationary surface of field equations); (ii) the three exterior derivatives along the gauge orbits, $d_a, a = 1, 2, 3$, have to define a exterior longitudinal tricomplex (K'', d_1, d_2, d_3) graduated in terms of (pgh, lev) and, moreover, the attached cohomologies to each d_a have to be isomorphic with the cohomology of the exterior longitudinal derivative from the standard BRST theory [13].

3 The construction of the exterior longitudinal tricomplex

Let us start with the construction of the exterior longitudinal complex (K'', d_1, d_2, d_3) graduated in terms of (pgh, lev). We will show that in the algebra K'' of the polynomials in ghosts with coeficients which are smooth functions on Σ, the total differential d splits as

$$d = d_1 + d_2 + d_3 \tag{14}$$

where each item satisfies (12).

In this respect we will start from the idea that in the $sp(3)$ BRST description, the gauge transformations are triplicated and the relation (1) can be extended in the form:

$$sq^i = R^i_{\alpha_0 1}(ghosts)^{\alpha_0 1} + R^i_{\alpha_0 2}(ghosts)^{\alpha_0 2} + R^i_{\alpha_0 3}(ghosts)^{\alpha_0 3} + \cdots \tag{15}$$

where

$$R^i_{\alpha_0 1} \equiv R^i_{\alpha_0 2} \equiv R^i_{\alpha_0 3} \equiv R^i_{\alpha_0}. \tag{16}$$

We can introduce the condensed notation

$$R^i_{A_0} \equiv (R^i_{\alpha_0}, R^i_{\alpha_0}, R^i_{\alpha_0}). \tag{17}$$

By that, s can be seen as the generator of a second order reducible theory. The reducibility relations are:

$$R^i_{A_0} Z^{A_0}_{B_0} = 0, \ Z^{A_0}_{B_0} Z^{B_0}_{\gamma_0} = 0. \tag{18}$$

We attach to the gauge generators (17) the ghosts

$$Q^{A_0} \equiv (Q^{\alpha_0 1}, Q^{\alpha_0 2}, Q^{\alpha_0 3}) \tag{19}$$

with the properties

$$\varepsilon(Q^{\alpha_0 a}) = \varepsilon_{\alpha_0} + 1, pgh(Q^{\alpha_0 a}) = 1, lev(Q^{\alpha_0 a}) = a - 1. \tag{20}$$

We also attach to the reducibility functions

$$Z^{A_0}_{B_0} \equiv \begin{pmatrix} 0 & \delta^{\alpha_0}_{\beta_0} & -\delta^{\alpha_0}_{\beta_0} \\ -\delta^{\alpha_0}_{\beta_0} & 0 & \delta^{\alpha_0}_{\beta_0} \\ \delta^{\alpha_0}_{\beta_0} & -\delta^{\alpha_0}_{\beta_0} & 0 \end{pmatrix} \tag{21}$$

and

$$Z^{B_0}_{\gamma_0} \equiv \begin{pmatrix} -\delta^{\beta_0}_{\gamma_0} \\ -\delta^{\beta_0}_{\gamma_0} \\ -\delta^{\beta_0}_{\gamma_0} \end{pmatrix} \tag{22}$$

the ghost type variables

$$\lambda^{A_0} \equiv (\lambda^{\alpha_0 1}, \lambda^{\alpha_0 2}, \lambda^{\alpha_0 3}) \tag{23}$$

$$\varepsilon(\lambda^{\alpha_0 a}) = \varepsilon_{\alpha_0}, pgh(\lambda^{\alpha_0 a}) = 2, lev(\lambda^{\alpha_0 a}) = 4 - a \tag{24}$$

and, respectively,

$$\eta^{A_0} \equiv \eta^{\alpha_0} \tag{25}$$

$$\varepsilon(\eta^{\alpha_0}) = \varepsilon_{\alpha_0} + 1, pgh(\eta^{\alpha_0}) = 3, lev(\eta^{\alpha_0}) = 3. \tag{26}$$

On the other hand, the gauge generators $R^i_{A_0}$ satisfy the reducibility relations

$$Z^{A_0}_{A_1} R^i_{A_0} = M^{ij}_{A_1} \frac{\delta S_0}{\delta q^j}. \tag{27}$$

Another association will be done by considering for

$$Z^{A_0}_{A_1} \equiv \begin{pmatrix} \frac{1}{3} Z^{\alpha_0}_{\alpha_1} & \frac{1}{3} Z^{\alpha_0}_{\alpha_1} & \frac{1}{3} Z^{\alpha_0}_{\alpha_1} \\ \frac{1}{3} Z^{\alpha_0}_{\alpha_1} & \frac{1}{3} Z^{\alpha_0}_{\alpha_1} & \frac{1}{3} Z^{\alpha_0}_{\alpha_1} \\ \frac{1}{3} Z^{\alpha_0}_{\alpha_1} & \frac{1}{3} Z^{\alpha_0}_{\alpha_1} & \frac{1}{3} Z^{\alpha_0}_{\alpha_1} \end{pmatrix} \tag{28}$$

the ghosts of ghosts

$$Q^{A_1} \equiv (Q^{\alpha_1 a|1}, Q^{\alpha_1 a|2}, Q^{\alpha_1 a|3}, a = 1, 2, 3) \tag{29}$$

with

$$\varepsilon(Q^{\alpha_1 a|b}) = \varepsilon_{\alpha_1}, pgh(Q^{\alpha_1 a|b}) = 2, lev(Q^{\alpha_1 a|b}) = a + b - 2, \ a, b = 1, 2, 3. \tag{30}$$

The matrix $M^{ij}_{A_1}$ have the form:

$$M^{ij}_{A_1} \equiv \begin{pmatrix} M^{ij}_{\alpha_1} \\ M^{ij}_{\alpha_1} \\ M^{ij}_{\alpha_1} \end{pmatrix}. \tag{31}$$

Not all the reducibility functions $Z^{A_0}_{A_1}$ are independent:

$$Z^{A_1}_{B_1} Z^{A_0}_{A_1} = 0 \tag{32}$$

where

$$Z^{A_1}_{B_1} = \begin{pmatrix} 0 & \delta^{\alpha_1}_{\beta_1} & -\delta^{\alpha_1}_{\beta_1} \\ -\delta^{\alpha_1}_{\beta_1} & 0 & \delta^{\alpha_1}_{\beta_1} \\ \delta^{\alpha_1}_{\beta_1} & -\delta^{\alpha_1}_{\beta_1} & 0 \end{pmatrix}. \tag{33}$$

Corresponding to these new reducibility functions, ghosts of ghosts of ghosts are introduced

$$\lambda^{A_1} \equiv (\lambda^{\alpha_1 a|1}, \lambda^{\alpha_1 a|2}, \lambda^{\alpha_1 a|3}, a = 1, 2, 3) \tag{34}$$

with

$$\varepsilon(\lambda^{\alpha_1 a|b}) = \varepsilon_{\alpha_1} + 1, pgh(\lambda^{\alpha_1 a|b}) = 3, lev(\lambda^{\alpha_1 a|b}) = a - b + 3, a, b = 1, 2, 3. \tag{35}$$

At their turn, the reducibility functions $Z_{B_1}^{A_1}$ are not independent, new reducibility relations occuring

$$Z_{A_1}^{B_1} Z_{B_1}^{\gamma_1} = 0. \tag{36}$$

We attach to the reducibility functions

$$Z_{B_1}^{\gamma_1} \equiv \begin{pmatrix} -\delta_{\beta_1}^{\gamma_1} \\ -\delta_{\beta_1}^{\gamma_1} \\ -\delta_{\beta_1}^{\gamma_1} \end{pmatrix} \tag{37}$$

new ghost-type variables

$$\eta^{A_1} \equiv (\eta^{\alpha_1|1}, \eta^{\alpha_1|2}, \eta^{\alpha_1|3}) \tag{38}$$

with

$$\varepsilon(\eta^{\alpha_1|a}) = \varepsilon_{\alpha_1}, pgh(\eta^{\alpha_1|a}) = 4, lev(\eta^{\alpha_1|a}) = a+2, a = 1,2,3. \tag{39}$$

The algebra $K'' = C^\infty(I) \otimes \mathbf{C}[Q^A]$ (I represents the space of all possible configurations of real fields) will be generated by the set of fields (real and ghost type)

$$Q^A \equiv \{q^i, Q^{\alpha_0 a}, \lambda^{\alpha_0 a}, \eta^{\alpha_0}, Q^{\alpha_1 a|b}, \lambda^{\alpha_1 a|b}, \eta^{\alpha_1|a}, a, b = 1,2,3\}. \tag{40}$$

It is easy to verify that in K'' the differential d is splited as in (14). It is clear that:

$$d^2 \approx 0 \Rightarrow d_a d_b + d_b d_a \approx 0,\ a, b = 1,2,3. \tag{41}$$

In a condensed form, the action of the operators $d_a,\ a = 1,2,3$ on the generators of K'' can be written:

$$d_a q^i = R_{\alpha_0}^i Q^{\alpha_0 b}\delta_{ba} + \frac{1}{2} M_{\alpha_1}^{ij}(q_{jb}^* Q^{\alpha_1\, b|c}\delta_{ca} + \overline{q}_{jb}\lambda^{\alpha_1\, c|b}\delta_{ca} + \overline{q}_j \eta^{\alpha_1\,|c}\delta_{ca}),$$

$$d_a Q^{\alpha_0 b} = \varepsilon_{adc}\delta^{db}\lambda^{\alpha_0 c} + \frac{1}{2}(-)^{\varepsilon_{\gamma_0}+1} c_{\beta_0\gamma_0}^{\alpha_0} Q^{\gamma_0 b} Q^{\beta_0 c}\delta_{ca} + Z_{\alpha_1}^{\alpha_0} Q^{\alpha_1 d|b}\delta_{da}, \tag{42}$$

$$d_a \lambda^{\alpha_0 b} = -\delta_a^b \eta^{\alpha_0} + \frac{1}{2}(-)^{\varepsilon_{\gamma_0}} c_{\beta_0\gamma_0}^{\alpha_0}\lambda^{\gamma_0 b} Q^{\beta_0 c}\delta_{ca} +$$

$$+\frac{1}{12}(-)^{\varepsilon_{\gamma_0}+1} c_{\beta_0\sigma_0}^{\alpha_0} c_{\gamma_0\rho_0}^{\sigma_0}\varepsilon_{bcd} Q^{\gamma_0 c} Q^{\beta_0 d} Q^{\rho_0 e}\delta_{ea} - \frac{1}{2} Z_{\alpha_1}^{\alpha_0}\lambda^{\alpha_1 b|d}\delta_{da}, \tag{43}$$

$$d_a \eta^{\alpha_0} = \frac{1}{2}(-)^{\varepsilon_{\gamma_0}+1} c_{\beta_0\gamma_0}^{\alpha_0} Q^{\gamma_0 b}\eta^{\beta_0}\delta_{ba} + \frac{1}{2} Z_{\alpha_1}^{\alpha_0}\eta^{\alpha_1|b}\delta_{ba}$$

$$+\frac{1}{12}(-)^{\varepsilon_{\gamma_0}}(c_{\beta_0\sigma_0}^{\alpha_0} c_{\gamma_0\rho_0}^{\sigma_0} - (-)^{\varepsilon_{\beta_0}(\varepsilon_{\gamma_0}+\varepsilon_{\rho_0})} c_{\gamma_0\sigma_0}^{\alpha_0} c_{\rho_0\beta_0}^{\sigma_0}) Q^{\rho_0 e} Q^{\gamma_0 c}\lambda^{\beta_0 c}\delta_{ea}, \tag{44}$$

$$d_a Q^{\alpha_1 b|c} = \varepsilon_{adb}\lambda^{\alpha_1 d|c}, d_a \lambda^{\alpha_1 b|c} = -\delta_a^b \eta^{\alpha_1|c},\ d_a \eta^{\alpha_1|c} = 0. \tag{45}$$

4 The construction of the Koszul-Tate tricomplex

In this section we intend to build the Koszul-Tate tricomplex so that this to realize a triresolution of $C^\infty(\Sigma)$. We note with K' the algebra of polynomial in fields and some objects (the antifields, which will be introduce later on) with coeficients which are functions on I. So, all closed non-exactely co-cycles from $\{\delta_a, a = 1,2,3\}$ homology have to be destroit. Firstly, we will introduce, like in the standard case [13], the antifields q_{ia}^* with $\varepsilon(q_{ia}^*) = 1$, $antigh(q_{ia}^*) = 1$, $lev(q_{ia}^*) = 1 - a$ so that

$$\delta_a q_{ib}^* = -\delta_{ab}\frac{\delta^R S_0}{\delta q^i}. \tag{46}$$

The existence of some non-trivial co-cycles in δ_a-homology, $a = 1,2,3$ asks for the introduction of new antifields, $\overline{q}_{ia}$, with $\varepsilon(\overline{q}_{ia}) = 0$, $antigh(\overline{q}_{ia}) = 2$ and $lev(\overline{q}_{ia}) = a - 4$ so that to assure

$$H_{(1,1-b)}(\delta_a) = 0,\ a, b = 1,2,3 \tag{47}$$

$$\delta_a(\varepsilon_{abc}q^*_{ib}) = 0,\ \delta_a\overline{q}_{ic} = \varepsilon_{abc}q^*_{ib}. \tag{48}$$

From (46) and (4) we observe that

$$\delta_a(\delta_{ab}R^i_{\alpha_0}q^*_{ib}) = -\frac{\delta^R S_0}{\delta q^i}R^i_{\alpha_0} = 0 \tag{49}$$

and we will introduce the antifields $Q^*_{\alpha_0 bc}$ with $\varepsilon(Q^*_{\alpha_0 bc}) = \varepsilon_{\alpha_0}$, $antigh(Q^*_{\alpha_0 bc}) = 2$ and $lev(Q^*_{\alpha_0 bc}) = 2 - b - c$ so that

$$\delta_a Q^*_{\alpha_0 bc} = \delta_{ab}R^i_{\alpha_0}q^*_{ic}. \tag{50}$$

The apparition of some non-trivial co-cycles at $(antigh = 2, lev = c - 4,\ c = 1,2,3)$

$$\delta_a(\delta_{ac}\overline{q}_{ic}) = 0, \tag{51}$$

$$\delta_a(Q^*_{\alpha_0 ac} - Q^*_{\alpha_0 ca} + R^i_{\alpha_0}\overline{q}_{ic}) = 0 \tag{52}$$

and of the non-trivial co-cycles

$$\delta_a(\varepsilon_{abd}Q^*_{\alpha_0 bc}) = 0 \tag{53}$$

$$\delta_a\left(\frac{1}{2}Z^{\alpha_0}_{\alpha_1}\left(Q^*_{\alpha_0 cb} + Q^*_{\alpha_0 bc}\right) + \frac{1}{2}M^{ij}_{\alpha_1}q^*_{ib}q^*_{jc}\right) = 0 \tag{54}$$

at $(antigh = 2, lev = 2 - b - c,\ b, c = 1,2,3)$ imply the introduction of the new antifields, $\overline{q}_i$, with $\varepsilon(\overline{q}_i) = 1$, $antigh(\overline{q}_i) = 3$ and $lev(\overline{q}_i) = -3$ s.t.

$$\delta_a\overline{q}_i = \delta_{ac}\overline{q}_{ic} \tag{55}$$

and respectively antifields $\lambda^*_{\alpha_0 ac}$, with $\varepsilon(\lambda^*_{\alpha_0 ac}) = \varepsilon_{\alpha_0}$, $antigh(\lambda^*_{\alpha_0 ac}) = 3$ and $lev(\lambda^*_{\alpha_0 ac}) = c - a - 3$ so that

$$\delta_a\lambda^*_{\alpha_0 bc} = \delta_{ab}(\varepsilon_{cde}Q^*_{\alpha_0 de} - R^i_{\alpha_0}\overline{q}_{ic}). \tag{56}$$

For assuring $H_{(2,2-b-c)}(\delta_a) = 0,\ a, b, c = 1,2,3$ we introduce the antifields $\overline{Q}_{\alpha_0 ab}$ with $\varepsilon(\overline{Q}_{\alpha_0 ab}) = \varepsilon_{\alpha_0}$, $antigh(\overline{Q}_{\alpha_0 ab}) = 3$ and $lev(\overline{Q}_{\alpha_0 ab}) = a - b - 3$ so that

$$\delta_a\overline{Q}_{\alpha_0 dc} = \varepsilon_{abd}Q^*_{\alpha_0 bc} \tag{57}$$

and antifields $Q^*_{\alpha_1\, bc|d}$ with $\varepsilon(Q^*_{\alpha_1\, bc|d}) = \varepsilon_{\alpha_1} + 1$, $antigh(Q^*_{\alpha_1\, bc|d}) = 3$ and $lev(Q^*_{\alpha_1\, bc|d}) = 3 - b - c - d$ so that

$$\delta_a Q^*_{\alpha_1\, bc|d} = \delta_{ab}\left(\frac{1}{2}Z^{\alpha_0}_{\alpha_1}\left(Q^*_{\alpha_0 cd} + Q^*_{\alpha_0 dc}\right) + \frac{1}{2}M^{ij}_{\alpha_1}q^*_{id}q^*_{jc}\right). \tag{58}$$

New closed non-exactely co-cycles appear

$$\delta_a(\delta_{ab}\overline{Q}_{\alpha_0 bc}) = 0 \tag{59}$$

$$\delta_a(\varepsilon_{abe}Q^*_{\alpha_1\, bc|d}) = 0 \tag{60}$$

and new antifields, $\overline{Q}_{\alpha_0 a}$, with $\varepsilon(\overline{Q}_{\alpha_0 a}) = \varepsilon_{\alpha_0} + 1$, $antigh(\overline{Q}_{\alpha_0 a}) = 4$ and $lev(\overline{Q}_{\alpha_0 a}) = -a - 2$ are necessary so that

$$\delta_a\overline{Q}_{\alpha_0 c} = \delta_{ab}\overline{Q}_{\alpha_0 bc} \tag{61}$$

The antifields $\overline{Q}_{\alpha_1\, bc|d}$ with

$$\varepsilon(\overline{Q}_{\alpha_1\, bc|d}) = \varepsilon_{\alpha_1}, antigh(\overline{Q}_{\alpha_1\, bc|d}) = 4, lev(\overline{Q}_{\alpha_1\, bc|d}) = b - c - d - 2 \tag{62}$$

are introduce so that

$$\delta_a\overline{Q}_{\alpha_1\, bc|d} = \varepsilon_{aeb}Q^*_{\alpha_1\, ec|d}. \tag{63}$$

The presence of some non-trivial co-cycles at $(antigh = 3, lev = a - b - 3,\ a, b = 1, 2, 3)$ asks the introduction of the antifields $\overline{\lambda}_{\alpha_0 ab}$ with $\varepsilon(\overline{\lambda}_{\alpha_0 ab}) = \varepsilon_{\alpha_0} + 1$, $antigh(\overline{\lambda}_{\alpha_0 ab}) = 4$ and $lev(\overline{\lambda}_{\alpha_0 ab}) = a + b - 8$ so that

$$\delta_a \overline{\lambda}_{\alpha_0 cd} = \varepsilon_{abc} \lambda^*_{\alpha_0 bd}. \tag{64}$$

At $(antigh = 3, lev = -3)$ appear non-trivial co-cycles of the form

$$\delta_a \left(\sum_{c=1}^{3} \left(\lambda^*_{\alpha_0 cc} - \overline{Q}_{\alpha_0 cc} \right) + R^i_{\alpha_0} \overline{q}_i \right) = 0,\ a = 1, 2, 3 \tag{65}$$

and the δ_a−closed modulo δ_a−exactely polynomial

$$\mu_{\alpha_1} = \sum_{b=1}^{3} \frac{1}{2} \left(Z^{\alpha_0}_{\alpha_1} \left(\lambda^*_{\alpha_0 bb} - \overline{Q}_{\alpha_0 bb} \right) + M^{ij}_{\alpha_1} q^*_{ib} \overline{q}_{jb} \right) \tag{66}$$

$$\delta_a \mu_{\alpha_1} = 0.$$

For their elimination from δ_a homology we introduce the antifields $\eta^*_{\alpha_0 a}$, with $\varepsilon(\eta^*_{\alpha_0 a}) = \varepsilon_{\alpha_0} + 1$, $antigh(\eta^*_{\alpha_0 a}) = 4$ and $lev(\eta^*_{\alpha_0 a}) = -a - 2$ so that

$$\delta_a \eta^*_{\alpha_0 b} = \delta_{ab} \left(\sum_{c=1}^{3} \left(\lambda^*_{\alpha_0 cc} - \overline{Q}_{\alpha_0 cc} \right) + R^i_{\alpha_0} \overline{q}_i \right) \tag{67}$$

and antifields $\lambda^*_{\alpha_1\, bc|c}$ with

$$\varepsilon(\lambda^*_{\alpha_1\, bc|c}) = \varepsilon_{\alpha_1}, antigh(\lambda^*_{\alpha_1\, bc|c}) = 4, lev(\lambda^*_{\alpha_1\, bc|c}) = -b - 1 \tag{68}$$

so that

$$\delta_a \lambda^*_{\alpha_1\, bc|c} = -\frac{1}{2} \delta_{ab} \mu_{\alpha_1}. \tag{69}$$

The following polynomials

$$\mu_{\alpha_1\, 1|2} \equiv -\frac{1}{2} Z^{\alpha_0}_{\alpha_1} \left(\lambda^*_{\alpha_0 12} - 2\overline{Q}_{\alpha_0 21} \right) + Q^*_{\alpha_1\, 31|1} - Q^*_{\alpha_1\, 11|3} + \frac{1}{2} M^{ij}_{\alpha_1} q^*_{i1} \overline{q}_{j2} \tag{70}$$

$$\mu_{\alpha_1\, 1|3} \equiv -\frac{1}{2} Z^{\alpha_0}_{\alpha_1} \left(\lambda^*_{\alpha_0 13} - 2\overline{Q}_{\alpha_0 31} \right) + Q^*_{\alpha_1\, 11|2} - Q^*_{\alpha_1\, 21|1} + \frac{1}{2} M^{ij}_{\alpha_1} q^*_{i1} \overline{q}_{j3} \tag{71}$$

$$\mu_{\alpha_1\, 2|3} \equiv -\frac{1}{2} Z^{\alpha_0}_{\alpha_1} \left(\lambda^*_{\alpha_0 23} - 2\overline{Q}_{\alpha_0 32} \right) + Q^*_{\alpha_1\, 12|2} - Q^*_{\alpha_1\, 22|1} + \frac{1}{2} M^{ij}_{\alpha_1} q^*_{i2} \overline{q}_{j3} \tag{72}$$

are δ_a-closed modulo δ_a-exactely and, for their elimination from δ_a homology we introduce the antifields $\lambda^*_{\alpha_1\, a1|2}, \lambda^*_{\alpha_1\, a1|3}$ and $\lambda^*_{\alpha_1\, a2|3}$ with properties

$$\varepsilon(\lambda^*_{\alpha_1\, a1|2}) = \varepsilon(\lambda^*_{\alpha_1\, a1|3}) = \varepsilon(\lambda^*_{\alpha_1\, a2|3}) = \varepsilon_{\alpha_1}, \tag{73}$$

$$antigh(\lambda^*_{\alpha_1\, a1|2}) = antigh(\lambda^*_{\alpha_1\, a1|3}) = antigh(\lambda^*_{\alpha_1\, a2|3}) = 4, \tag{74}$$

$$lev(\lambda^*_{\alpha_1\, a1|2}) = lev(\lambda^*_{\alpha_1\, a1|3}) = lev(\lambda^*_{\alpha_1\, a2|3}) = -a - 1 \tag{75}$$

so that

$$\delta_a \lambda^*_{\alpha_1\, b1|2} = \delta_{ab} \mu_{\alpha_1\, 1|2}, \delta_a \lambda^*_{\alpha_1\, b1|3} = \delta_{ab} \mu_{\alpha_1\, 1|3}, \delta_a \lambda^*_{\alpha_1\, b2|3} = \delta_{ab} \mu_{\alpha_1\, 2|3}. \tag{76}$$

Similarly, we introduce the antifields $\lambda^*_{\alpha_1\, a2|1}, \lambda^*_{\alpha_1\, a3|1}$ and $\lambda^*_{\alpha_1\, a3|2}$ with

$$\varepsilon(\lambda^*_{\alpha_1\, a2|1}) = \varepsilon(\lambda^*_{\alpha_1\, a3|1}) = \varepsilon(\lambda^*_{\alpha_1\, a3|2}) = \varepsilon_{\alpha_1}, \tag{77}$$

$$antigh(\lambda^*_{\alpha_1\, a2|1}) = antigh(\lambda^*_{\alpha_1\, a3|1}) = antigh(\lambda^*_{\alpha_1\, a3|2}) = 4, \tag{78}$$

$$lev(\lambda^*_{\alpha_1\, a2|1}) = lev(\lambda^*_{\alpha_1\, a3|1}) = lev(\lambda^*_{\alpha_1\, a3|2}) = -a - 1 \tag{79}$$

so that the non-trivial polynomials from δ_a homology:

$$\mu_{\alpha_1 2|1} \equiv -\frac{1}{2} Z^{\alpha_0}_{\alpha_1} \left(\lambda^*_{\alpha_0 21} - 2\overline{Q}_{\alpha_0 12}\right) + Q^*_{\alpha_1 22|3} - Q^*_{\alpha_1 32|2} + \frac{1}{2} M^{ij}_{\alpha_1} q^*_{i2} \overline{q}_{j1} \tag{80}$$

$$\mu_{\alpha_1 3|1} \equiv -\frac{1}{2} Z^{\alpha_0}_{\alpha_1} \left(\lambda^*_{\alpha_0 31} - 2\overline{Q}_{\alpha_0 13}\right) + Q^*_{\alpha_1 23|3} - Q^*_{\alpha_1 33|2} + \frac{1}{2} M^{ij}_{\alpha_1} q^*_{i3} \overline{q}_{j1} \tag{81}$$

$$\mu_{\alpha_1 3|2} \equiv -\frac{1}{2} Z^{\alpha_0}_{\alpha_1} \left(\lambda^*_{\alpha_0 32} - 2\overline{Q}_{\alpha_0 23}\right) + Q^*_{\alpha_1 33|1} - Q^*_{\alpha_1 13|3} + \frac{1}{2} M^{ij}_{\alpha_1} q^*_{i2} \overline{q}_{j3} \tag{82}$$

to be eliminated

$$\delta_a \lambda^*_{\alpha_1 b2|1} = \delta_{ab} \mu_{\alpha_1 2|1}, \delta_a \lambda^*_{\alpha_1 b3|1} = \delta_{ab} \mu_{\alpha_1 3|1}, \delta_a \lambda^*_{\alpha_1 b3|2} = \delta_{ab} \mu_{\alpha_1 3|2}. \tag{83}$$

The relations (69), (76) and (83) can be write in the condensed form

$$\delta_a \lambda^*_{\alpha_1 bc|d} = \delta_{ab} \mu_{\alpha_1 c|d}, \; c \neq d \tag{84}$$

$$\delta_a \lambda^*_{\alpha_1 bc|c} = -\frac{1}{2} \delta_{ab} \mu_{\alpha_1}, c = d \tag{85}$$

where

$$\mu_{\alpha_1 c|d} = -\frac{1}{2} Z^{\alpha_0}_{\alpha_1} \left(\lambda^*_{\alpha_0 cd} - 2\overline{Q}_{\alpha_0 dc}\right) + \varepsilon_{cde} (Q^*_{\alpha_1 ec|c} - Q^*_{\alpha_1 cc|e}) + \frac{1}{2} M^{ij}_{\alpha_1} q^*_{ic} \overline{q}_{jd}. \tag{86}$$

The δ_a-closed co-cycles which not are δ_a-exactely from $H_{(4,b+c-8)}(\delta_a)$, $a, b, c = 1, 2, 3$ will be eliminated from δ_a homology by introduction of the antifields $\overline{\lambda}_{\alpha_0 a}$ with $\varepsilon(\overline{\lambda}_{\alpha_0 a}) = \varepsilon_{\alpha_0}$, $antigh(\overline{\lambda}_{\alpha_0 a}) = 5$ and $lev(\overline{\lambda}_{\alpha_0 a}) = a - 7$ so that

$$\delta_a \overline{\lambda}_{\alpha_0 c} = \delta_{ab} \overline{\lambda}_{\alpha_0 bc}. \tag{87}$$

We observe that

$$\delta_a \left(\varepsilon_{abc} \eta^*_{\alpha_0 b}\right) = 0 \tag{88}$$

$$\delta_a (\overline{Q}_{\alpha_1 ac|d}) = 0 \tag{89}$$

and we introduce the antifields $\overline{\eta}_{\alpha_0 a}$ with $\varepsilon(\overline{\eta}_{\alpha_0 a}) = \varepsilon_{\alpha_0}$, $antigh(\overline{\eta}_{\alpha_0 a}) = 5$ and $lev(\overline{\eta}_{\alpha_0 a}) = a - 7$ so that

$$\delta_a \overline{\eta}_{\alpha_0 b} = \varepsilon_{acb} \eta^*_{\alpha_0 c}$$

and the antifields $\overline{Q}_{\alpha_1 c|d}$ with

$$\varepsilon(\overline{Q}_{\alpha_1 c|d}) = \varepsilon_{\alpha_1} + 1, antigh(\overline{Q}_{\alpha_1 c|d}) = 5, \; lev(\overline{Q}_{\alpha_1 c|d}) = -c - d - 1$$

so that

$$\delta_a \overline{Q}_{\alpha_1 c|d} = \delta_{ab} \overline{Q}_{\alpha_1 bc|d}.$$

The existence of non-trivial co-cycles $\varepsilon_{abc} \lambda^*_{\alpha_1 bd|e}$ imply $H_{(4,b-c-d-2)}(\delta_a) \neq 0, \; a, b, c, d = 1, 2, 3$ and for assuring $H_{(4,b-c-d-2)}(\delta_a) = 0$ we introduce the antifields $\overline{\lambda}_{\alpha_1 cd|e}$ with $\varepsilon(\overline{\lambda}_{\alpha_1 cd|e}) = \varepsilon_{\alpha_1} + 1$, $antigh(\overline{\lambda}_{\alpha_1 cd|e}) = 5$ and $lev(\overline{\lambda}_{\alpha_1 cd|e}) = c - d + e - 7$ so that

$$\delta_a \overline{\lambda}_{\alpha_1 cd|e} = \varepsilon_{abc} \lambda^*_{\alpha_1 bd|e}.$$

For assuring $H_{(5,c-d+e-7)}(\delta_a) = 0, a, c, d, e = 1, 2, 3$ we introduce the antifields $\overline{\lambda}_{\alpha_1 d|e}$ with $\varepsilon(\overline{\lambda}_{\alpha_1 d|e}) = \varepsilon_{\alpha_1}, antigh(\overline{\lambda}_{\alpha_1 d|e}) = 6$ and $lev(\overline{\lambda}_{\alpha_1 d|e}) = e - d - 6$ so that

$$\delta_a \overline{\lambda}_{\alpha_1 d|e} = \delta_{ac} \overline{\lambda}_{\alpha_1 cd|e}.$$

Non-trivial co-cycles from δ_a homology of the form $\delta_{ab} \overline{\eta}_{\alpha_0 b}$ will be destroit by introduction of the antifields $\overline{\eta}_{\alpha_0}$ with $\varepsilon(\overline{\eta}_{\alpha_0}) = \varepsilon_{\alpha_0} + 1$, $antigh(\overline{\eta}_{\alpha_0}) = 6$ and $lev(\overline{\eta}_{\alpha_0}) = -6$ so that

$$\delta_a \overline{\eta}_{\alpha_0} = \delta_{ab} \overline{\eta}_{\alpha_0 b}. \tag{90}$$

We remark the existence of polynoamials

$$\nu_{\alpha_1\,|c} \equiv \frac{1}{2}Z^{\alpha_0}_{\alpha_1}\eta^*_{\alpha_0 c} - \lambda^*_{\alpha_1\,cd|d} + \frac{1}{2}M^{ij}_{\alpha_1}\left(\frac{1}{2}\varepsilon_{cde}\overline{q}_{id}\overline{q}_{je} - q^*_{ic}\overline{q}_j\right) \tag{91}$$

in δ_a homology

$$\delta_a\nu_{\alpha_1\,|c} = 0$$

and for their destroire we introduce the antifields $\eta^*_{\alpha_1\,c|c}$ with $\varepsilon(\eta^*_{\alpha_1\,c|c}) = \varepsilon_{\alpha_1} + 1$, $antigh(\eta^*_{\alpha_1\,c|c}) = 5$ and $lev(\eta^*_{\alpha_1\,c|c}) = -2c - 1$ so that

$$\delta_a\eta^*_{\alpha_1\,c|c} = \delta_{ac}\nu_{\alpha_1\,|c}. \tag{92}$$

From the last relation we observe the existence of some non-trivial co-cycles in $H_{(5,-2c-1)}(\delta_a),\ a, c = 1, 2, 3$ and for their killing we introduce the antifields $\overline{\eta}_{\alpha_1\,b|c}$ with $\varepsilon(\overline{\eta}_{\alpha_1\,b|c}) = \varepsilon_{\alpha_1}$, $antigh(\overline{\eta}_{\alpha_1\,b|c}) = 6$ and $lev(\overline{\eta}_{\alpha_1\,b|c}) = b - c - 6$ so that

$$\delta_a\overline{\eta}_{\alpha_1\,b|c} = \varepsilon_{abc}\eta^*_{\alpha_1\,c|c}, a + b + c = 6. \tag{93}$$

The elimination of non-trivial co-cycles from $H_{(6,b-c-6)}(\delta_a),\ a, b, c = 1, 2, 3$ is done with help of new antifields $\overline{\eta}_{\alpha_1\,|c}$ with $\varepsilon(\overline{\eta}_{\alpha_1\,|c}) = \varepsilon_{\alpha_1} + 1$, $antigh(\overline{\eta}_{\alpha_1\,|c}) = 7$ and $lev(\overline{\eta}_{\alpha_1\,|c}) = -c - 5$ so that

$$\delta_a\overline{\eta}_{\alpha_1\,|c} = \delta_{ab}\overline{\eta}_{\alpha_1\,b|c}. \tag{94}$$

It is easy to verify that other non-trivial co-cycles not appear in δ_a homology:

$$H_{(j,l)}(\delta_a) = 0,\ j > 1, l \leq 0,\ a = 1, 2, 3. \tag{95}$$

With the other words, $H_{(0,0)}(\delta_a)$ contain whole δ_a homology:

$$H_{(0,0)}(\delta_1) = H_{(0,0)}(\delta_2) = H_{(0,0)}(\delta_3) = C^\infty(\Sigma) = H_0(\delta). \tag{96}$$

In conclusion, we succeded to introduce the complete spectrum of antifields

$$Q^*_{Aa} \equiv \{q^*_{ia}, Q^*_{\alpha_0 ab}, \lambda^*_{\alpha_0 ab}, \eta^*_{\alpha_0 a}, Q^*_{\alpha_1\,ab|c}, \lambda^*_{\alpha_1\,ab|c}, \eta^*_{\alpha_1\,a|b}\}, \tag{97}$$

$$\overline{Q}_{Aa} \equiv \{\overline{q}_{ia}, \overline{Q}_{\alpha_0 ab}, \overline{\lambda}_{\alpha_0 ab}, \overline{\eta}_{\alpha_0 a}, \overline{Q}_{\alpha_1\,ab|c}, \overline{\lambda}_{\alpha_1\,ab|c}, \overline{\eta}_{\alpha_1\,a|b}\}, \tag{98}$$

$$\overline{Q}_A \equiv \{\overline{q}_i, \overline{Q}_{\alpha_0 a}, \overline{\lambda}_{\alpha_0 a}, \overline{\eta}_{\alpha_0}, \overline{Q}_{\alpha_1\,a|b}, \overline{\lambda}_{\alpha_1\,a|b}, \overline{\eta}_{\alpha_1\,|a}\}. \tag{99}$$

so that the tricomplex $(K', \delta_1, \delta_2, \delta_3)$ graduated in terms $(antigh, lev)$ to realized a triresolution of $C^\infty(\Sigma)$.

5 Conclusions

A consistent $sp(3)$ BRST description of the 1-reducible gauge theories in a Lagrangian form is possible using for variables and operators a bi-graduation (gh, lev). It has to be done in an extended space generated by:

* fields (real and ghost-type):

$$Q^A \equiv \{q^i, Q^{\alpha_0 a}, \lambda^{\alpha_0 a}, \eta^{\alpha_0}, Q^{\alpha_1 a|b}, \lambda^{\alpha_1 a|b}, \eta^{\alpha_1|a}, a, b = 1, 2, 3\}. \tag{100}$$

* antifields:

$$Q^*_{Aa} \equiv \{q^*_{ia}, Q^*_{\alpha_0 ab}, \lambda^*_{\alpha_0 ab}, \eta^*_{\alpha_0 a}, Q^*_{\alpha_1\,ab|c}, \lambda^*_{\alpha_1\,ab|c}, \eta^*_{\alpha_1\,a|b}\}, \tag{101}$$

$$\overline{Q}_{Aa} \equiv \{\overline{q}_{ia}, \overline{Q}_{\alpha_0 ab}, \overline{\lambda}_{\alpha_0 ab}, \overline{\eta}_{\alpha_0 a}, \overline{Q}_{\alpha_1\,ab|c}, \overline{\lambda}_{\alpha_1\,ab|c}, \overline{\eta}_{\alpha_1\,a|b}\}, \tag{102}$$

$$\overline{Q}_A \equiv \{\overline{q}_i, \overline{Q}_{\alpha_0 a}, \overline{\lambda}_{\alpha_0 a}, \overline{\eta}_{\alpha_0}, \overline{Q}_{\alpha_1\,a|b}, \overline{\lambda}_{\alpha_1\,a|b}, \overline{\eta}_{\alpha_1\,|a}\}. \tag{103}$$

It is easy to note that there is not a one-to-one correspondence between ghosts (100) and antifields (101), (102) and (103). There are much more antifields generated by the acyclicity requirement for $\{\delta_a, a=1,2,3\}$. To define "canonical" pairs would impose more ghosts, but, as our approach allowed to see, this enlargement is not necessary. We can keep the ghost spectrum at a minimum size and to see δ_a as a sum between a "canonical" and a "noncanonical" part

$$\delta_a * = \delta_a^{can} * + \delta_a^{noncan} *, \; a=1,2,3 \tag{104}$$

where

$$\delta_a^{noncan} * = (-)^{\varepsilon(Q^A)} \varepsilon_{abc} Q^*_{Ac} \frac{\delta^R}{\delta \overline{Q}_{Ab}} * + (-)^{\varepsilon(Q^A)+1} \delta_{ab} \overline{Q}_{Ab} \frac{\delta^R}{\delta \overline{Q}_A} * \tag{105}$$

and the canonical part is defined in respect to the antibrackets $(,)_a, \; a=1,2,3$

$$\delta_a^{can} * = (*, S)_a|_{ghosts=0} \,. \tag{106}$$

In the previous relations, S represents the generator of the $sp(3)$ BRST Lagrangian symmetry in the anticanonical structures of the antibrackets. The noncanonical part will act nontrivially on the non-paired antifields:

$$\delta_a^{noncan} \overline{Q}_{Aa} \neq 0, \; \delta_a^{noncan} \overline{Q}_A \neq 0. \tag{107}$$

In conclusion, the $sp(3)$ BRST Lagrangian differentials will be decomposed as

$$s_a^{can} * = (*, S)_a + \delta_a^{noncan} *, \; a=1,2,3 \tag{108}$$

The master equations will be of the form

$$\frac{1}{2}(S,S)_a + \delta_a^{noncan} S = 0. \tag{109}$$

For reducible theories, the acyclicity of δ_a is not achieved by killing some ghost variables with new generators. Some non-trivial co-cycles are given now by some special polynomials of the "star" and "bar" antifields. We presented in this paper the concrete form of all these polynomials for the 1-reducible case.

On the basis of the equivalence between the Lagrangian and the Hamiltonian formalisms based on the (gh, lev) graduation, like in the irreducible case ([14]), a new simple and efficient gauge fixing procedure can be proposed.

A complete construction of a $sp(3)$ BRST Lagrangian theory and the equivalence between this formulation and the $sp(3)$ BRST Hamiltonian one will be done in forthcoming papers.

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