Physics AUC, vol. 18, 178-187 (2008) **PHYSICS AUC**

Dust Acoustic Solitons in a Dusty Plasma with Dust Particle Charge Variation

A. Grecu, D. Grecu
Department of Theoretical Physics
National Institute of Physics and Nuclear Engineering
"Horia Hulubei"
Bucharest-Măgurele, Romania

Abstract

A dusty plasma is a complex system containing electrons, ions and massive dust grains. After a short introduction of the main characteristics of a dusty plasma, the charging process of the dust grains is analyzed. For reasonable plasma parameters this is a very fast process compared to the slow motion of the dust acoustic waves(DAW). This fact allows us to introduce the "local equilibrium approximation"(LEA) in the study of the charging process. According to LEA the same equation giving the dust particle charge is used both in the equilibrium case, and in the situation when a slow DAW is moving through the plasma. Then both the linear and nonlinear DAW are studied taking into account the dust particle charge variation. Using a multiple scales approach the nonlinear DAW are described by the KdV equation, and the influence of charge variation on the soliton parameters is determined. Few comments about the limitation of this approach are finally given.

1 Introduction

A dusty plasma is a complex medium composed of electrons, different species of positive, negative ions and also massive particles of dust which not only perturb the plasma properties but also can introduce new phenomena. Even in the simplest case of one type of spherical dust particles of mass m_d and radius r_d, the problem is complicated by the fact that the grain charge q_d is a dynamical variable depending on the local characteristic of the plasma (local potential φ). Although the dust particulates density, n_d, is several orders smaller than the densities of the other plasma species, the charge accumulated on the dust grains is high enough to give the charge density $q_d n_d$ of the dust fluid the same order of magnitude as the other charge densities present. This is the reason why dusty plasmas can have different and new properties with respect to dust free (pure) plasmas. In the following we shall consider a plasma containing only one species of positive, single ionized ions. Without the influence of other external, ionizing factors (an external radiation field producing photoionizations, energetic particles, etc.), due to impinging electrons and ions on the dust particles, these become negatively charged. Denoting by n_{e0}, n_{i0}, n_{d0} the equilibrium densities of electrons, ions and dust particles respectively, the macroscopic neutrality condition writes

$$n_{i0} = Z_d n_d + n_{e0} \tag{1.1}$$

If $Z_d n_{d0}$ is of order of n_{i0}, one sees that $n_{e0} \ll n_{i0}$ and the electron fluid is strongly depleted with respect to an usual plasma ($n_{i0} \simeq n_{e0}$). The missing electrons are those fixed on and creating the negative charge of the dust particles.

One of the general properties of a plasma is the Debye shielding of a charged impurity. This is characterized by the Debye shielding length λ_D. In a region far from the charged impurity (the volume close the impurity has low contribution to the screening) the following relation for λ_D can be derived

$$\frac{1}{\lambda_D^2} = \frac{1}{\lambda_{De}^2} + \frac{1}{\lambda_{Di}^2} = \frac{e^2 n_{e0}}{\varepsilon_0 k_B T_e} + \frac{e^2 n_{i0}}{\varepsilon_0 k_B T_i} \tag{1.2}$$

where T_e, T_i are the equilibrium temperatures of the electron and ion fluid respectively. If $n_{e0} \ll n_{i0}$ one sees that $\lambda_D \simeq \lambda_{Di}$, and thus, the screening properties are determined by ions. If a is the mean distance between two dust particles, the system is a real dusty plasma if and only if $a < \lambda_D$ and the dust particles are directly interacting and participating to the collective properties of the plasma. Otherwise, when $a > \lambda_D$, the system is composed of dust particles in a plasma and their influence can be treated perturbatively (plasma with dust).

When a plasma is disturbed from its equilibrium state, the resulting internal space charged field gives rise to collective motions of the plasma constituents, which tend to restore the original charge neutrality. These collective oscillations are characterized by a natural frequency, called the plasma frequency. For each species $s = e, i, d$, one can write a continuity equation

$$\frac{\partial n_s}{\partial t} + \nabla \left(n_s \vec{v_s}\right) = 0 \tag{1.3}$$

and a momentum equation

$$\frac{\partial \vec{v_s}}{\partial t} + \left(\vec{v_s} \cdot \nabla\right) \vec{v_s} = -\frac{q_s}{m_s} \nabla \Phi. \tag{1.4}$$

This system of equations is closed by the Poisson's equation

$$\nabla^2 \Phi = -\frac{1}{\varepsilon_0} \sum_{\{s\}} q_s n_s \tag{1.5}$$

Under the assumption that the amplitude of the oscillations are small, equations (1.3) and (1.4) can be substituted by their linear approximations and eliminating the speeds from the equation system (1.3)-(1.5), then integrating twice over space with equilibrium boundary conditions at infinity, a harmonic oscillator equation for the internal field is obtained

$$\frac{d^2 \Phi}{dt^2} + \omega_p^2 \Phi = 0, \tag{1.6}$$

$$\omega_p^2 = \sum_{\{s\}} \omega_{ps}^2 = \sum_{\{s\}} \frac{n_{s0} q_s^2}{\varepsilon_0 m_s}$$

where ω_{ps} represents the plasma frequency associated to the plasma s-type constituent. It should be noted that for the case of a dusty plasma, the dust fluid frequency $\omega_{pd} \sim m_d^{-1/2}$ is much smaller that the frequencies associated to the other constituents and this is the basis of the "*local equilibrium approximation*" that will be used in the followings for the treatment of dust acoustic waves and solitons.

Another parameter characterizing a dusty plasma is the Coulomb coupling parameter, defined as the ratio between the interaction potential energy among two dust particles separated by a distance a, and the mean dust thermal kinetic energy

$$\Gamma_C = \frac{q_d^2}{4\pi\varepsilon_0 a k_B T_d} \exp\left(-\frac{a}{\lambda_D}\right) \tag{1.7}$$

When $\Gamma_C \ll 1$ one gets the case of a weakly coupled system which is the usual situation in space and laboratory conditions as well as the subject of the present research report. At the opposite limit $\Gamma_C \gg 1$ ($\Gamma_C \geq 170$), the strong coupling may lead, as it has already been realized in laboratories in special conditions, to a phase transition from the disordered fluid state to an ordered crystalline phase (Wigner's crystal).

If dust is a nuisance in every day life, it is a common and unavoidable presence in space conditions and also in many laboratory devices. Let us give some examples: interstellar and circumstellar clouds (the collapse of these clouds give birth to stellar clusters), in planetary rings, in comet tails (when in vicinity of Sun or other *heat* source), in Earth's atmosphere as noctilucent clouds observed during polar summer mesopause. It is present in many laboratory devices such as DC and RF dischargers, in plasma processing reactors and fusion plasma devices (tokamaks and stellerators) and so on. Several review articles and books [1] - [7], to mention just a few of them, are a good introduction to this rapidly developing field of plasma physics.

2 Dust Charging Process

As mentioned before, without any external ionizing processes, the dust particles become negatively charged since the electron thermal speed is much larger than the ion thermal speed. In the orbit-limited motion approximation, the electronic and ionic currents which fall onto the dust grain are given by the following expressions

$$I_e = -4\pi r_d^2 n_e e \left(\frac{k_B T_e}{2\pi m_e}\right)^{1/2} \exp\left(\frac{e\Phi_d}{k_B T_e}\right) \tag{2.1}$$

for the electron current (also for any other single ionized, negatively charged species in the plasma) and

$$I_i = 4\pi r_d^2 n_i e \left(\frac{k_B T_i}{2\pi m_i}\right)^{1/2} \left(1 - \frac{e\Phi_d}{k_B T_i}\right) \tag{2.2}$$

for the positive ionic current. Here $\Phi_d = \Phi_g - \Phi_p$ is the difference between grain potential Φ_g and the plasma one Φ_p. As negative charges accumulate on the dust grain surface the negative constituents of the plasma are more and more repelled and finally an equilibrium situation is attained when

$$I_e + I_i = 0 \tag{2.3}$$

Together with the neutrality condition (1.1), it leads to the following equation determining the equilibrium value of Φ_d [1],[3],[5],[6]

$$\left(\frac{T_i}{T_e}\frac{m_e}{m_i}\right)^{1/2} \left(1 - \frac{e\Phi_d}{k_B T_i}\right) \exp\left(-\frac{e\Phi_d}{k_B T_e}\right) = 1 - Z_{d0}\frac{n_{d0}}{n_{i0}} \tag{2.4}$$

For spherical shaped dust grains of radius r_d the potential Φ_d is related to the charge $q_d = -Z_d e$ by

$$\Phi_d = -\frac{e}{4\pi\varepsilon_0}\frac{Z_{d0}}{r_d} \tag{2.5}$$

and thus the equilibrium value of Z_{d0} can be determined for given parameters of the dusty plasma.

With the notations

$$\begin{aligned} \sigma &= \frac{T_i}{T_e} \\ Y &= -\frac{e\Phi_d}{k_B T_i} = \frac{e^2}{4\pi\varepsilon_0 r_d}\frac{1}{k_B T_i} Z_{d0} \\ P &= -\frac{Z_{d0}}{Y}\frac{n_{d0}}{n_{i0}} = \frac{4\pi\varepsilon_0 r_d}{e^2} k_B T_i \frac{n_{d0}}{n_{i0}} \end{aligned} \tag{2.6}$$

the relation (2.4) writes

$$\sqrt{\sigma}(1+Y) = \sqrt{\frac{m_i}{m_e}}\,(1 - PY)\exp(-\sigma Y) \tag{2.7}$$

which can be solved numerically for each value of P. One has to mention that these expressions are valid if the streaming speeds of electrons and ions are much smaller than their thermal velocities.

In order to understand better the charging process it is necessary to evaluate the time characterizing the evolution of the dust grain charge to its equilibrium value after a perturbation occurs (the charging – relaxation time). As perturbation we shall consider a small sudden change in the potential Φ_d but taking P constant (corresponding to the equilibrium values of n_d and n_i). The time evolution of the dust grain charge is given by

$$\frac{dZ_d}{dt} = -\frac{I_i + I_e}{e} \tag{2.8}$$

Using the expressions (2.1) and (2.2) for the currents I_e and I_i (with $T_e = T_i = T$) and writing

$$Y \rightarrow Y_0 + y, \tag{2.9}$$

where Y_0 is the equilibrium value, solution of equation (2.8), and $y \ll Y_0$ a small perturbation, one obtains

$$\frac{dy}{dt} = -\frac{2+Y_0}{\sqrt{2\pi}}\frac{\omega_{pi}^2 r_d}{c_i} y, \tag{2.10}$$

where $c_i = \sqrt{\frac{k_B T}{m_i}}$ is the thermal velocity of ions and $\omega_{pi} = \left(\frac{e^2 n_{i0}}{\varepsilon_0 m_i}\right)^{1/2}$ is the plasma frequency of the ionic fluid. Assuming $y \sim \exp(-t/\tau)$, where τ is the charging (relaxation) time, one gets

$$\frac{1}{\tau} = \frac{2+Y_0}{\sqrt{2\pi}}\frac{\omega_{pi}^2 r_d}{c_i} \tag{2.11}$$

A numerical evaluation for reasonable physical conditions gives $\tau \sim 10^{-7} - 10^{-5}$ sec.. Compared to the slow time scale characteristic for the evolution of dust acoustic waves, one can conclude that the charging process is a fast one.

3 Dust Acoustic Waves in a Dusty Plasma

It is well known that a great variety of collective wave phenomena may arise in a plasma, due to the coherent motion of its constituents. In the absence of a magnetic field, one will consider only longitudinal waves produced by densities and potential fluctuations. The presence of charged dust grains can modify, or even dominate the wave phenomena. This is especially true in the low-frequency regime, where linear and nonlinear dust acoustic (DA) waves appear. They are produced by variations from the quasi-neutrality condition and determined by the dust particle dynamics. In the following we shall discuss briefly the properties of these low-frequency longitudinal waves, in a weakly coupled plasma, taking into account the dust charge variation.

The DA waves were predicted theoretically by Rao, Shukla and Yu in 1990 [8], [1], [3]. The phase velocity of the DA waves is much smaller than the electron and ion thermal speeds, and consequently in a first, but very good, approximation, one may assume that electrons and ions are in permanent thermal equilibrium with the local potential (the movement of the charged dust fluid is so slow that at any moment, the electronic and ionic fluids have enough time to restore the local equilibrium). Then the first equations describing the system are

$$n_e = n_{e0} \exp\left(\frac{e\Phi}{k_B T_e}\right) \tag{3.1}$$

$$n_i = n_{i0} \exp\left(-\frac{e\Phi}{k_B T_i}\right)$$

where T_e, T_i are the temperatures of the electron and ion fluid respectively and n_{e0}, n_{i0} the equilibrium values of their densities. As regards the movement of the dust particle fluid, this is described by a continuity equation

$$\frac{\partial n_d}{\partial t} + \nabla\left(n_d \vec{u}_d\right) = 0 \tag{3.2}$$

and a momentum equation

$$m_d\left(\frac{\partial}{\partial t} + \vec{u}_d \nabla\right)\vec{u}_d = -q_d \nabla\Phi - \frac{3k_B T_d}{n_d}\nabla n_d \tag{3.3}$$

In these relations n_d is the dust number density, $\vec{u}_d$ the velocity of dust fluid of temperature T_d, and $q_d < 0$ the negative charge of the dust particle. In the right-hand side of equation (3.3), the first term is the electric force, while the second one represents the force due to the pressure gradient. The set of these equations is completed by Poisson's equation

$$\nabla^2\Phi = \frac{e}{\varepsilon_0}(n_e + Z_d n_d - n_i) \tag{3.4}$$

Assuming equilibrium conditions at infinity, the boundary conditions of these quantities are

$$n_j \to n_{j0}, (j = e, i, d), \quad q_d \to q_{d0} = -eZ_{d0}, \quad \Phi = 0 \tag{3.5}$$

and the equilibrium values n_{j0}, Z_{d0} are satisfying the neutrality condition (1.1).

In the previous section it was shown that the charging process of dust particles is a fast one. Therefore one can assume that during the slow propagation of dust acoustic waves the electronic and ionic local currents have enough time to charge the dust particles at their local equilibrium values. Otherwise said, one considers the equation (2.8) true not only for the equilibrium values, but also for the local values $n_j(\vec{r}, t)$ taken by these densities during the slow evolution of the acoustic wave. It is the simplest assumption that can be done and it is similar to that invoked for justifying the equations (3.1). In the followings, it will be called "the local equilibrium approximation"(LEA). The results obtained for linear and nonlinear DA waves will be compared to existing results [9],[10],[11],[1], found using similar but more complex approximations.

Using the notations σ, Y, P defined previously (2.6), the local equilibrium condition resumes to the relation (2.7). The equations (3.1)–(3.4) and (2.7) together with the boundary conditions (3.5) constitute a complete set of equations from which the linear and nonlinear (dust acoustic) waves can be derived.

3.1 Linear Dust Acoustic Waves

Let us discuss first the linear DA waves. In the linear approximation, the equations (3.1) - (3.4) become

$$\begin{gathered}
n_{e1} = n_{e0}\frac{e\Phi}{k_B T_e}, \qquad n_{i1} = -n_{i0}\frac{e\Phi}{k_B T_i} \\
\frac{\partial n_{d1}}{\partial t} + n_{d0}\nabla\vec{u}_d = 0 \\
m_d\frac{\partial \vec{u}_d}{\partial t} = eZ_{d0}\nabla\Phi - \frac{3k_B T_d}{n_{d0}}\nabla n_{d1} \\
\nabla^2\Phi = \frac{e}{\varepsilon_0}(n_{e1} - n_{i1} + n_{d0}Z_{d1} + Z_{d0}n_{d1})
\end{gathered} \tag{3.6}$$

Here we considered $n_j = n_{j0} + n_{j1}$, $Z_d = Z_{d0} + Z_{d1}$ with $n_{j1} \ll n_{j0}$ and $Z_{d1} \ll Z_{d0}$. Also one writes $Y = Y_0 + y$, $P = P_0 - p$, with P_0, Y_0 defined by (2.6) and satisfying the equilibrium condition (2.7). According to LEA, Y and P obey the same relation (2.7) and therefore, the following expression for y and p is deduced

$$\left[\frac{1 + P_0}{1 + Y_0} + \sigma(1 - P_0 Y_0)\right] y = Y_0 p \tag{3.7}$$

But from their definitions (2.6) we get

$$\begin{gathered}
y = \frac{e^2/4\pi\varepsilon_0 r_d}{k_B T_i} Z_{d1} \\
p = \frac{k_B T_i}{e^2/4\pi\varepsilon_0 r_d}\left(\frac{n_{i1}}{n_{i0}} - \frac{n_{d1}}{n_{d0}}\right)\frac{n_{d0}}{n_{i0}}
\end{gathered} \tag{3.8}$$

and using (3.7), Z_{d1} is easily calculated in terms of the density fluctuations n_{i1} and n_{d1}. One obtains

$$n_{d0}Z_{d1} = A\left(\frac{n_{d0}}{n_{i0}}n_{i1} - n_{d1}\right) \tag{3.9}$$

where the dimensionless parameter A is given by

$$A = Y_0\left(\frac{k_B T_i}{e^2/4\pi\varepsilon_0 r_d}\right)^2\left[\frac{1 + P_0}{1 + Y_0} + \sigma(1 - P_0 Y_0)\right]^{-1}\frac{n_{d0}}{n_{i0}} \tag{3.10}$$

Looking for plane wave solutions, $\exp\left[i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right]$, one gets the following dispersion relation

$$\omega^2 = 3v_{Td}^2 k^2 + \frac{c_d^2\left(1-\frac{A}{Z_{d0}}\right)k^2}{1-\frac{\lambda_D^2}{\lambda_{Di}^2}\frac{n_{d0}}{n_{i0}}A+\lambda_D^2 k^2} \tag{3.11}$$

where $v_{Td} = \left(\frac{k_B T_d}{m_d}\right)^{1/2}$ is the thermal speed of dust particles, λ_D is the Debye screening length, $c_d = \omega_{pd}\lambda_D$ is the speed of the DAW and $\omega_{pd} = \left(\frac{e^2 Z_{d0}^2 n_{d0}}{\varepsilon_0 m_d}\right)^{1/2}$ is the plasma frequency of the dust fluid. Neglecting the effect of the dust particle charge variation (taking $A = 0$) (3.11) transforms into the well known result [1],[8]

$$\omega^2 = 3v_{Td}^2 k^2 + \frac{c_d^2 k^2}{1+\lambda_D^2 k^2} \tag{3.12}$$

It is worth mentioning that a similar LEA was used by Ma and Liu [11] in their study on DAW in a dusty plasma. Comparing the charging time τ_{ch} with a hydrodynamic time $\tau_h \sim \omega_{pd}^{-1}$, for reasonable values of plasma parameters, they found $\tau_{ch} \ll \tau_h$ and concluded that during slow motion of a DA wave the charge on the dust particles has enough time to reach the local equilibrium conditions.

Low frequency DA waves (in the range of few Hz) were experimentally observed [12], confirming the hypothesis of the local equilibrium approximation. As A given by (3.10) is a small quantity, the effect of dust particle charge variation on the linear DAW frequency is also small. More elaborated discussions [1],[9][13]-[15], show a damping of the DAW due to the delay in the charging process of dust particles. Such an effect cannot be obtained in our simplified treatment.

3.2 Dust Acoustic Solitons

The fluid equations describing a plasma are intrinsically nonlinear. The effect of nonlinearity is a cumulative process, manifesting at large space and time values, and leading to the formation of localized robust nonlinear structures – solitary waves and solitons in completely integrable situations. In plasma physics the solitons are well known for a long time [16] - [18]. They are described by the Korteweg - de Vries equation, which is completely solvable using the "inverse scattering method" [16], [17]. The Korteweg-de Vries equation is found applying an adequate asymptotic method to the nonlinear fluid equations describing the plasma [19], [16]-[18]. In the present section this problem is discussed for a dusty plasma. Different kinds of solitons appear in different frequency regimes. For dust acoustic solitons (DAS) the dust particle motion is considered, the electrons and ions being in thermal equilibrium with the local potential. In another frequency range, for dust ion-acoustic solitons (DIAS) - when the motion of ions is considered, the dust particles can be considered at rest. The topic of this section is the study of DAS, taking into account the charge variation of the dust particles.

It is convenient to use dimensionless variables and quantities. For simplicity the same temperature for all the plasma constituents ($T_e = T_i = T_d = T$) is assumed. The space coordinate x will be measured in units of Debye length λ_d, $\lambda_d = \left(\frac{\varepsilon_0 k_B T}{e^2 Z_{d0} n_{d0}}\right)^{1/2}$, the time variable t in units of ω_{pd}^{-1}, where ω_{pd} is the plasma frequency for the dust fluid, $\omega_{pd} = \left(\frac{q_{d0}^2 n_{d0}}{\varepsilon_0 m_d}\right)^{1/2}$, the dust fluid velocity u_d in units of $c_d = \left(\frac{k_B T}{m_d} Z_{d0}\right)^{1/2} = \omega_{pd}\lambda_d$, the dust particle thermic velocity, the electrostatic potential in units of $k_B T/e$, the charge number Z_d in units of equilibrium value Z_{d0} and the number densities n_s $(s = e, i, d)$ in units of their equilibrium values n_{s0} respectively. Then the dust particle fluid is described by a continuity equation and an equation of motion

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}\left(u_d n_d\right) = 0 \tag{3.13}$$

$$\frac{\partial u_d}{\partial t} + u_d\frac{\partial u_d}{\partial x} = Z_d\frac{\partial \varphi}{\partial x} \tag{3.14}$$

(only the electric force is considered) and Poisson equation

$$\frac{\partial^2 \varphi}{\partial x^2} = -\mu_i n_i + \mu_e n_e + Z_d n_d \tag{3.15}$$

where

$$\mu_s = \frac{n_{s0}}{n_{d0} Z_{d0}}, \qquad s = e, i \tag{3.16}$$

The electrons and ions are considered in thermal equilibrium with the local potential, and using the previously defined dimensionless variables, one has

$$n_e = \exp(\varphi) \quad n_i = \exp(-\varphi) \tag{3.17}$$

To these one has to add the equation (2.7) (with $\sigma = 1$) giving the dust particle charge in the local equilibrium approximation adopted here

$$(1 + Y) = \sqrt{\frac{m_i}{m_e}} (1 - PY) \exp(-Y) \tag{3.18}$$

where

$$\begin{aligned} Y &= Y_0 Z_d, \quad Y_0 = \frac{e^2/4\pi\varepsilon_0 r_d}{k_B T} Z_{d0} \\ P &= P_0 \frac{n_d}{n_i}, \quad P_0 = \frac{k_B T}{e^2/4\pi\varepsilon_0 r_d} \frac{n_{d0}}{n_{i0}} \end{aligned} \tag{3.19}$$

To study the dynamics of small, but finite amplitude DAS, the reductive technique [19] (multiple scale analysis) is employed. One introduces the stretched variables

$$\xi = \varepsilon^{1/2}(x - v_0 t), \quad \tau = \varepsilon^{3/2} t \tag{3.20}$$

and expand n_s, u_d, φ, Z_d in power series of ε

$$\begin{aligned} n_s &= 1 + \varepsilon n_s^{(1)} + \varepsilon^2 n_s^{(2)} + \dots, \quad s = e, i, d \\ u_d &= \varepsilon u_d^{(1)} + \varepsilon^2 u_d^{(2)} + \dots \\ \varphi &= \varepsilon \varphi^{(1)} + \varepsilon^2 \varphi^{(2)} + \dots \\ Z_d &= 1 + \varepsilon Z_d^{(1)} + \varepsilon^2 Z_d^{(2)} + \dots \end{aligned} \tag{3.21}$$

Introducing these in the equations (3.13)-(3.15) and (3.18), in order zero of ε from the Poisson equation one gets

$$\mu_i = 1 + \mu_e$$

which is merely another form to express the neutrality condition at equilibrium. In the first order of ε the following relations are obtained

$$\begin{aligned} & u_d^{(1)} = -\frac{1}{v_0}\varphi^{(1)}, \quad n_d^{(1)} = -\frac{1}{v_0^2}\varphi^{(1)} \\ & n_e^{(1)} = \varphi^{(1)}, \quad n_i^{(1)} = -\varphi^{(1)} \\ & \left(\mu_i + \mu_e - \frac{1}{v_0^2}\right)\varphi^{(1)} + Z_d^{(1)} = 0 \end{aligned} \tag{3.22}$$

where the last expression was obtained from the Poisson equation. Taking into account (3.21) and the definitions (3.19), one has

$$\begin{aligned} Y &= Y_0 \left(1 + \varepsilon Z_d^{(1)} + \varepsilon^2 Z_d^{(2)} + \dots\right) \\ P &= P_0 \left\{1 + \varepsilon \left(n_d^{(1)} - n_i^{(1)}\right) + \varepsilon^2 \left[n_d^{(2)} - n_i^{(2)} + \left(n_i^{(1)}\right)^2 - n_d^{(1)} n_i^{(1)}\right] + \dots\right\} \end{aligned} \tag{3.23}$$

and in first order of ε, the following relation between $Z_d^{(1)}$ and $\varphi^{(1)}$ is found from (3.18)

$$
\begin{aligned}
Z_d^{(1)} &= -B\left(1-\frac{1}{v_0^2}\right)\varphi^{(1)} \\
\frac{1}{B} &= 1+\frac{(2+Y_0)(1-P_0Y_0)}{P_0(1+Y_0)}
\end{aligned}
\tag{3.24}
$$

Introducing (3.24) into the last relation (3.22), one gets the expression for v_0

$$
v_0 = \left[\frac{1-B}{\mu_i+\mu_e-B}\right]^{1/2} \tag{3.25}
$$

Taking $B=0$, (3.25) writes

$$
v_0 = (\mu_i+\mu_e)^{-1/2}
$$

which is the result when the charge of the dust particle is constant [1].

In order ε^2 one obtains

$$
\frac{\partial}{\partial\xi}\left(-v_0 n_d^{(2)} + u_d^{(2)}\right) - \frac{1}{v_0^2}\frac{\partial\varphi^{(1)}}{\partial\tau} + \frac{2}{v_0^3}\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\xi} = 0
$$

$$
-\frac{\partial}{\partial\xi}\left(\frac{1}{v_0}\varphi^{(2)} + u_d^{(2)}\right) - \frac{1}{v_0^2}\frac{\partial\varphi^{(1)}}{\partial\tau} + \frac{1}{v_0^3}\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\xi} - \frac{B}{v_0}\left(1-\frac{1}{v_0^2}\right)\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\xi} = 0
$$

from the continuity and the momentum equation respectively. Here the relations (3.21) - (3.22) were used to express all the first order quantities with respect to $\varphi^{(1)}$. Eliminating $u_d^{(2)}$ from the above equations one remains with

$$
\frac{\partial}{\partial\xi}\left(\frac{1}{v_0^2}\varphi^{(2)} + n_d^{(2)}\right) + \frac{2}{v_0^3}\frac{\partial\varphi^{(1)}}{\partial\tau} - \left[\frac{3}{v_0^4} - \frac{B}{v_0^2}\left(1-\frac{1}{v_0^2}\right)\right]\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\xi} = 0 \tag{3.26}
$$

The Poisson equation in order ε^2 gives

$$
\frac{\partial^2\varphi^{(1)}}{\partial\xi^2} = (\mu_i+\mu_e)\,\varphi^{(2)} - \frac{1}{2}\left(\varphi^{(1)}\right)^2 + n_d^{(2)} + Z_d^{(2)} - \frac{B}{v_0^2}\left(1-\frac{1}{v_0^2}\right)\left(\varphi^{(1)}\right)^2 \tag{3.27}
$$

Here we used equation (3.24) and

$$
n_{e,i}^{(2)} = \pm\varphi^{(2)} + \frac{1}{2}\left(\varphi^{(1)}\right)^2
$$

But from (3.18), in order ε^2, one obtains

$$
\begin{aligned}
Z_d^{(2)} &= -B\left(n_d^{(2)} + \varphi^{(2)}\right) - BD\left(\varphi^{(1)}\right)^2 \\
D &= -\frac{1}{2} + \left(1-\frac{1}{v_0^2}\right)\left[1 + B\left(1-\frac{1}{v_0^2}\right)\left(1 + BY_0\frac{(1-P_0Y_0)(3+Y_0)}{2P_0(1+Y_0)}\right)\right]
\end{aligned}
\tag{3.28}
$$

Introducing (3.28) into (3.27) we get

$$
\frac{\partial^2\varphi^{(1)}}{\partial\xi^2} = (\mu_e+\mu_i-B)\varphi^{(2)} + (1-B)n_d^{(2)} - \frac{1}{2}K\left(\varphi^{(1)}\right)^2 \tag{3.29}
$$

where

$$
K = 1 + 2B\left[\frac{1}{v_0^2}\left(1-\frac{1}{v_0^2}\right) + D\right] \tag{3.30}
$$

Then, using the definition of v_0, the equation (3.29) can be written

$$\frac{1}{1-B}\frac{\partial^3\varphi^{(1)}}{\partial\xi^3}+\frac{K}{1-B}\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\xi}=\frac{\partial}{\partial\xi}\left(n_d^{(2)}+\frac{1}{v_0^2}\varphi^{(2)}\right)$$

which combined with (3.29) allows the second order terms elimination.
Finally $\varphi^{(1)}$ is satisfying the following Korteweg - de Vries equation

$$\begin{aligned}\frac{2}{v_0^3}\frac{\partial\varphi^{(1)}}{\partial\tau}+\frac{1}{1-B}\frac{\partial^3\varphi^{(1)}}{\partial\xi^3}-M\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\xi}=0\\ M=\frac{3}{v_0^4}-\frac{B}{v_0^2}\left(1-\frac{1}{v_0^2}\right)-\frac{K}{1-B}\end{aligned}\tag{3.31}$$

For $B=0$, the equation (3.31) writes

$$\begin{aligned}\frac{\partial\varphi^{(1)}}{\partial\tau}-a_s\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\xi}-b_s\frac{\partial^3\varphi^{(1)}}{\partial\xi^3}=0\\ a_s=\frac{v_0^3}{3}\left(\frac{3}{v_0^4}-1\right)>0,\qquad b_s=\frac{v_0^3}{3}\end{aligned}\tag{3.32}$$

and this corresponds to the result obtained when the electrical charge on the dust particles is constant. As $\mu_i>1$, we have $v_0<1$ and consequently $a_s>0$ [1]. For small B we expect that these conclusions remain true also for equation (3.31). Defining

$$a_s=\frac{v_0^3}{2}M>0,\qquad b_s=\frac{v_0^2}{2(1-B)}\tag{3.33}$$

the one soliton solution of (3.31) is

$$\begin{aligned}\varphi^{(1)}(\xi,\tau)=-\,\varphi_m^{(1)}\mathrm{sech}^2\left[(\xi-u_0\tau)/\Delta\right]\\ \varphi_m^{(1)}=\frac{3u_0}{a_s},\qquad\Delta=\sqrt{\frac{4b_s}{u_0}}\end{aligned}\tag{3.34}$$

describing a propagating soliton with velocity u_0 and with $n_d^{(1)}(\xi,\tau)>0$ (from (3.22) $n_d^{(1)}>0$ if $\varphi^{(1)}<0$) i.e. a compressive solitary wave.

4 Concluding remarks

In the previous sections, the "*local equilibrium approximation*" was used to determine the linear and the nonlinear (soliton) dust-acoustic wave solutions. It is in authors' opinion the simplest way to take into account the charge fluctuations on the dust particles. The effect is a (real) shift of the characteristic properties of the linear and nonlinear solutions with respect to the unperturbed ones. In their work on the effect of the dust grain charge fluctuations on the dust acoustic solitons, Rao and Shukla [10],[1] used a quite similar approximation. They determine a Sagdeev (nonlinear) potential $V(\varphi,Z_d)$ controlling the stationary evolution of plasma potential

$$\frac{1}{2}\left(\frac{d\varphi}{d\xi}\right)^2+V(\varphi,Z_d)=0\tag{4.1}$$

and the effect of charge fluctuation is investigated numerically. It is worth to compare their results with the present ones for given parameters of the dusty plasma.

A better analysis of the effect of the dust particle charge fluctuations can be done using a kinetic approach [20]. In this way the effect of collisions between plasma constituents can be taken into account. Usually the effect of these collisions is a damping factor for plasma excitations, and the comparison with experimental facts is better. Such an approach should be better for dust ion acoustic (DIA) waves [1], [21]-[25], where a local equilibrium approximation is questionable.

References

[1] P. K. Shukla, A. A. Mamun - "Introduction to Dusty Plasma Physics" (IOP Publishing, Bristol, 2002)

[2] K. Avinash - Phys. Plasmas, 14, 012904 (2007)

[3] P. K. Shukla - Phys. Plasmas, 8, 1791 (2001)

[4] U. de Angelis - Phys. Plasmas, 13, 012514 (2006)

[5] U. de Angelis - Physica Scripta, 45, 465 (1992)

[6] T. G. Northrop - Physica Scripta, 45, 475 (1992)

[7] F. Verheest - Planet. Space Sci., 1, 1 (1992)

[8] N. N. Rao, P. K. Shukla, M. Y. Yu - Planet. Space Sci., 38, 543 (1990)

[9] F. Melandsø, T. Aslaksen, O. Havnes - Planet. Space Sci., 41, 321 (1993)

[10] N. N. Rao, P. K. Shukla - Planet. Space Sci., 42, 221 (1994)

[11] J. X. Ma, J. Liu - Phys. Plasmas, 4, 253 (1997)
Y. F. Li, J. X. Ma, D-L Xiao - Phys. Plasmas, 11, 5108 (2004)

[12] A. Barkan, R. L. Merlino, N. D'Angelo - Phys. Plasmas, 2, 3563 (1995)
J. B. Pieper, J. Goree - Phys. Rev. Lett., 15, 3137 (1996)

[13] M. Rosenberg - Planet. Space Sci., 41, 229 (1993)

[14] M. R. Jana, A. Sen, P. K. Kaw - Phys. Rev. E, 48, 3930 (1993)

[15] P. K. Shukla, M. Rosenberg - Phys. Plasmas, 6, 1038 (1999)

[16] R. K. Dodd, J. C. Eilbeck, J. D. Gibbon, H. C. Morris - "Solitons and Nonlinear Wave Equations" (Acad. Press, London, 1982)

[17] P. G. Drazen, R. S. Johnson - "Solitons. An Introduction" (Cambridge Univ. Press, 1989)

[18] A. Scott - "Nonlinear Science. Emergence and Dynamics of Coherent Structures" (Oxford Univ. Press, 2003)

[19] T. Taniuti, C. C. Wei - J.Phys. Soc. Japan, 24, 941 (1968)
T. Taniuti - Suppl. Prog. Theor. Phys., 55, 1 (1974)

[20] V. N. Tsytovich, U. de Angelis - Phys. Plasmas, 6, 1093 (1999);7, 554 (2000);8, 1141 (2001);9, 2497 (2002)

[21] P. K. Shukla, V. P. Silin - Physica Scripta, 45, 508 (1992)

[22] S. Ghosh, S. Sarkar, M. Khan, M. R. Gupta - Phys. Lett. A, 274, 162 (2000)

[23] W. Masood, A. Mushtaq, R. Khan - Phys. Plasmas, 14, 123702 (2007)

[24] S. I. Popel et al. - Phys. Rev. E, 67, 056402 (2003)

[25] D. Grecu, A. T. Grecu - J. Opt. Adv. Mat., 10, 80 (2008)