Physics AUC, vol. 18, 166-177 (2008) **PHYSICS AUC**

On tachyon-like phenomena in classical and quantum physics

D. D. Dimitrijević G. S. Djordjević Lj. D. Nešić
Department of Physics, University of Niš,
P.O.Box 224, 18000 Niš, Serbia

Abstract

In this paper we review some tachyon or tachyon-like phenomena classical and quantum physics. We consider cosmological aspects on tachyons on real and p-adic number fields. A brief overview on tachyons in string theory is given and their classical analogue is considered.

1 Introduction

The first association on tachyons is probably hypothetical particles that travel faster than light. The first issue about the object, now called tachyons was due to A. Sommerfeld one century ago [1]. Even though there is no experimental evidence of tachyons, there are several reasons why tachyons are still of interest today. In fact, interest in tachyons increase.

Tachyons captured some interest in the physics community in the 1960s and 70s, when they were given this name by G. Feinberg, from the Greek "$\tau\alpha\chi\iota\sigma$" meaning "swift" or "speedy" [2], but they have since fallen somewhat from fashion because direct experimental evidence has not been found to support their existence, and also because of concerns about causality.

Let us remind the relativistic relation between the velocity v, the spatial momenta k and mass m of a particle (we use the system where $c = h = 1$)

$$v = \frac{k}{\sqrt{k^2 + m^2}}. \tag{1}$$

It is obvious that for real k a tachyon ($v > 1$) must have negative mass squared

$$v > 1 \iff m^2 < 0. \tag{2}$$

A modern view of tachyon phenomena is related not to the superluminal speed ($v > 1$) but to the negative mass squared. Field theories (classical and quantum) offer a much better insight into the role of tachyons. If we carry out perturbative quantization of the canonical[1] scalar field by expanding a potential around extremum configuration, we will find a particle like state with either $m^2 > 0$ if this configuration is minimum, or a particle with negative mass squared (if this configuration is maximum), i.e. a tachyon. The existence of a tachyon in a quantum field theory is associated with an instability of the system which causes a breakdown of the perturbation theory. We simply need to expand the potential around a new point where one has a minimum, and carry out perturbative quantization of the theory around this point. This in turn will give a particle with positive mass squared in the spectrum. Spontaneous symmetry breaking in the standard model of elementary particles is sometime considered as a fact that Higgs scalar field has a tachyonic mass term. This is reviewed in Section 2.

One feature of the 26-dimensional bosonic string theory has been problematic one, since the early days of the string theory: both open and closed bosonic strings have tachyons in their spectra, indicating that the usual perturbative vacua used for these theories are unstable. Concerning Dp-branes, A. Sen [3] has assumed that tachyon condensation occurs in unstable configurations of Dp-branes. Action for the effective theory around the tachyon vacua is given by noncanonical lagrangian [4]. Such

[1]Canonical field is the one with lagrangian $L = T - V$.

systems have tachyonic open string states which condense. The resulting stable vacuum state is a closed string vacuum state, whereas the D-branes have decayed and therefore open strings are absent. This is the topic of Section 3.

Effective tachyon field action has been used in cosmology to consider dark energy, dark matter [5] and inflation. In the Section 4 we discuss this point.

Use of p-adic numbers Q_p is still of some interests in cosmology. p-Adic inflation is one of such ideas [6]. Also, it has been shown there is some considerable similarity between the p-adic action for the de Sitter model in $(2+1)$ dimensions and action for the tachyon field in the zero dimensional model [7]. Sections 5 and 6 are devoted to p-adic inflation and a zero dimensional tachyon model.

Section 7 is reserved for conclusion.

2 Tachyons in field theory

2.1 Canonical scalar fields

Let us consider canonical scalar field Φ with conventional kinetic term, and a potential $V(\Phi)$ which has an extremum at the origin ($V(\Phi=0)=0$, $\frac{dV(0)}{d\Phi}=0$)

$$L = \frac{1}{2}(\partial_\mu \Phi)^2 - V(\Phi). \tag{3}$$

If we carry out perturbative quantization of the scalar field by expanding the potential around $\Phi = 0$

$$V(\Phi) = \frac{1}{2}\frac{d^2V}{d\Phi^2}|_{\Phi=0}\Phi^2 + ..., \tag{4}$$

and ignore the cubic and higher order terms in the action, we find a particle like state with $m^2 = V''(0)$. For $V''(0)$ positive this describes a particle with positive mass squared. But, for $V''(0) < 0$ we have a particle with negative mass squared, i.e. a tachyon.

The existence of the tachyon here has a clear physical interpretation. For $V''(0) < 0$, the potential $V(\Phi)$ has a maximum at the origin, and hence a small displacement of Φ away from the origin will make it grows exponentially in time. Thus perturbation theory, in which we treat the cubic and higher order terms in the potential to be small, breaks down. Therefore, the existence of a tachyon in a quantum field theory is associated with an instability of the system. We simply need to expand the potential around a new point where it has a minimum, and carry out perturbative quantization of the theory around this point. This in turn will give a particle with positive mass squared in the spectrum.

If we include higher order term such as $\frac{d^4V}{d\Phi^4}$ and use tachyonic mass term, then it is possible to induce spontaneous symmetry breaking (SSB), which is the basis for the standard model theory of elementary particles. In the following we will illustrate "appearance" of a tachyon-like state in the well-known Higgs mechanism.

2.2 Spontaneous symmetry breaking as a tachyon-like phenomenon

Phenomenon, where the solutions of the equations of motion break the symmetries of the equations, is called spontaneous symmetry breaking. One of the simplest example of SSB is the Higgs mechanism [8]. A set of scalar fields which transform nontrivially under the symmetry group is introduced into the model. If the vacuum expectation value (VEV) of one of these fields is nonzero, then the gauge symmetry will be broken, and all of the gauge bosons for which the field with nonzero VEV has nonzero charge will become massive. Since the Lagrangian itself is still gauge invariant, the theory is still renormalizable; however, the vacuum state (i.e. the solution of the equations of motion) explicitly breaks the symmetry, allowing the vector bosons to acquire mass.

To sketch it, we will use $U(1)$ gauge invariant scalar field theory with tachyonic mass term coupled to a massless vector field A_μ

$$L = (D_\mu\Phi)^\dagger(D^\mu\Phi) - \Big(m^2\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2\Big) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{5}$$

where

$$D_\mu = \partial_\mu - ieA_\mu, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{6}$$

After SSB, which is possible because we are dealing with tachyonic scalar field, the lagrangian (5) became

$$L' = \frac{1}{2}e^2\sigma^2 A_\mu A^\mu - e\sigma A_\mu \partial^\mu \chi' - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + ..., \tag{7}$$

where σ is VEV of the scalar field ($< \Phi >= \sigma$). Notice that $m_{A_\mu} \neq 0$, i.e. vector field became massive.

3 Tachyons in string theory

3.1 Classical bosonic strings

The action for a relativistic string can be written as here (we will consider the open strings only)

$$S = -T \int_\Sigma d\sigma d\tau \sqrt{-det[\eta_{\mu\nu}\partial_i X^\mu \partial_j X^\nu]}, \tag{8}$$

where σ i τ are (local) coordinate of the world sheet Σ, $\eta_{\mu\nu}$ is Minkowski metric tensor $\eta_{\mu\nu} = diag(-,+,+,+,...+)$, σ and τ are world sheet coordinates, T is string tension and X^μ are string coordinates. This is the Nambu-Goto action, which is the direct generalization of the action for a massive relativistic particle.

In the light-cone coordinates and light-cone gauge, using appropriate Neumann boundary condition, solutions of the equation of motion for the open string are

$$X^+(\tau, \sigma) = \sqrt{2\alpha'}\alpha_0^+ \tau, \tag{9}$$

$$X^-(\tau, \sigma) = x_0^- + \sqrt{2\alpha'}\alpha_0^- \tau + i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n}\alpha_n^- \exp{(-in\tau)} \cos{(n\sigma)}, \tag{10}$$

$$X^I(\tau, \sigma) = x_0^I + \sqrt{2\alpha'}\alpha_0^I \tau + i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n}\alpha_n^I \exp{(-in\tau)} \cos{n\sigma}, \tag{11}$$

where $\alpha' = \frac{1}{2\pi T}$ is slope parameter. Hamiltonian of the classical bosonic string is of the form [9]

$$\mathcal{H} = L_0^\perp, \tag{12}$$

where the transverse Virasoro mode $L_0^\perp$ is defined through momentum components p^+, p^- and slope parameter α'

$$L_0^\perp = 2\alpha' p^+ p^-. \tag{13}$$

The mass of a string which is performing an arbitrary motion can be calculated using the relativistic equation

$$M^2 = -p^2 = 2p^+p^- - p^I p^I = \frac{1}{\alpha'}L_0^\perp - p^I p^I, \tag{14}$$

which yields to

$$M^2 = \frac{1}{\alpha'} \sum_{n \in N} (\alpha_n^I)^\star \alpha_n^I = \frac{1}{\alpha'} \sum_{n \in N} |\alpha_n^I|^2 \geq 0. \tag{15}$$

This means that the mass of the classical open string is a real number:

$$M^2 \geq 0 \Rightarrow M = +\sqrt{M^2} \in R^+. \tag{16}$$

3.2 Quantum theory of bosonic strings

When quantizing bosonic strings all coefficients α in (9), (10) and (11) became operators, satisfying suitable commutation relations. In addition, the transverse Virasoro mode becomes a Virasoro operator

$$L_0^{\perp} \rightarrow : \hat{L_0^{\perp}} : +a, \tag{17}$$

where we used normal ordering prescription and added normal-ordered constant a which need to be added to satisfy condition of Lorentz invariance.

The constant a of normal ordering may be determined by constructing the Lorentz generators for the D-dimensional string theory, and requiring the commutators of the Lorentz generators do not have any quantum anomalies spoiling the Lorentz invariance [10]. The vanishing of these anomalies requires

$$D = 26, \qquad a = -1. \tag{18}$$

Hamiltonian operator is now of the form

$$\hat{\mathcal{H}} =: \hat{L_0^{\perp}} : -1, \tag{19}$$

and the mass squared operator becomes

$$\hat{M}^2 = \frac{1}{\alpha'}\left(-1 + \hat{N}^{\perp}\right), \tag{20}$$

where we introduce number operator

$$\hat{N}^{\perp} = \sum_{n=1}^{\infty} n \hat{a}_n^{\dagger I} \hat{a}_n^I, \tag{21}$$

and creation and annihilation operators

$$\hat{\alpha}_n^I \rightarrow \hat{a}_n^I = \frac{1}{\sqrt{n}} \hat{\alpha}_n^I, \tag{22}$$

$$\hat{\alpha}_n^{\dagger I} = \hat{\alpha}_{-n}^I \rightarrow \hat{a}_n^{\dagger I} = \frac{1}{\sqrt{n}} \hat{\alpha}_n^{\dagger I}. \tag{23}$$

Now, we can see the mass spectra of quantum bosonic string is shifted by the factor -1, which will cause the appearance of tachyonic ground state:

$$M^2 = -\frac{1}{\alpha'} < 0, \qquad \hat{N}^{\perp} = 0. \tag{24}$$

One can say the condition of Lorentz invariance of "quantum" string theory simultaneously fixes number of dimensions of spacetime and the constant shift in the masses of the particles introducing tachyonic ground state in the string spectra.

3.3 *D*-branes

Tachyons do not only occur in the ground state of bosonic string theories, but also in some D-branes configurations. A. Sen [3] conjectured that tachyon condensation occurs in unstable configurations of D-branes. Such systems have tachyonic open string states which condense. The resulting stable vacuum is the closed string vacuum, whereas the D-branes have decayed and therefore open strings are absent.

For the two parallel Dp-branes configuration, located at positions $x^a_{(1)}$ and $x^a_{(2)}$, the new feature is that there are strings which start and end on different branes. For such strings there is an additional term in the mass formula, which accounts for the stretching (compare with (20)):

$$\hat{M}^2 = \frac{1}{\alpha'}\left(-1 + \hat{N}^{\perp} + (\frac{|x^a_{(1)} - x^a_{(2)}|}{2\pi\sqrt{\alpha'}})^2\right). \tag{25}$$

The ground state ($N^{\perp}=0$) becomes tachyonic one if two branes come close enough. It is a signal of instability of the D-brane configuration

$$M^2 < 0 \iff |x^a_{(1)} - x^a_{(2)}| \to 0. \tag{26}$$

It is known that both the open and closed bosonic strings have tachyons in their spectra, indicating the usual perturbative vacua used for these theories are unstable. It has been suggested [11] that the open bosonic string should be thought of as ending on a space-filling $D25$-brane. This D-brane is unstable in the bosonic theory and the open bosonic string tachyon should be interpreted as the instability mode of the $D25$-brane. The three following conjectures are made:

1) The effective potential for the tachyon mode has a minimum, and the difference in energy between the perturbative vacuum and the minimum of the potential cancels the mass of the 25-dimensional space filling D-brane.

2) In the minimum of the potential there are no open string excitations. This is so because we expect that at the true vacuum the brane has decayed and only closed string excitations are present.

3) There should be lump solutions of the tachyon potential which correspond to lower dimensional branes.

Effective field theory of tachyon matter is of noncanonical form [3] (see also [4]). The action is given as:

$$S = -\int d^{D+1}x V(T)\sqrt{1+\eta^{\mu\nu}\partial_\mu T \partial_\nu T}, \tag{27}$$

where $\eta_{00} = -1$ and $\eta_{\alpha\beta} = \delta_{\alpha\beta}$, $\alpha, \beta = 1, 2, 3, ..., D$, $T(x)$ is the scalar tachyon field and $V(T)$ is the tachyon potential which "unusually" appears in the action as a multiplicative factor and has exponential dependence with respect to the tachyon field $V(T) \sim e^{-\alpha T/2}$. This potential $V(T)$ has a maximum at $T = 0$ and a minimum at infinity.

Action (27) attracted considerable interest in cosmology in attempts to describe both dark components as well as inflation.

The bosonic string theory is not entirely satisfactory, first because it does not possess any fermionic states, and second because the string ground state is tachyonic.

As for the bosonic string, normal-ordering constants may be determined by requiring that the commutators of the Lorentz generators are free from quantum anomalies. For the superstring, this requires

$$D = 10, \qquad a = -\frac{1}{2}. \tag{28}$$

The problem of tachyonic ground states may be avoided, and a (10 dimensional) space-time supersymmetric theory obtained at the same time, by applying the Gliozzi-Scherk-Olive (GSO) projection [12].

4 Tachyons in cosmology

4.1 Basic facts

Standard big-bang cosmology is based upon the cosmological principle [13], which requires that the Universe is homogeneous and isotropic on the large scales. Then the metric takes the Friedmann-Robertson-Walker (FRW) form

$$ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\varphi^2)], \tag{29}$$

where $a(t)$ is the scale factor with t being the cosmic time. The constant k is the spatial curvature, where positive, zero or negative values correspond to the closed, flat or hyperbolic spatial Universe, respectively.

Classical dynamics of the universe is used to be described by the Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{M^2_{pl}}T_{\mu\nu} + g_{\mu\nu}\Lambda, \tag{30}$$

where $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, Λ is the cosmological constant and $M_{pl} = (8\pi G_N)^{-1/2}$ is reduced Planck mass (G_N is Newton gravitational constant). The energy momentum tensor $T_{\mu\nu}$ has the form of ideal (perfect) fluid

$$T_\mu^\nu = (-\rho, P, P, P), \tag{31}$$

where ρ and P are the energy density and the pressure density of the fluid, respectively.

From the Einstein equations (30) for the background FRW metric (29), we obtain the Friedmann and Raychaudhuri equations, respectively ($k = 0$, $\Lambda = 0$)

$$H^2 = \frac{\rho}{3M_p^2}, \tag{32}$$

$$2\frac{\ddot{a}}{a} = -\frac{1}{3M_p^2}(\rho + 3P), \tag{33}$$

where $H = \frac{\dot{a}(t)}{a(t)}$ is the Hubble expansion rate. Combination of these equations implies energy conservation

$$\dot{\rho} = -3H(\rho + P). \tag{34}$$

The evolution of the Universe is dependent on the content within, where a key role plays the equation of state connecting energy density and pressure (w is a state parameter)

$$P(t) = w\rho(t). \tag{35}$$

Beside some "old" problems, this model is confronted with observations made in 1998 [14], strongly favourize an accelerating expansion of the Universe in last 4.5 billion of years.

4.2 Dark components

Cosmological observations show that the Universe is homogeneous and isotropic at large scales (larger than about 200 Mpc). Most of observations related to large scale structure suggest that the Universe is populated by a non-luminous component of matter, called dark matter (DM), perhaps made of weakly interacting massive particles. This component contributes about $20 - 25\%$ of the total energy budget of the Unverse and has the simple equation of state $P_{DM} = 0$.

As we said, analysis of the type Ia supernovae, galaxy clusters measurements and WMAP data brings out an evidence of the accelerated expansion of the Universe [14], [15]. It indicates that the present day Universe is dominated by a smoothly distributed dark energy (DE), with the present time DE state parameter $w_{DE} \approx -1$. This component is unclustered and has negative pressure and contributes by $60 - 75\%$ of the total energy budget. The simplest choice for such dark energy with negative pressure is the cosmological constant. This term acts like a fluid with an equation of state $P_{DE} = -\rho_{DE}$.

The most popular alternative to cosmological constant is a scalar field with a suitably chosen potential. The hope is that one can find a model in which the current value can be explained naturally without a fine tuning. There are two possibilities based on the lagrangians ($g_{\mu\nu} = sign(-, +, +, +)$):

$$\mathcal{L}_{quin} = -\frac{1}{2}g_{\mu\nu}\partial^\mu\Phi\partial^\nu\Phi - V(\Phi), \tag{36}$$

$$\mathcal{L}_{tach} = -V(T)\sqrt{1 + g_{\mu\nu}\partial^\mu T\partial^\nu T}. \tag{37}$$

Both these lagrangians involve one arbitrary function - potential V. The first one, $\mathcal{L}_{quin}$, which is a natural generalization of the lagrangian for a nonrelativistic particle

$$L = (1/2)q^2 - V(q), \tag{38}$$

is usually called quintessence. The structure of the lagrangian (37) can be understood by an analogy with a relativistic particle described by the lagrangian

$$L = -m\sqrt{1-q^2}. \tag{39}$$

One can now constructs tachyon field theory by upgrading $q(t)$ to a field T and treating the mass parameter m as a function of T to obtain the lagrangian (37). This provides a rich gamut of possibilities in the context of cosmology [16], [17].

Energy-momentum tensor of tachyonic perfect fluid has the form

$$T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_{tach} + V(T)\frac{\partial_\mu T\partial_\nu T}{\sqrt{1+g_{\mu\nu}\partial^\mu T\partial^\nu T}}. \tag{40}$$

In a flat FRW background the energy density ρ and the pressure density P are given by

$$\rho = -T^0_0 = \frac{V(T)}{\sqrt{1+g_{\mu\nu}\partial^\mu T\partial^\nu T}}, \tag{41}$$

$$P = T^N_N = -V(T)\sqrt{1+g_{\mu\nu}\partial^\mu T\partial^\nu T}. \tag{42}$$

From eqs. (32) and (34) for homogenous tachyon field ($T = T(t)$) we obtain the following equations

$$H^2 = \frac{1}{3M_{pl}^2}\frac{V(T)}{\sqrt{1-\dot{T}^2}}, \tag{43}$$

$$\frac{\ddot{T}}{1-\dot{T}^2} + 3H\dot{T} + \frac{1}{V}\frac{dV}{dT} = 0. \tag{44}$$

The equation of state of the tachyon is given by

$$\frac{P}{\rho} = w = \dot{T}^2 - 1. \tag{45}$$

We see from (45) that tachyon field is very different from the standard one. Irrespective of the steepness of the tachyon potential, the state parameter varies between 0 to 1.

Interesting feature of eqs. (41) and (42) is that it could be considered as the sum of two components: dark energy and dark matter

$$\rho = \rho_{DM} + \rho_{DE}, \qquad P = P_{DM} + P_{DE}, \tag{46}$$

where

$$\rho_{DM} = \frac{V(T)g_{\mu\nu}\partial^\mu T\partial^\nu T}{\sqrt{1+g_{\mu\nu}\partial^\mu T\partial^\nu T}}, \qquad P_{DM} = 0, \tag{47}$$

$$\rho_{DE} = V(T)\sqrt{1+g_{\mu\nu}\partial^\mu T\partial^\nu T}, \qquad P_{DE} = -\rho_{DE}. \tag{48}$$

This means that the stress tensor can be thought of as made up of two components one behaving like a pressure-less fluid, while the other having a negative pressure [5].

It is widely accepted that a period of inflation, even short one, is unavoidable in any satisfactory cosmological model.

4.3 Inflation

In order to generate enough inflation, it is necessary for the inflaton field to roll slowly enough. This is characterized by two dimensionless parameters ϵ and η. For the conventional inflaton with canonically normalized kinetic energy term, these are given by [13]

$$\epsilon = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta = M_{pl}^2\left(\frac{V''}{V}\right). \tag{49}$$

The conditions for slow roll inflation are

$$\epsilon \ll 1, \qquad |\eta| \ll 1. \tag{50}$$

These formulas are not directly applicable to case of the tachyon field. While the inflation satisfies one slow roll condition (at least, initially), it fails to satisfy the other one. In this case Universe fails to inflate enough [18].

Tachyonic inflation (considered on real numbers) can not be not responsible for the last 60 e-folds of inflation. This last stage of inflation determined the large scale structure of the observable part of the Universe. It might be possible, however, that tachyonic inflation is responsible for an earlier stage of inflation. The existence of this short stage of inflation at a nearly Planckian density may be important for the resolution of major cosmological problems such as the homogeneity, flatness and isotropy problems [19]. But one would need to have a second stage of inflation after that.

5 Tachyons in p-adic theories

5.1 Introduction

Any norm must satisfy three conditions: nonnegativity, homogeneity and triangle inequality. The completion of the field of rational numbers Q with respect to the absolute value, or standard norm $|\cdot|_\infty$, gives the field of real numbers $R = Q_\infty$. Besides this norm there are other norms which satisfy the first two conditions, and the third one in a stronger way

$$\parallel x + y \parallel \leq max(\parallel x \parallel, \parallel y \parallel), \tag{51}$$

so called strong triangle inequality. The most important of them is p-adic norm $|\cdot|_p$ (p denotes a prime number) [20]. The feature (51), also called ultrametricity, is one of the most important characteristics of the p-adic norm. The number fields obtained by completion of Q with respect to this norm are called p-adic number fields Q_p. Let us note that because Q_p is local compact commutative group the Haar measure can be introduced, which enables integration.

Simultaneous treatment of real and p-adic numbers can be realized by concept of adeles. An adele $a \in A$ is an infinite sequence

$$a = (a_\infty, a_2, a_3, ..., a_{137}, ..., a_p, ...), \tag{52}$$

where $a_\infty \in R$ and $a_p \in Q_p$, with the restriction that ap $a_p \in Z_p$ for all but a finite set S of primes p.

5.2 p-adic quantum mechanics

p-Adic quantum mechanics (QM) has been developed in two different ways: in the first one, wave function is complex valued function of p-adic variable [20], and in the second one, p-adic wave function depends on p-adic variable [21]. If we want simultaneous (adelic) treatment of standard quantum mechanics and all p-adics, including usual probabilistic interpretation developed in standard QM, then the first formulation is the preferable one. Because the fields Q_p are totally disconnected, dynamics of p-adic quantum models is described by a unitary evolution operator $U(t)$ without using the Hamilton operator and infinitesimal displacement (for alternative formulation of p-adic QM see [20], [22]).

It was shown that in many cases p-adic quantum cosmological models can be considered as quantum mechanical models in p-adic and adelic approach. Let us introduce this approach.

5.3 p-adic quantum cosmology

In the path integral approach to quantum cosmology over the field of real numbers R, the starting point is the idea that the amplitude to go from one state with intrinsic metric h'_{ij}, and matter configuration ϕ' on an initial hypersurface Σ', to another state with metric h''_{ij}, and matter configuration ϕ'' on a final hypersurface Σ'', is given by a functional integral of the form

$$\langle h''_{ij}, \phi'', \Sigma'' | h'_{ij}, \phi', \Sigma' \rangle = \int \mathcal{D}g_{\mu\nu} \mathcal{D}\Phi e^{-S[g_{\mu\nu}, \Phi]}, \tag{53}$$

over all four-geometries $g_{\mu\nu}$, and matter configurations Φ, which interpolate between the initial and final configurations. In this expression $S[g_{\mu\nu}, \Phi]$ is an Einstein-Hilbert action for the gravitational and matter fields (which can be massless, minimally or conformally coupled with gravity). This expression remains valid in p-adic and adelic case too, because of its form invariance under change of real to the p-adic number fields [23].

p-Adic and adelic wave functions of the Universe may be found by solving eigenvalue problem of the evolution operator $U(t)$. The corresponding adelic eigenstates have the form

$$\Psi(q^\alpha) = \psi_\infty(q^\alpha_\infty) \prod_{p\in S} \psi_p(q^\alpha_p) \prod_{p\notin S} \Omega(|q^\alpha_p|_p), \tag{54}$$

where $\psi_\infty(q^\alpha_\infty)$ and $\psi_p(q^\alpha_p)$ are the corresponding real and p-adic counterparts of the wave functions of the Universe, S is a finite set of primes p and a function $\Omega(|q^\alpha_p|_p)$ is defined as follows

$$\Omega(|q^\alpha_p|_p) = \begin{cases} 1, & |q^\alpha_p|_p \leq 1, \\ 0, & |q^\alpha_p|_p > 1. \end{cases} \tag{55}$$

Among many cosmological models, there is a very important one, so-called de Sitter model. The de Sitter model is a model with the cosmological constant Λ and without matter fields. Models of this type are exactly soluble models and because of that, they play a role similar to a linear harmonic oscillator in ordinary quantum mechanics. The general form of the metric for these models is [24]

$$ds^2 = \sigma^2[-N^2 dt^2 + a^2(t) d\Omega^2_{D-1}], \tag{56}$$

where $d\Omega^2_{D-1}$ denotes the metric on the unit $(D-1)$-sphere, $\sigma^{D-2} = 8\pi G/\ V^{D-1}(D-1)(D-2)$, V^{D-1} is the volume of the unit $(D-1)$-sphere. In the $D = 3$ case, this model is related to the multiple sphere configuration and wormhole solutions. υ-Adic ($\upsilon = \infty$ for the real, and $\upsilon = p$ in the p-adic cases) classical action for this model is

$$\bar{S}_\upsilon(a'', N; a', 0) = \frac{1}{2\sqrt{\lambda}} \left[N\sqrt{\lambda} + \lambda \left(\frac{2a''a'}{\sinh(N\sqrt{\lambda})} - \frac{a'^2 + a''^2}{\tanh(N\sqrt{\lambda})} \right) \right], \tag{57}$$

where a denotes a scale factor and λ denotes here the appropriately rescaled cosmological constant Λ, i.e. $\lambda = \sigma^2\Lambda$. This model was investigated in all aspects (p-adic, real and adelic) in Ref. [25]. We will see later that it can be related to the quantum tachyons.

5.4 p-adic string theory

Various amplitudes in ordinary bosonic open string theory are written as integrals over the boundary of the world-sheet which is the real line R. Now replace the integrals over R by integrals over the p-adic field Q_p with appropriate measure, and the norms of the functions in the integrand by the p-adic norms. These rules were derived from a local action defined on the "world-sheet" of the p-adic string [26].

This leads to an exact action for the open string tachyon Φ in D-dimensional p-adic string theory [27]

$$S = \frac{1}{g^2} \frac{p^2}{p^2 - 1} \int d^D x \left(-\frac{1}{2} \Phi p^{-\frac{\triangle}{2}} \Phi + \frac{1}{p+1} \Phi^{p+1} \right), \tag{58}$$

where $\triangle$ denotes the D-dimensional Laplacian and g is the open string coupling constant. Term $p^{-\frac{1}{2}\triangle}$ should be understood as

$$p^{-\frac{1}{2}\triangle} = \exp\left(-\frac{1}{2} \ln(p) \triangle \right) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{1}{2} \ln(p) \right)^n (\triangle)^n. \tag{59}$$

Tachyonic potential is of the form

$$V(\Phi) = \left(\frac{m_s^4}{g_p^2} \right) \left(\frac{1}{2} \Phi^2 - \frac{1}{p+1} \Phi^{p+1} \right). \tag{60}$$

The equation of motion for this tachyon field is,

$$p^{-\frac{1}{2}\triangle}\Phi = \Phi^p. \tag{61}$$

In the limit $p \to 1$ the equation of motion becomes a local equation

$$\triangle\Phi = 2\Phi \ln \Phi. \tag{62}$$

Solution $\Phi = 1$ is a critical point of the tachyonic potential, representing the unstable tachyonic maximum. For odd p one also has another unstable point at $\Phi = 1$. There is also the stable vacuum of the tachyon, at $\Phi = 0$. For both even and odd p the potential is unbounded from below.

5.5 p-adic inflation

A recent paper on p-adic inflation [6] gives rise to the hopes that nonlocal inflation can succeed where the real string theory fails. Starting from the action (58) it has been shown that a p-adic tachyon drives a sufficiently long period of inflation while rolling away from the maximum of its potential. p-Adic string tachyon can roll slowly enough to give many e-foldings of inflation. There are two regimes which allow for successful inflation. There is a region with small prime number ($p = O(1)$) for which the p-adic field potential is flat and slow roll inflation proceeds in the usual manner. The other one is for $p >> 1$ for which the potential is extremely steep but the p-adic scalar field nevertheless rolls slowly. This remarkable behavior relies on the nonlocal nature of the theory: the effect of the higher derivative terms in the action is to slow down the field sufficiently, despite its steep potential.

Even though this result is constrained by $p >> 1$ and obtained by an approximation, it is a good motivation to consider p-adic inflation for different tachyonic potentials. In particular, it would be interesting to study p-adic inflation in quantum regime and in adelic framework to overcome the constraint $p >> 1$, with an unclear physical meaning.

6 Tachyons: from field theory to classical analogue

It is very useful to understand and to investigate lower dimensional analogs of the tachyon field theory (37). The corresponding zero dimensional analogue of a tachyon field can be obtained by the correspondence: $x^i \to t, T \to x, V(T) \to V(x)$. The action reads [28], [29]

$$S = -\int dt V(x)\sqrt{1-\dot{x}^2}. \tag{63}$$

In what follows, all variables and parameters can be treated as real or p-adic without a formal change in the obtained forms. It is not difficult to see that action (63), with some appropriate replacement and an exponential potential $V(x) \sim \exp(-\alpha x)$ leads to the equation of motion for a particle with mass m, under a constant external force, in the presence of quadratic damping:

$$m\ddot{y} + \beta\dot{y}^2 = mg. \tag{64}$$

This equation of motion can be obtained from two Lagrangians [30]:

$$L(y,\dot{y}) = \left(\frac{1}{2}m\dot{y}^2 + \frac{m^2 g}{2\beta}\right) e^{2\frac{\beta}{m}y}, \tag{65}$$

$$L(y,\dot{y}) = -e^{-\frac{\beta}{m}y}\sqrt{1-\frac{\beta}{mg}\dot{y}^2}. \tag{66}$$

Despite the fact that different Lagrangians can give rise to nonequivalent quantization, we will choose the form (65) that can be handled easily. The first one is better because of the presence of the square root in the second one. The general solution of the equation of motion is

$$y(t) = C_2 + \frac{m}{\beta}\ln[\cosh(\sqrt{\frac{g\beta}{m}}\,t + C_2)]. \tag{67}$$

For the initial and final conditions $y' = y(0)$ and $y'' = y(T)$, for the v-adic classical action we obtain

$$\bar{S}_v(y'', T; y', 0) = \frac{\sqrt{mg\beta}}{2\sinh(\sqrt{\frac{g\beta}{m}}T)} \left[(e^{\frac{2\beta}{m}y'} + e^{\frac{2\beta}{m}y''}) \cosh(\sqrt{\frac{g\beta}{m}}T) - 2e^{\frac{\beta}{m}(y'+y'')} \right]. \tag{68}$$

In the p-adic case, we get a constraint [7] which arises from the investigation of the domain of a convergence analytical function which appears during the derivation of the formulae (67). This constraint is $|\dot{y}|_p \leq \frac{1}{p}|\sqrt{\frac{gm}{\beta}}|_p$.

By the transformation $X = \frac{m}{\beta}e^{\frac{\beta}{m}y}$, we can convert Lagrangian (65) in a more suitable, quadratic form

$$L(X, \dot{X}) = \frac{m\dot{X}^2}{2} + \frac{g\beta X^2}{2}. \tag{69}$$

For the conditions $X' = X(0)$, and $X'' = X(T)$, action for the classical v-adic solution $\bar{X}(t)$ is

$$\bar{S}_v(X'', T; X', 0) = \frac{\sqrt{mg\beta}}{2\sinh(\sqrt{\frac{g\beta}{m}}T)} \left[(X'^2 + X''^2) \cosh(\sqrt{\frac{g\beta}{m}}T) - 2X'X'' \right]. \tag{70}$$

Because this action is quadratic one (with respect to the initial and final point), the corresponding kernel of the operator of evolution in quantum mechanics can be computed in p-adic and adelic case [7], [31].

We note that the action (70) is different from the action (57) only in one constant term. There is considerable similarity of the action for the de Sitter model in $(2+1)$ dimensions with the action for the tachyon field in the zero dimensional model, i.e. "quadratically damped particle under gravity".

7 Conclusion

Tachyon phenomenon became a part of physical theories since the theory of special relativity. At the beginning, tachyons were considered as particles with velocity greater than velocity of light in vacuum. From (quantum) field theory point of view, tachyons have been recognized as unstable configurations which signal instability in the theory. The effective action for tachyons obtained in string theory has been widely used in many applications in cosmology (dark matter, dark energy, inflation). These applications are especially interesting in frame of p-adic approach. Consideration of tachyon field theory through its classical and quantum one-dimensional analogies could shad light on this problem and use as a good starting point for more realistic model.

Acknowledgements The authors would like to thank the organizers for the kind invitation to give talk at the "6th Spring School and Workshop on QFT and Hamiltonian Systems" (6-11 May 2008, Calimanesti, Romania). D. D. Dimitrijević is thankful to Prof. S.-Odile Saliu and Prof. R. Constantinescu for their kind hospitality during the Workshop. This work is partially supported by the Ministry of Science of the Republic of Serbia under Grants 144014 and 141016. Work on this paper is also supported in part by the UNESCO-BRESCE/IBSP grant No. 875.922.8 within the framework of the Southeastern European Network in Mathematical and Theoretical Physics (SEENET-MTP).

References

[1] A. Sommerfeld, K. Akad. Wet. Amsterdam Proc. **8**, 346 (1904).

[2] G. Feinberg, Physical Review **159**, 1089 (1967).

[3] A. Sen, JHEP **9809**, 023 (1998); A. Sen, JHEP **0204**, 048 (2002).

[4] M. R. Garousi, Nucl.Phys. **B 584**, 284 (2000).

[5] T. Padmanabhan, T. R. Choudhury, Phys. Rev. **D 66**, 081301 (2002).

[6] N. Barnaby, T. Biswas and J. M. Cline, JHEP **0704**, 056 (2007).

[7] D. D. Dimitrijevic, G. S. Djordjevic, and Lj. Nesic, Fortschr. Phys. **56**, No. 4–5, 412-417 (2008).

[8] P. W. Higgs, Phys. Rev. Lett. **13**, 508 (1964); F. Englert and R. Brout, Phys. Rev. Lett. **13**, 321 (1964).

[9] B. Zweibach, *A First Course in String Theory*, Cambridge University Press, Cambridge, UK (2004).

[10] P. Goddard, J. Goldstone, C. Rebbi and C. B. Thorn, Nucl. Phys. **B 56**, 109 (1973).

[11] A. Sen, JHEP **9912**, 027 (1999).

[12] F. Gliozzi, J. Scherk and D. Olive, *Phys. Lett.* **B65**, 282 (1976); *Nucl. Phys.* **B122**, 253 (1977).

[13] A. R. Liddle, and D. H. Lyth, *Cosmological Inflation and Large-Scale Structure*, Cambridge University Press, Cambridge, 2000.

[14] S. J. Perlmutter et al., Astrophys. J. **517**, 565 (1999); A. Riess et al., Astron. J. **116**, 1009 (1998).

[15] A. Riess et al., Astrophys. J. **607**, 665 (2004); D. N. Spergel et al., Astrophys. J. Suppl. **148**, 175 (2003).

[16] T. Padmanabhan, Phys. Rev. **D 66**, 021301 (2002).

[17] G. W. Gibbons, Phys. Lett. **B 537**, 1 (2002).

[18] D. Choudhury and D. Ghoshal, D. P. Jatkar, S. Panda Phys. Lett. **B 544**, 231–238 (2002).

[19] L. Kofman and A. Linde, JHEP **0207**, 004 (2002).

[20] V. S. Vladimirov, I. V. Volovich and E. I. Zelenov, *p-Adic Analysis and Mathematical Physics*, World Scientific, Singapore, 1994.

[21] A. Khrennikov, *Non-Archimedean Analysis: Quantum Paradoxes, Dynamical Systems and Biological Models*, Kluwer Acad. Publ., Dordrecht, 1997.

[22] D. Dimitrijevic, G. S. Djordjevic and B. Dragovich, Bul. Jour. Phys. **3** Vol. 27, 50–53 (2000).

[23] I. V. Volovich, CERN-TH 4781 (1987).

[24] J. J. Halliwell and R. C. Myers, Phys. Rev. **D 40**, 4011 (1989).

[25] G. S. Djordjevic, B. Dragovich, Lj. D. Nesic and I. V. Volovich, Int. J. Mod. Phys. **A17**, 1413 (2002).

[26] P. G. O. Freund, M. Olson, Phys. Lett. **B 199**, 186 (1987); P. G. O. Freund, E. Witten, Phys. Lett. **B 199**, 191 (1987);

[27] L. Brekke, P. G. O. Freund, M. Olson, E. Witten, Nucl. Phys. **B 302**, 365 (1988).

[28] S. Kar, [hep-th/0210108].

[29] D. D. Dimitrijevic and G. S. Djordjevic, in The Sixth International Conference of the Balkan Physical Union, edited by S. A. Cetin and I. Hikmet, (American Institute of Physics, 2007), pp. 359-360.

[30] D. Jain, A. Das and S. Kar, [quant-ph/0611239].

[31] G. Djordjevic, B. Dragovich and Lj. Nesic, Infnite Dimensional Analysis, Quantum Probability and Related Topics **6**, 179 (2003).