Direct methods of construction of conservation laws

Roman Popovych*

Institute of Mathematics of NAS of Ukraine, 3 Tereshchenkivs'ka Str., Kyiv-4, 01601 Ukraine

Fakultät für Mathematik, Universität Wien, Nordbergstraße 15, A-1090 Wien, Austria

Abstract

The basic notions and statements of conservation laws of differential equations are reviewed. The main attention is paid to constructive methods of finding conservation laws for general systems of differential equations, which are not Euler–Lagrange equations of a functional and, therefore, do not admit application of the symmetry approach based on the Noether theorem. Recently introduced notions of equivalence of conservation laws with respect to Lie symmetry groups for fixed systems of differential equations and with respect to equivalence groups or sets of admissible transformations for classes of such systems are considered. To construct conservation laws, we develop and apply a modification of the most direct method, which is effective to construct both local and potential conservation laws, especially, in the case of two independent variables. Classification of potential conservation laws of diffusion–convection equations with respect to the associated equivalence group and exhaustive list of locally inequivalent potential systems corresponding to these equations are adduced as an example on calculation of complete hierarchy of potential conservation laws. More details are presented on simpler classification of local conservation laws of variable coefficient diffusion–reaction equations.

---

*E-mail: rop@imath.kiev.ua