Renormalization group equations and geometric flows

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Abstract

The quantum field theory of two-dimensional sigma models with bulk and boundary couplings provides a natural framework to realize and unite different species of geometric flows that are of current interest in mathematics. In particular, the bulk renormalization group equation gives rise to the Ricci flow of target space metrics, to lowest order in perturbation theory, whereas the boundary renormalization group equation gives rise to the mean curvature flow for embedded branes. Together they form a coupled system of parabolic non-linear second order differential equations that can be further generalized to include non-trivial fluxes. Some closely related higher order curvature flows, such as the Calabi and Willmore flows associated to quadratic curvature functionals, are also briefly discussed as they arise in physics and mathematics. However, there is no known interpretation for them, as yet, in the context of quantum field theory.

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